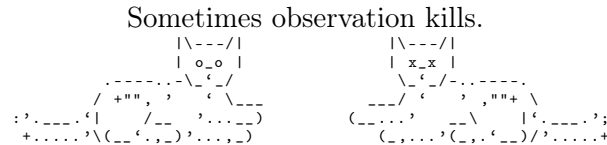


# Quantum mechanics II, Chapter 2 : Entanglement (Part 1)

*TA : Slimane Thabet, Sofia Brizigotti, Alba Miren Taddei, Reyhaneh Aghaei Saem, Mehrad Sahebi, Ricard Puig, Sacha Lerch*



## Problem 1 : Bell States

Consider two spin-1/2 particles  $A$  and  $B$  with :

$$|\psi(A)\rangle = c_0^{(A)}|0\rangle_A + c_1^{(A)}|1\rangle_A$$

and

$$|\psi(B)\rangle = c_0^{(B)}|0\rangle_B + c_1^{(B)}|1\rangle_B.$$

The states  $|0\rangle$  and  $|1\rangle$  are the eigenstates of the  $\hat{S}_z = \frac{\hbar}{2}\sigma_z$  operator with eigenvalues  $+\hbar/2$  and  $-\hbar/2$ , respectively.

1. Write down the four possible basis states for the composite system  $|\psi(A)\rangle \otimes |\psi(B)\rangle$  in terms of the basis vectors  $\{|0\rangle_A, |1\rangle_A\}$  and  $\{|0\rangle_B, |1\rangle_B\}$ .
2. The Bell states are two-particle states given by :

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B).$$

Show that these four Bell states form an orthonormal basis of the Hilbert space of the two spins  $H = H_A \otimes H_B$ .

3. Are the four Bell states separable?
4. We now consider the Bell state  $|\Psi^+\rangle$ .
  - (a) What is the probability of measuring  $-\hbar/2$  when measuring the spin  $\hat{S}_z^{(B)}$  of particle B?
  - (b) Let's assume now that the measurement of the spin B gives us  $-\hbar/2$ . What is the probability of subsequently measuring  $+\hbar/2$  for the spin A?
5. We now consider  $|\theta\rangle = \frac{1}{2}|\Phi^+\rangle + \frac{\sqrt{3}}{2}|\Psi^-\rangle$ . Is this state correctly normalized? What is the probability of obtaining  $\pm\hbar/2$  upon measuring  $\hat{S}_z^{(A)}$  on the first spin?

6. What is the probability of obtaining  $\pm\hbar/2$  upon measuring  $\hat{S}_x^{(A)}$  (the  $x$ -axis spin of the first particle) for all of the four Bell states?

Problem 2 : Composite system of two spin-1/2 particles

Consider the following Hamiltonian operator for two spin-1/2 particles :

$$\hat{H} = \mu_x \hat{S}_x^{(A)} \otimes \hat{S}_x^{(B)} + \mu_y \hat{S}_y^{(A)} \otimes \hat{S}_y^{(B)}$$

where  $\hat{S}_x^{(A)}$  and  $\hat{S}_y^{(A)}$  are the spin operators for the first spin and  $\hat{S}_x^{(B)}$  and  $\hat{S}_y^{(B)}$  are the spin operators for the second spin.

1. What are the conditions on the coefficients  $\mu_x$  and  $\mu_y$  such that  $\hat{H}$  is a valid observable?
2. Write down the matrix elements of  $\hat{H}$  in the basis of  $\hat{S}_z$  (i.e.  $\{|0\rangle, |1\rangle\}^{\otimes 2}$ ).
3. Diagonalize the Hamiltonian in this basis and find its eigenvalues and the corresponding eigenvectors.
4. Are the eigenvectors of the Hamiltonian separable between the two particles?