Today's agenda:

Magnetic Fields.

You must understand the similarities and differences between electric fields and field lines, and magnetic fields and field lines.

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Magnetism

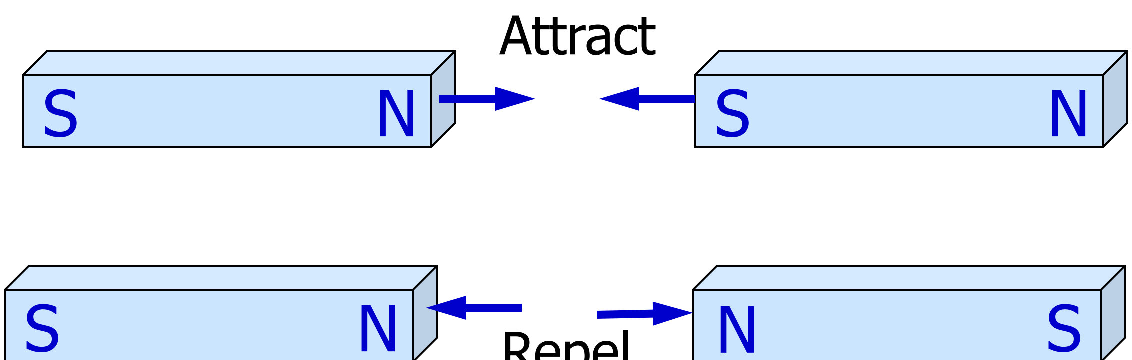
Recall: (lecture 1)

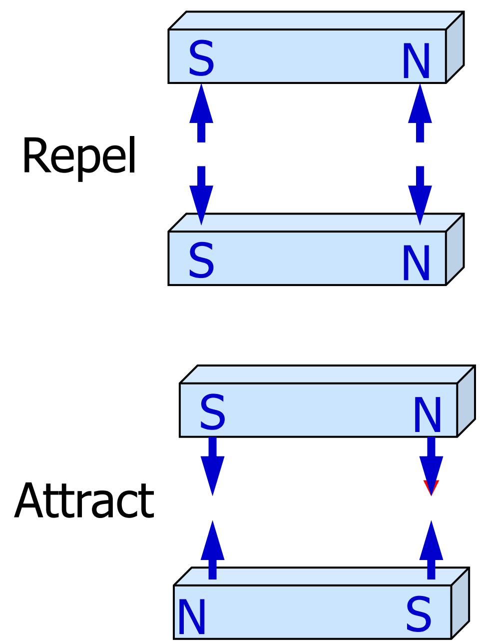
- electric (Coulomb) force between electric charges (+ and -), like charges repel, opposite charges attract

Analogously:

- two kinds of magnetic poles (north and south), like poles repel, opposites attract

Example: bar magnets:

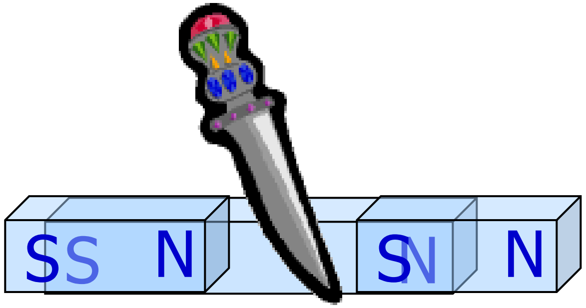




Important difference:

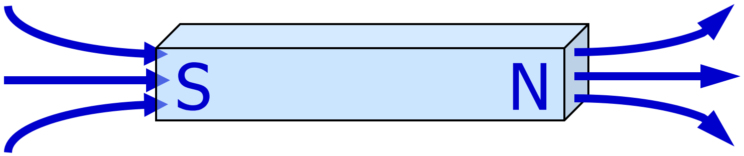
- isolated + or – electric charges exist
- isolated N and S poles (magnetic monopoles) are **not** observed (topic of active physics research*)

- every magnet has BOTH a N and a S pole
- basic magnetic structure is a dipole



Magnetic Fields

- magnetic dipoles create **magnetic fields**
- magnetic field lines point away from north and towards south



