

Classical Electrodynamics

Week 11

1. Lorentz transformation of electromagnetic fields

Show that a Lorentz boost leads to the following transformation:

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} \quad (1)$$

$$\mathbf{E}'_{\perp} = \gamma (\mathbf{E}_{\perp} + c\boldsymbol{\beta} \times \mathbf{B}) \quad (2)$$

$$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} \quad (3)$$

$$\mathbf{B}'_{\perp} = \gamma \left(\mathbf{B}_{\perp} - \frac{1}{c} \boldsymbol{\beta} \times \mathbf{E} \right) \quad (4)$$

Consider now a constant electromagnetic field $\{\mathbf{E}, \mathbf{B}\}$ in the reference frame \mathcal{R} .

- Find a reference frame \mathcal{R}' where the electromagnetic fields $\{\mathbf{E}', \mathbf{B}'\}$ are parallel to each other: $\mathbf{E}' \parallel \mathbf{B}'$.
- Is this always possible? Is the solution unique?
- Compute the magnitudes of \mathbf{E}' and \mathbf{B}' in the reference frame \mathcal{R}' .

2. Synchrotron radiation

Consider a non-relativistic electron ($v \ll c$) in circular movement due to a magnetic field \mathbf{B} orthogonal to the plane of the movement.

- Calculate the Poynting vector $\mathbf{S} = \epsilon_0 c^2 \mathbf{E}_e \times \mathbf{B}_e$, using the radiative part of the electromagnetic fields $\mathbf{E}_e, \mathbf{B}_e$ produced by the accelerated electron. You can use the formulas of Liénard-Wiechert in the non-relativistic limit ($v \ll c$).
- Calculate the time average of the Poynting vector

$$\langle \mathbf{S} \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \mathbf{S}(t), \quad (5)$$

and determine the total radiated power.

- Study the angular distribution of the radiation.

3. Classical atom

Consider the classical model of the hydrogen atom:

- The proton, of charge $e = 1.60 \times 10^{-19}$ C and mass $m_p = 1.67 \times 10^{-27}$ kg, is at rest at the center of the atom.

- The electron, of charge $-e$ and mass $m_e = 9.11 \times 10^{-31}$ kg $\ll m_p$, moves around the proton in a circular orbit of radius $r_0 = 5.29 \times 10^{-11}$ m.
- a) Calculate the frequency ν of this rotation.
- b) Calculate the total power radiated by the system. Recall the formula

$$\mathbf{S}(t) = \frac{e^2}{16\pi^2\epsilon_0 c^3} \frac{|\mathbf{a}|^2 \sin^2 \alpha(t)}{r^2} \mathbf{e}_r \quad (6)$$

for the Poynting vector of a non-relativistic electron. Here, $\alpha(t)$ is the angle between the acceleration vector \mathbf{a} of the electron and the observation direction \mathbf{e}_r .

- c) Estimate the life time of the classical atom. Why are you still alive?