Exercise sheet #12

Problem 1. What does "short range order" mean in the context of molecules in a fluid? How does that compare to the structure of a liquid?

Problem 2. Liquid crystals are phases of matter that are intermediate between solids and liquids. In the nematic phase, molecules have no positional order (they can flow like a liquid), but they exhibit long-range orientational order: their long axes tend to align along a common direction called the director. At first glance, this seems surprising. Pure liquids are disordered, why would molecules in a "liquid" spontaneously align? Explain qualitatively why the nematic phase can be thermodynamically stable, even though the molecules are still mobile like in a liquid. Why does raising the temperature eventually destroy the nematic order, leading to an isotropic liquid?

Problem 3. In a nematic liquid crystal, the rod-like molecules tend to point roughly in the same direction, called the director. The amount of alignment is measured by the order parameter S, which is close to 1 when molecules are well aligned and close to 0 when they are almost random.

Light polarized parallel to the director experiences a refractive index $n_{\parallel}$, while light polarized perpendicular to it experiences $n_{\perp}$. The birefringence[1] is

$$\Delta n = n_{\parallel} - n_{\perp}.$$

Explain qualitatively why a higher order parameter (stronger molecular alignment) leads to a larger birefringence.

Problem 4. In a nematic liquid crystal, the rod-like molecules tend to align along a common direction called the director. However, in many situations-such as when the material is confined, rubbed surfaces impose different orientations, or temperature changes occur-this alignment cannot be maintained everywhere. As a result, topological defects (also called disclinations) appear, where the director field becomes undefined.

Consider a thin film of nematic liquid crystal confined between two parallel plates. The plates impose different anchoring directions: the bottom plate forces the director to lie horizontally, while the top plate forces the director to lie vertically. Somewhere in the film, the director field must change from one direction to the other.

a) Explain why defects must form in this situation.

b) A common defect in nematics is a $\pm\frac{1}{2}$ disclination, where the director rotates by $\pm 180°$ around a loop enclosing the defect core. Explain qualitatively why such half-integer defects are allowed in nematic liquid crystals, and why they occur more frequently than integer defects.

c) Describe how the elastic energy of the liquid crystal tends to influence the size and structure of the region near a defect, and why the core of the defect often behaves more like an isotropic fluid.

[1] Birefringence, also known as double refraction, is the optical property of certain materials where light is refracted in two different directions, resulting in two distinct rays.