- symbol for magnetic field: $\vec{B}$

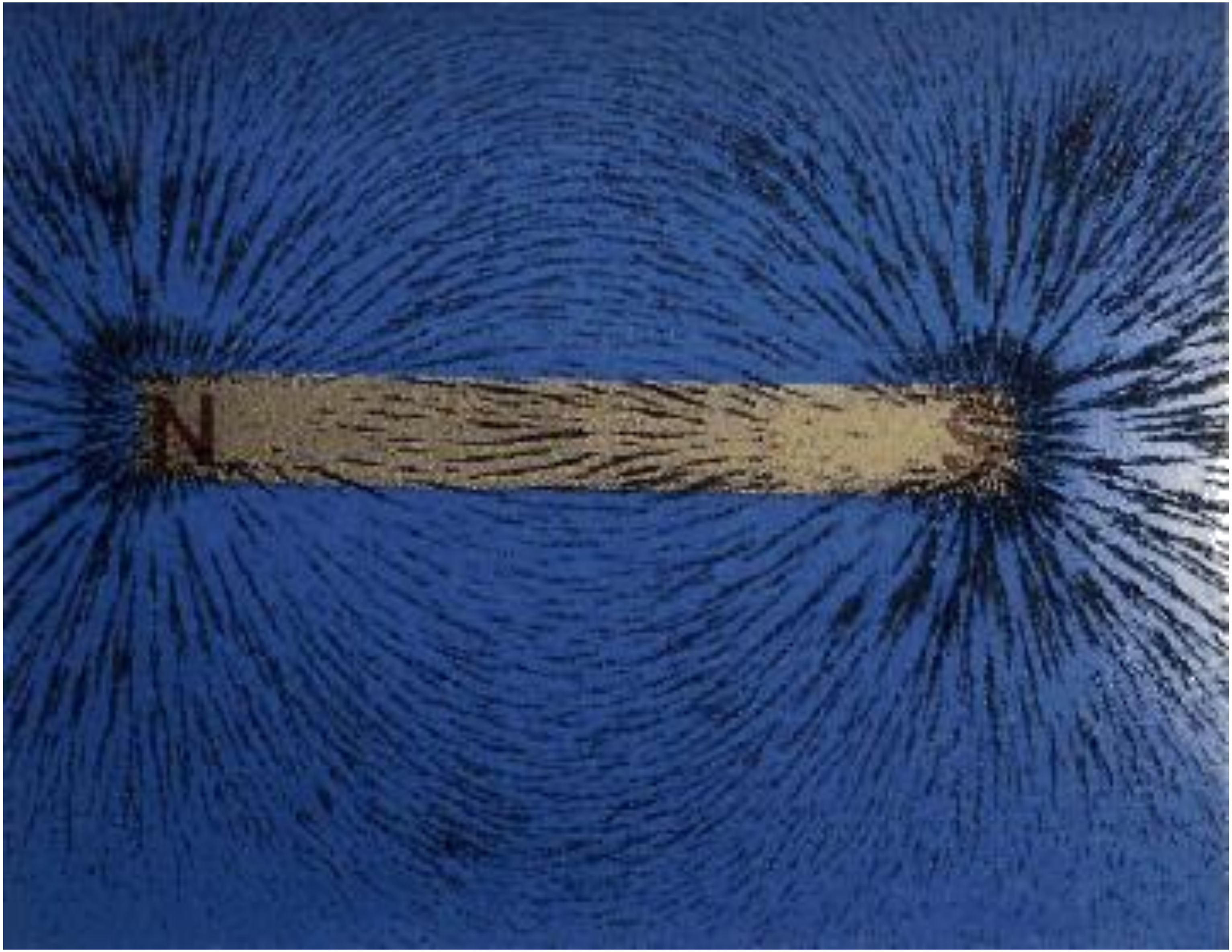
- SI unit* for magnetic field: T (Tesla)

$$1\ \text{T} = \frac{1\ \text{kg}}{\text{C}\cdot\text{s}}$$

These units come from the magnetic force equation, which appears three slides from now.

*Old unit, still sometimes used: 1 Gauss = 10^{-4} Tesla.

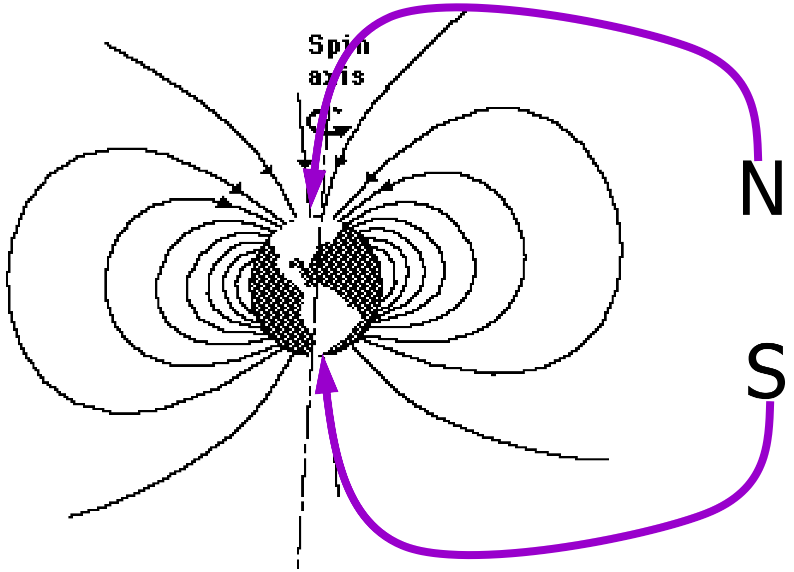
magnetic field of a bar magnet, visualized using iron filings



Earth's Magnetic Field

- earth has magnetic field, with poles near the geographic poles

Magnetic compass:

