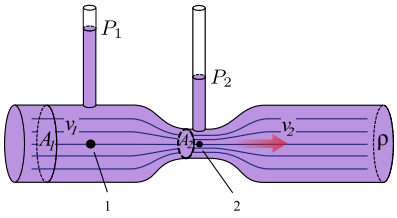
Exercise sheet #2

Problem 1. In one of his note books Leonardo da Vinci tries to propose a mechanism for how water circulates in the earth by making an analogy between the earth and the human body, in particular the circulatory system. Specifically, he wrote "... so does the water which is moved from the deep sea up to the summits of the mountains, and through the burst veins [by this he means the mountain springs] it falls down again to the shallows of the sea, and so rises again to the height where it burst through, and then returns in the same descent (Arundel codex)." In other words, mountains springs originate from a network of fine capillaries inside the mountains which are capable of lifting water. Given what know about surface tension and gravity, what, if anything, is wrong with Leonardo's mechanism? (*Hint:* Consider the balance between surface tension and gravity in a thin cylindrical channel)

Problem 2. A Venturi tube is a simple device often used to measure flow velocities. A schematic of a Venturi tube is shown below. Consider the flow of an ideal, steady, incompressible, fluid with density ρ through the Venturi tube. The tube is oriented horizontally, so gravity can be neglected. Use Bernoulli's equation to derive an expression for the upstream velocity v_1 in terms of the measured pressure difference $P_1 - P_2$, the fluid density ρ, and the cross-sectional areas A_1 and A_2.



Problem 3. The two-dimensional steady flow of a fluid with density ρ is given by

$$\boldsymbol{v}(x,y) = K\frac{-y\hat{\boldsymbol{x}} + x\hat{\boldsymbol{y}}}{x^2+y^2},$$

where K is a constant.

a) Can this flow correspond to the flow of an incompressible fluid?

b) Determine and sketch the streamlines of this flow and the acceleration field.

c) Determine the pressure difference between two points, a distance r_1 and $r_2 > r_1$ away from the origin. (Hint: Start by taking two points on the positive x-axis and then generalize using a symmetry argument.) What happens at the origin?