Exercise sheet #1

Problem 1. Both the atmosphere and the oceans are discrete collections of molecules that we can reimagine as a continuum described by a density field $\rho(\mathbf{r}, t)$. What is the spacing between the molecules of air and water for the density fields of the atmosphere and ocean respectively? According to this spacing, can you justify approximating the ocean and atmosphere as continua?

Use the following assumptions:

- mass density of air: $\rho_{\mathrm{m,air}} \approx 1\ \mathrm{kg\ m}^{-3}$
  molar mass of air $M_{\text{air}} \approx 30\ \mathrm{g\ mol}^{-1}$
- mass density of water: $\rho_{\text{m, water}} = 1\ \mathrm{g\ cm}^{3}$
  molar mass of water $M_{\text{water}} \approx 20\ \mathrm{g\ mol}^{-1}$.
- Avogadro's number $N_A \approx 6 \times 10^{23}\ \mathrm{mol}^{-1}$.

Problem 2. Find the bulk modulus of an ideal gas that is compressed by a hydraulic strain. Recall that for an ideal gas: $PV = N k_{\mathrm{B}} T$

Problem 3. In this problem, we'll analyze the Poisson effect for the extension of a homogeneous and isotropic elastic body which originally has the shape of a cube with sides of length L. We choose coordinates such that the origin is at the center of the cube, and the axes are aligned with its sides. We apply a tensile stress along the x axis, causing the cube to deform into a rectangular block, with the dimension along the x axis now $L + 2\Delta L$, and along the y and z axes now $L - 2\Delta L'$.

a) Consider a point inside the cube at position $\boldsymbol{r} = (x, y, z)$. After the deformation, the point has moved to $\boldsymbol{r}' = \boldsymbol{r} + \mathrm{d}\boldsymbol{r}$. As the origin stays fixed and the sides of the cube move, the deformation is not constant throughout the block. Argue why the local strain is given by

$$\mathrm{d}\gamma_x = \frac{\mathrm{d}x}{x}, \quad \mathrm{d}\gamma_y = -\frac{\mathrm{d}y}{y}, \quad \mathrm{d}\gamma_z = -\frac{\mathrm{d}z}{z}.$$

where we've taken the deformations $\mathrm{d}x$, $\mathrm{d}y$ and $\mathrm{d}z$ to be all positive.

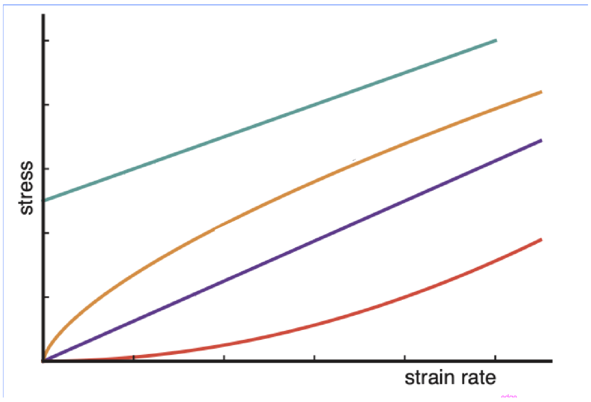
b) By definition of the Poisson ratio ν (and symmetry in y and z), we have $-\nu \mathrm{d}\gamma_x = \mathrm{d}\gamma_y = \mathrm{d}\gamma_z$. As our material is homogeneous, we can integrate the resulting differential equation, to get

$$-v \int_L^{L+\Delta L} \frac{\mathrm{d}x}{x} = -\int_{L-\Delta L'}^{L} \frac{\mathrm{d}y}{y} = \int_L^{L-\Delta L'} \frac{\mathrm{d}y}{y}$$

(We're leaving out the integral over z as it is identical to the one over y). Carry out the integration, to find an equation containing v and the ratios $\Delta L/L$ and $\Delta L'/L$.

c) Using the expansion $(1+x)^n = 1 + nx + O\left(x^2\right)$ (valid for small x, and even for n not an integer and/or negative), simplify the relation you found in (b) to get the approximate expression for the Poisson ratio we use mostly in practice.

Problem 4. The following plot shows the stress-strain rate relation for different fluids. Which of the curves describes: a newtonian fluid, a shear thinning fluid, a shear thickening fluid and a Bingham plastic (a type of non-Newtonian fluid that behaves like a solid until a certain minimum stress is applied, and then flows like a viscous fluid).


stress
strain rate