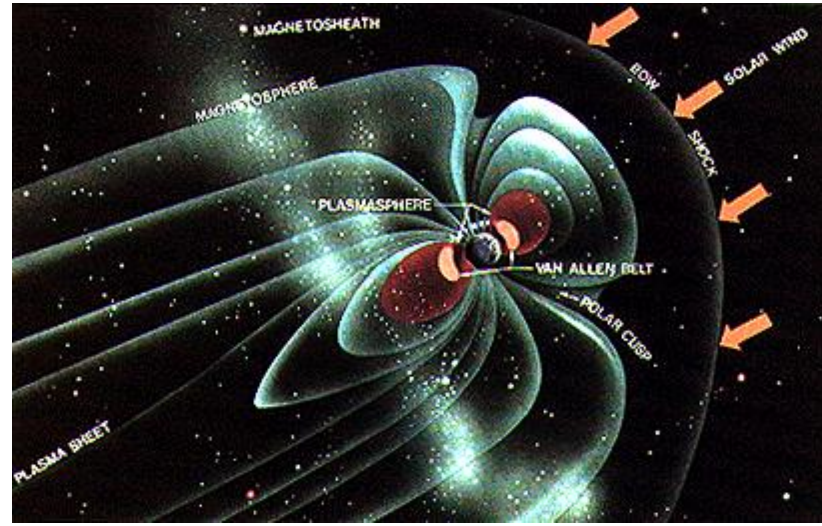


- magnetic poles of compass needle (small bar magnet) are attracted to and repelled by earth's magnetic poles

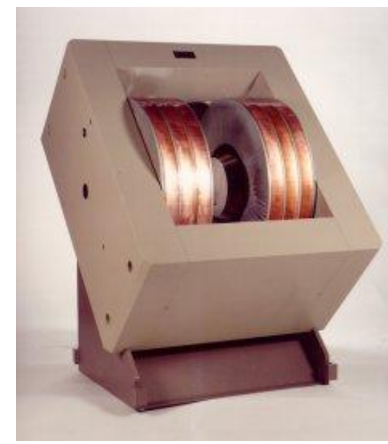
- magnetic north pole of needle is attracted to geographic N pole
- magnetic south pole of needle is attracted to geographic S pole

Magnetic north pole of earth is (close to) geographic south pole and vice versa

The earth's magnetic field has a magnitude of roughly 0.5 G, or 0.00005 T. A powerful perm-anent magnet, like the kind you might find in headphones, might produce a magnetic field of 1000 G, or 0.1 T.



The electromagnet that used to be in the basement of the Physics Bldg produced a field of 26000 G = 26 kG = 2.6 T.



Superconducting magnets can produce a field of over 10 T. Never get near an operating super-conducting magnet while wearing a watch or belt buckle with iron in it!

Magnetic Fields

Two questions:

- How can one create magnetic fields?
  (permanent magnets and ???)

- What are the effects of magnetic fields?

We start with the second question!

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Applications: galvanometers, electric motors, rail guns.

You must be able to use your understanding of magnetic forces and magnetic fields to describe how electromagnetic devices operate.

Magnetic force on a charge

- magnetic field $\vec{B}$ creates force acting on **moving** charged particle

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

force on particle

velocity of charged particle

magnetic field vector

What is the magnetic force if the charged particle is at rest?

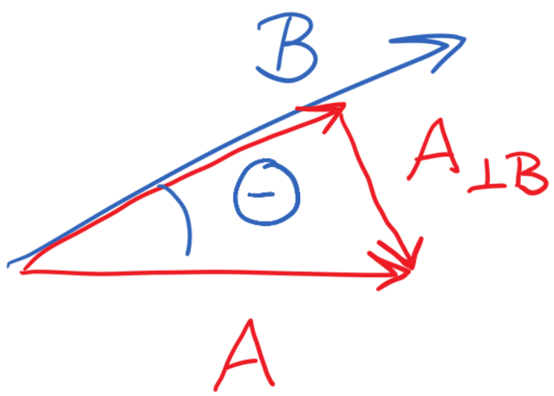
Question: motion with respect to what? You, the earth, the sun?
Highly nontrivial, leads to Einstein's theory of relativity.

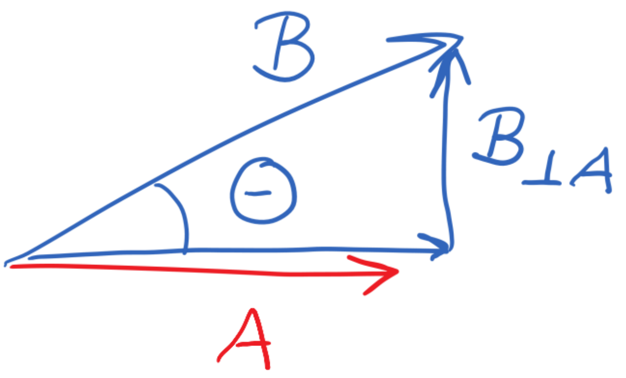
Recall:

