Exercise sheet #5

Problem 1. Dimensional analysis and the drag on a car

a) Estimate the order of magnitude of the Reynolds number for a car driving at 100 km/h.

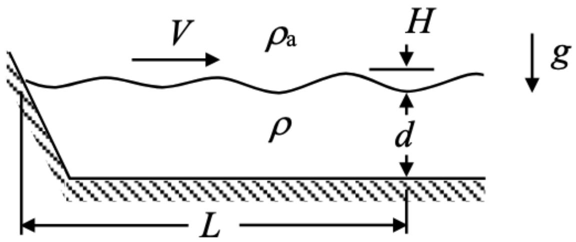
b) Give the dimensions of the drag force F_{drag} , the speed and the air density.

c) Use dimensional analysis to argue that $F_{\text{drag}} = \rho U^2 A$, where $[A] = l^2$.

d) Argue that it makes sense to write $A = C_{\mathrm{d}} A_{\text{cross}}$, where A_{cross} is the head-on cross-sectional area of the car, and where C_d is a dimensionless drag coefficient.

e) Look for values of C_{d} of various cars on the internet, and convince yourself that air resistance a Porsche experiences at 200 km/h is comparable to that of a Hummer at 100 km/h, illustrating that cross-section and design matter.

Problem 2. Use Buckingham pi to determine a functional relationship for the wavelength, H of waves created when wind blows across a lake (See figure below). The wavelength, H, is assumed to be a function of the wind speed, V, the water density, ρ, the air density, ρ_{a}, the water depth, d, the distance from the shore, L, and the acceleration of gravity, g.



Problem 3. In this problem you will derive in a few simple steps the logarithmic law for the stream-wise velocity near the wall in the case of high-Reynolds turbulent flow past a plate or in a pipe.

Very close to the wall, there is a small layer where the flow is laminar. For large enough Reynolds numbers, one will have well-developed turbulence on outer scales. We derive here the Von Kármán logarithmic wall law from dimensional arguments in the intermediate range between the viscous layer and the outer scale. We consider a flat wall at $y = 0$ with mean flow in the x direction, so y is measuring the distance from the wall.

a) We denote the wall stress by σ_{w}, and the fluid density by ρ_0 (be aware that in most of the literature the wall stress is indicated as τ). As always, ν is the kinematic viscosity. Write down the dimensions $[\sigma_{\mathrm{w}}]$, $[\rho_0]$, $[y]$ and $[\nu]$.

b) Introduce the velocity scale $u_\sigma = \sigma_{\mathrm{w}}/\rho_0$. Show that this has indeed the dimension of velocity, and show that ν/u_σ has the dimension of length.

c) We postulate that the mean flow $U(y)$ can only depend on the quantities considered in step a) in the intermediate range between the viscous layer and the outer scale. Write down the only possible law for $\mathrm{d}U(y)/\mathrm{d}y$ that is allowed on dimensional grounds.

d) Integrate the law derived in c) to derive the logarithmic law

$$U^+(y) = \frac{1}{\kappa} \ln\left(y^+\right) + A$$

where $U^+ = U/u_\sigma$ and $y/\left(\nu/u_\sigma\right)$ are the dimensionless mean velocity and distance from the wall, and where κ is a dimensionless constant, referred to as the Von Kármán constant.

This logarithmic law has been extensively confirmed in experiments and simulations. For smooth walls, it turns out that $A \approx 5.0$ and $\kappa \approx 0.4$ (See *Smits et. 2010 al posted on moodle*) for an overview.