Vector cross product: Magnitude

$$\vec{A} \times \vec{B} = \vec{C}$$

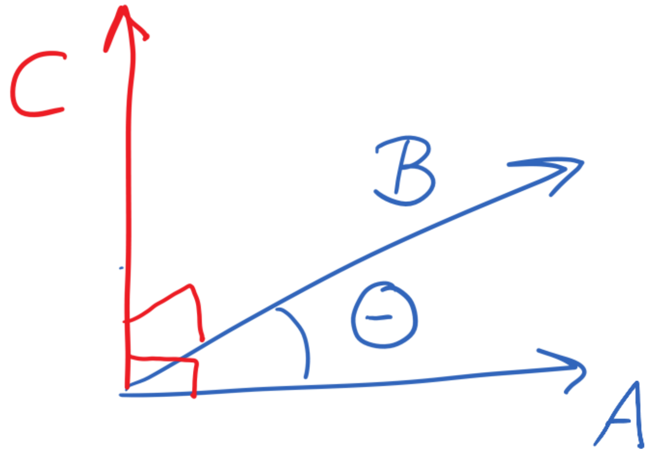
$$C = AB\, sin\theta = A_{\perp B}B = AB_{\perp A}$$





Vector cross product: Direction

$$\vec{A} \times \vec{B} = \vec{C}$$

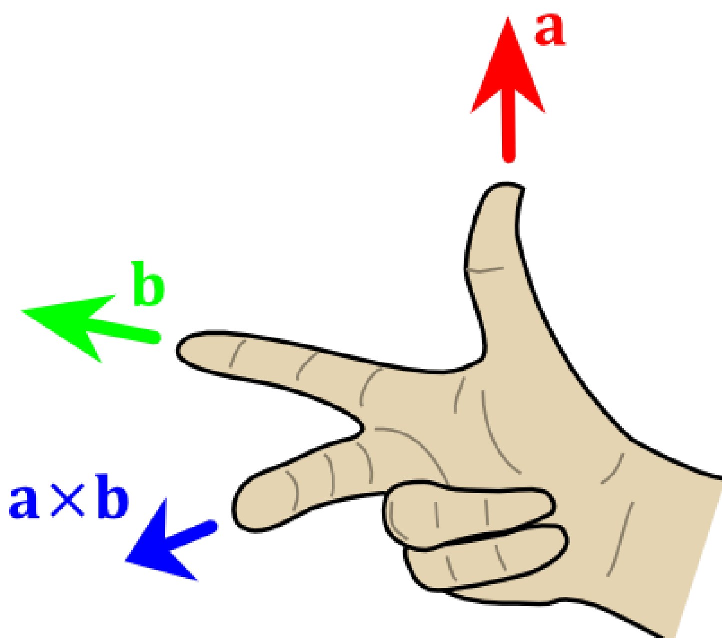


$\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$

Direction: right hand rule

$$\vec{A} \quad \times \quad \vec{B} \qquad = \quad \vec{C}$$

thumb × index finger = middle finger



Many alternative versions of the right hand rule exist!

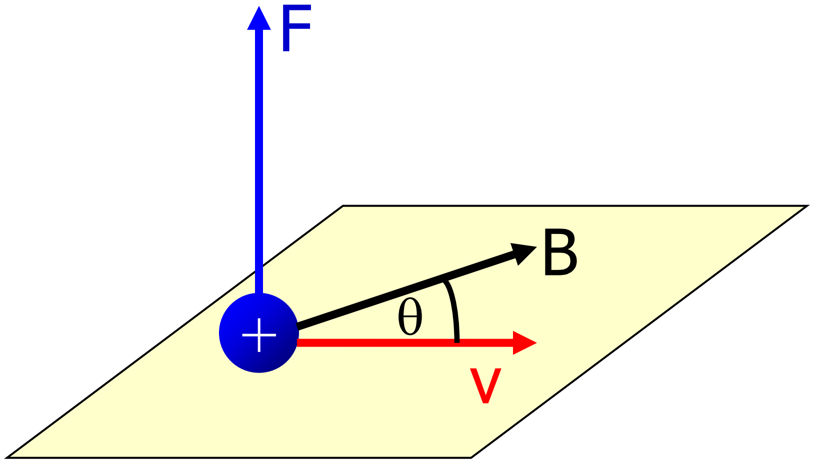
- textbook presents several versions
- other classes may use different right hand rules
- Wikipedia, Youtube

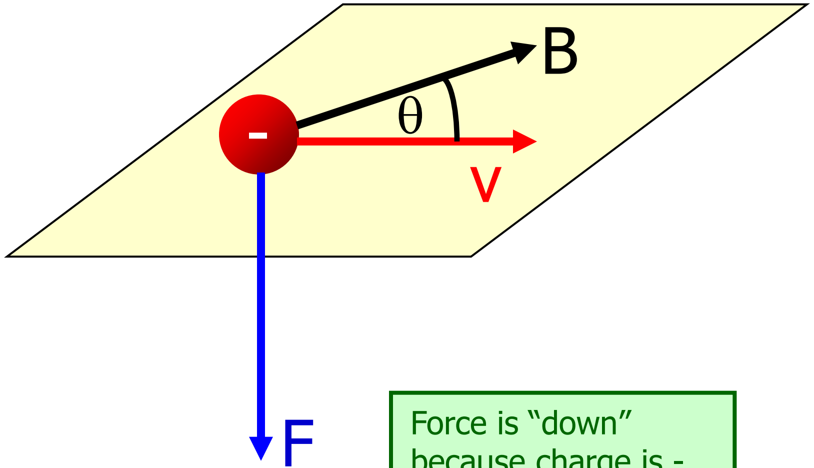
All versions are equivalent!

Recommendation:

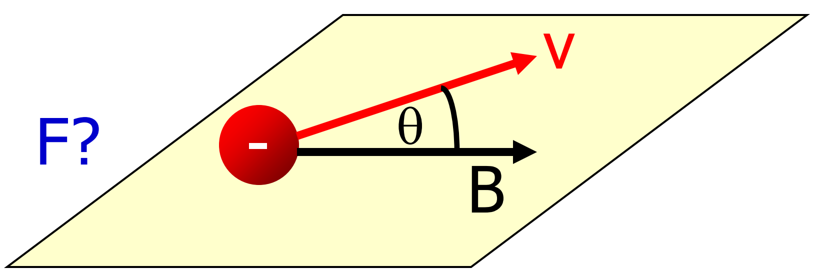
Memorize one version and stick to it!

Examples: Direction of the magnetic force





Force is “down” because charge is -.



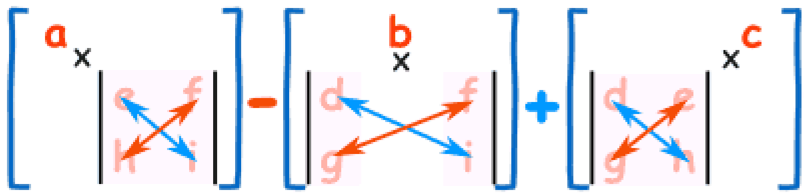
Cross product in terms of vector components

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q \ \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{pmatrix}$$

All of the right-hand rules are just techniques for determining the direction of vectors in the cross product without having to do any actual math.

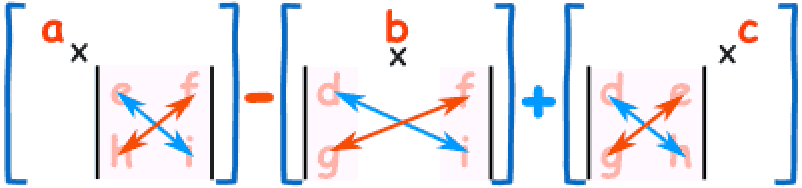
Example: a proton is moving with a velocity $\vec{v} = v_0\hat{j}$ in a region of uniform magnetic field. The resulting force is $\vec{F} = F_0\hat{i}$. What is the magnetic field (magnitude and direction)?



Blue is +, red is -. Image from http://www.mathsisfun.com/algebra/matrix-determinant.htm

Example: a proton is moving with a velocity $\vec{v} = v_0\hat{j}$ in a region of uniform magnetic field. The resulting force is $\vec{F} = F_0\hat{i}$. What is the magnetic field (magnitude and direction)?

$$\vec{v} = v_0\hat{j} \qquad \vec{F} = F_0\hat{i}$$



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = F_0\hat{i} = q\left[\det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{pmatrix}\right] = (+e)\left[\det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & v_0 & 0 \\ B_x & B_y & B_z \end{pmatrix}\right]$$

$$F_0\hat{i} = (+e)\left[\hat{i}\left(v_0 \cdot B_z - B_y \cdot 0\right) - \hat{j}\left(0 \cdot B_z - B_x \cdot 0\right) + \hat{k}\left(0 \cdot B_y - B_x \cdot v_0\right)\right]$$

Example: a proton is moving with a velocity $\vec{v} = v_0\hat{j}$ in a region of uniform magnetic field. The resulting force is $\vec{F} = F_0\hat{i}$. What is the magnetic field (magnitude and direction)?

$$F_0\hat{i} = ev_0B_z\hat{i} - 0\hat{j} - eB_xv_0\hat{k} = \left(ev_0B_z\right)\hat{i} + \left(-0\right)\hat{j} + \left(-eB_xv_0\right)\hat{k}$$

$$F_0 = ev_0B_z \quad \text{and} \quad 0 = -eB_xv_0$$

$$B_z = \frac{F_0}{ev_0} \quad \text{and} \quad B_x = 0$$

What is B_y???? It could be anything. Not enough information is provided to find B_y. We can't find the magnitude and direction of the magnetic field using only the information given!

Example: a proton is moving with a velocity $\vec{v} = v_0\hat{j}$ in a region of uniform magnetic field. The resulting force is $\vec{F} = F_0\hat{i}$. What is the minimum magnitude magnetic field that can be present?

$$B_z = \frac{F_0}{ev_0} \quad \text{and} \quad B_x = 0$$

$B_y = 0$ for a minimum magnitude magnetic field, so...

$$B_{min} = \frac{F_0}{ev_0}$$

Vector notation conventions:

⊙

⊙ is a vector pointing out of the paper/board/screen (looks like an arrow coming straight for your eye).

⊗

⊗ is a vector pointing into the paper/board/screen (looks like the feathers of an arrow going away from eye).

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Motion of a charged particle in a uniform magnetic field

Example: an electron travels at 2×10^7 m/s in a plane perpendicular to a 0.01 T uniform magnetic field. Describe its path.

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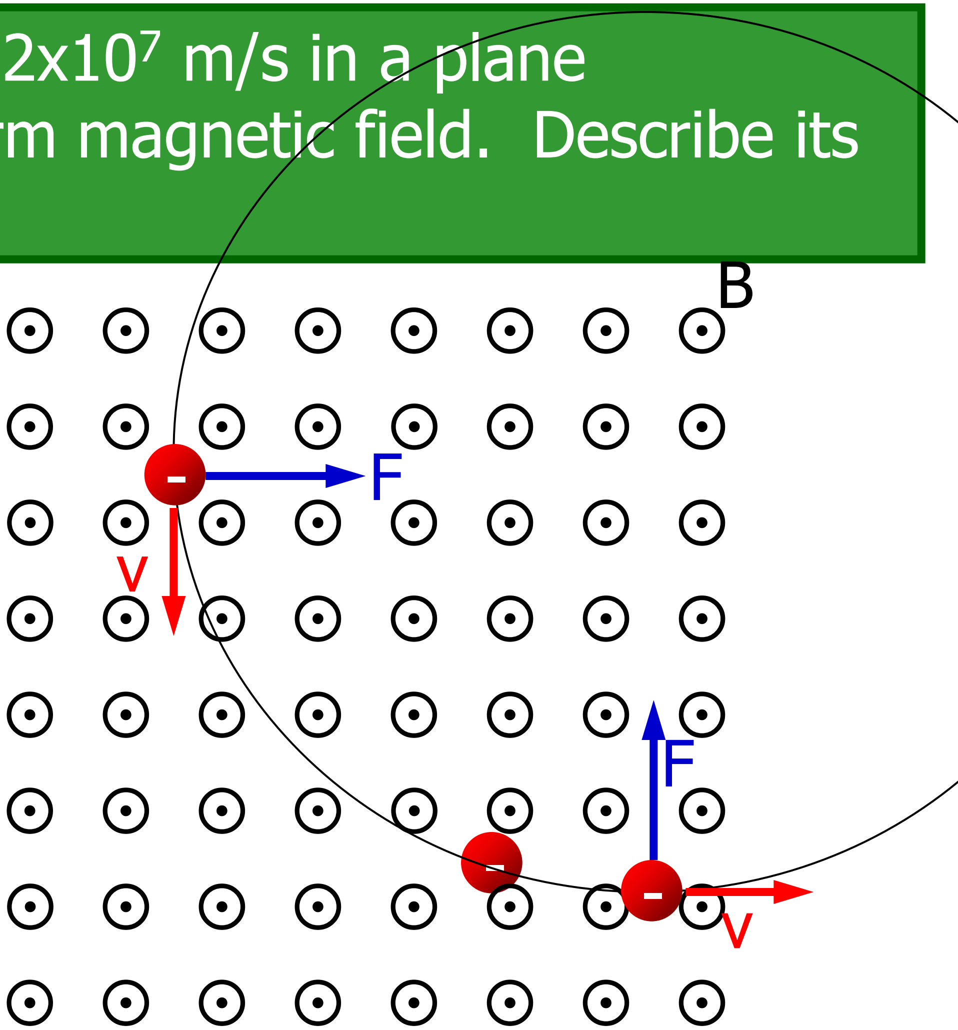
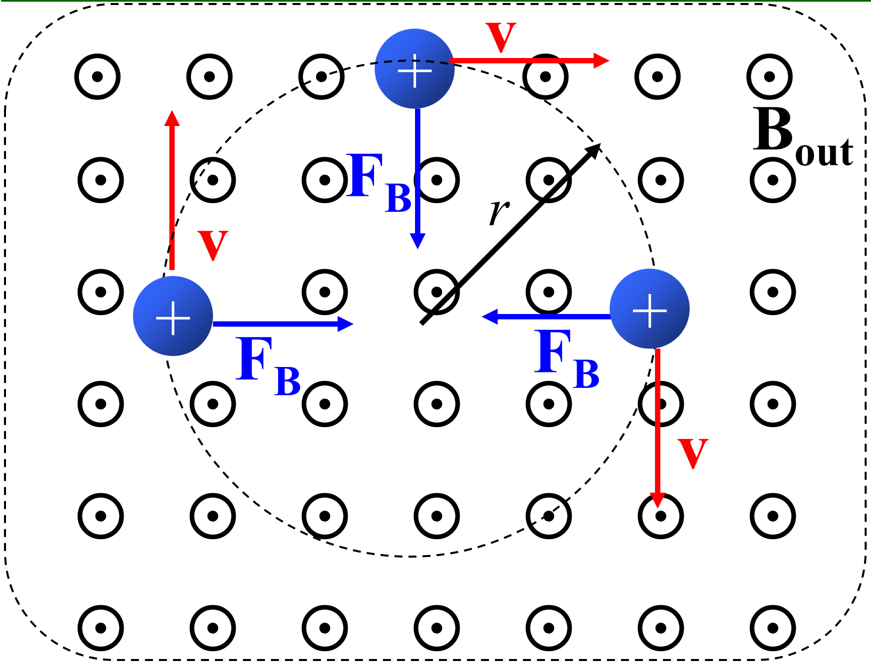


- force on electron is always perpendicular to the velocity
- force does not change speed of electron
- force only changes direction of electron

Electron will move in a **circular path with a constant speed and acceleration = v^2/r,** where r is the radius of the circle.

Example: a proton of speed v travels in a plane perpendicular to a uniform magnetic field. Find the radius of its orbit.



- force is in radial direction
- magnitude $F=|q|vB$
- for circular motion, $a = v^2/r$

$$F = |q|vB = \frac{mv^2}{r}$$

$$v = \frac{|q|rB}{m} \qquad r = \frac{mv}{|q|B}$$

period T of orbit:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}$$

The rotational frequency f is called the cyclotron frequency

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$

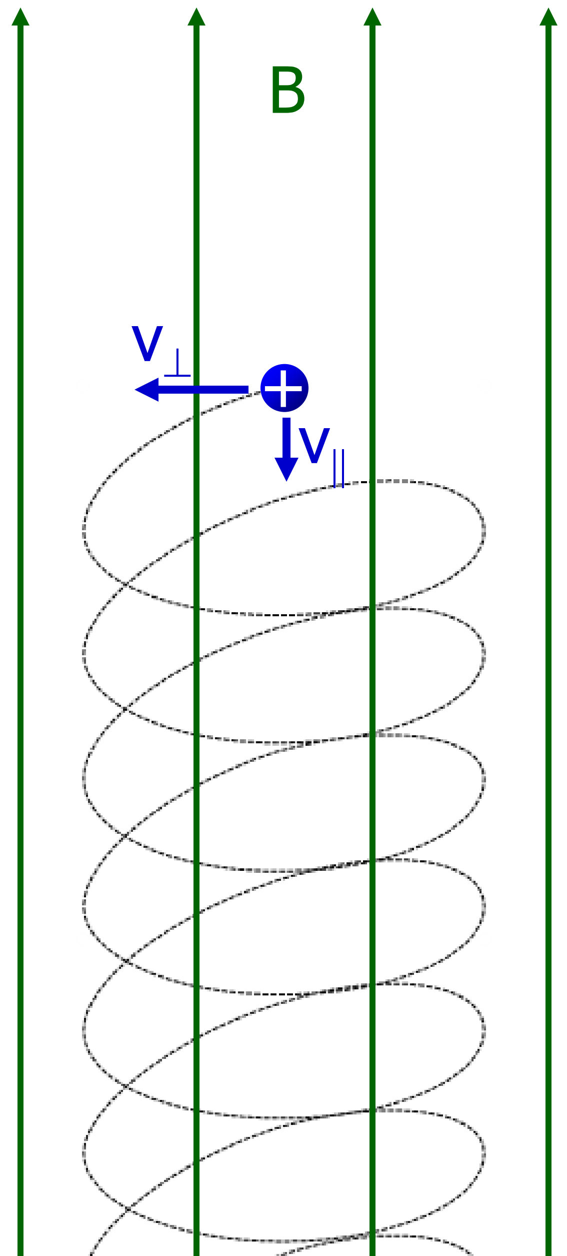
Helical motion in a uniform magnetic field

If $\vec{v}$ and $\vec{B}$ are perpendicular, a charged particle travels in a circular path. v remains constant but the direction of $\vec{v}$ constantly changes.

If $\vec{v}$ has a component parallel to $\vec{B}$, then $v_{\parallel}$ remains constant, and the charged particle moves in a helical path.

There won't be any test problems on helical motion.

*or antiparallel

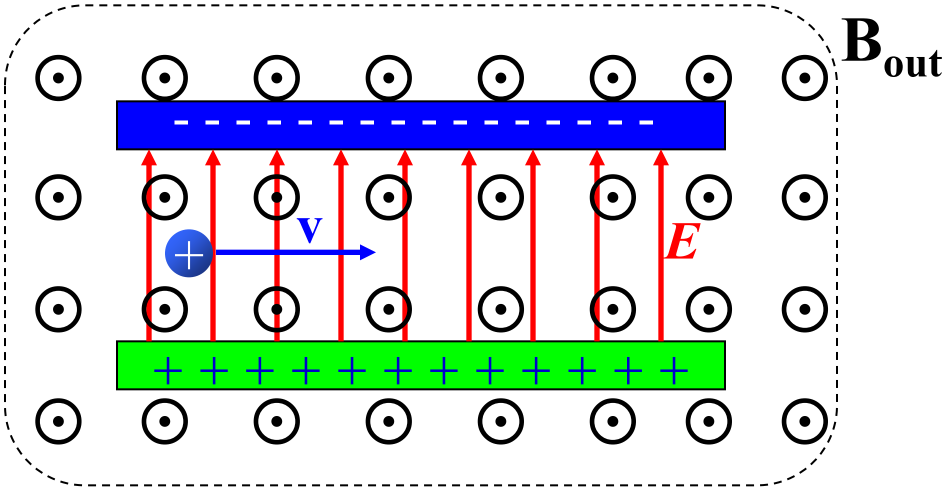


Lorentz Force Law

If both electric and magnetic fields are present,

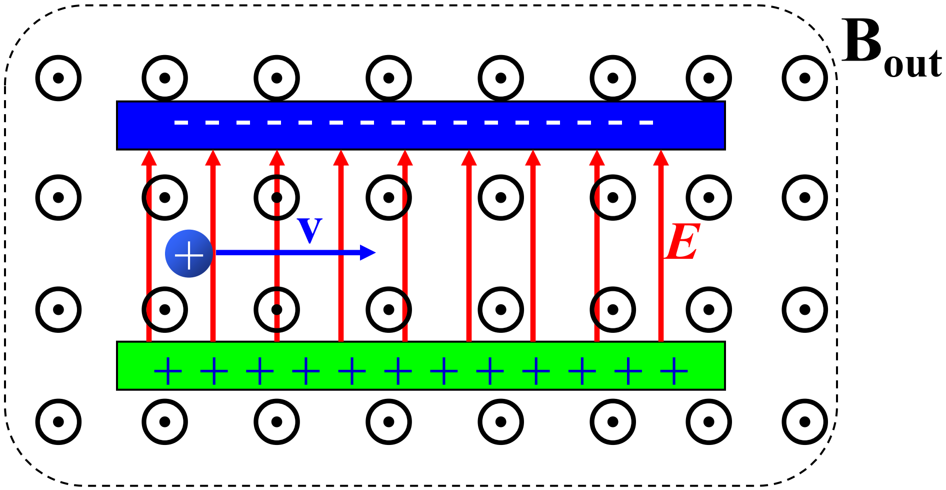
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right).$$

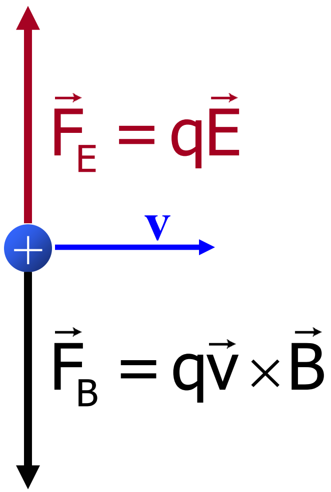
Application: Velocity Selector



The magnetic field is uniform inside the dashed-line region. What speed will allow the proton to pass undeflected through region of uniform electric field?

Velocity Selector



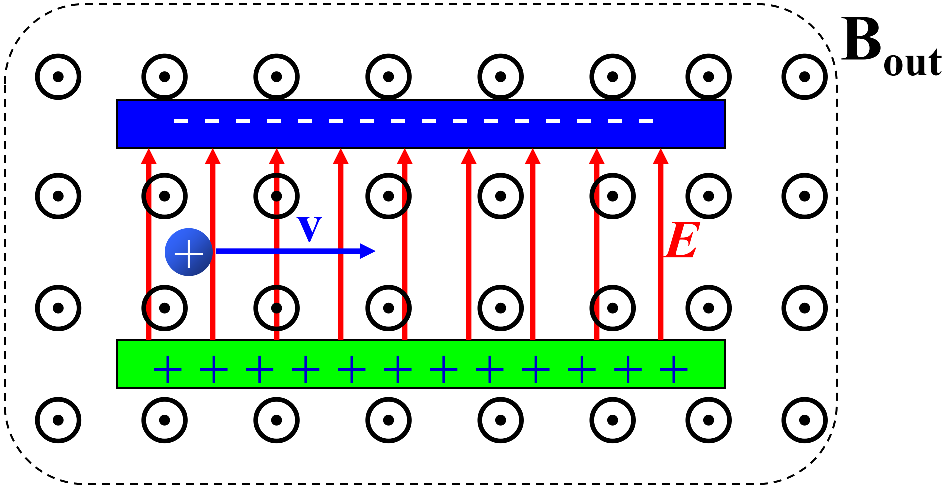


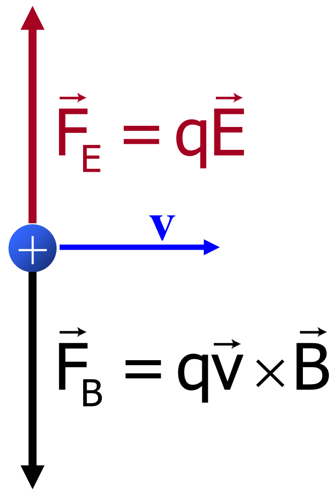
When the electric and magnetic forces balance then the charge will pass straight through.

$$\left|\vec{F}_E\right| = \left|\vec{F}_B\right|$$

$$|q|E = |q|vB \qquad \text{or} \qquad v = \frac{E}{B}$$

Velocity Selector





This only works if the electric and magnetic fields are perpendicular and oriented so that $q\vec{v}\times\vec{B}$ is antiparallel to $\vec{E}$.

Also, we simplified the calculation by making $\vec{v}$ perpendicular to $\vec{B}$.

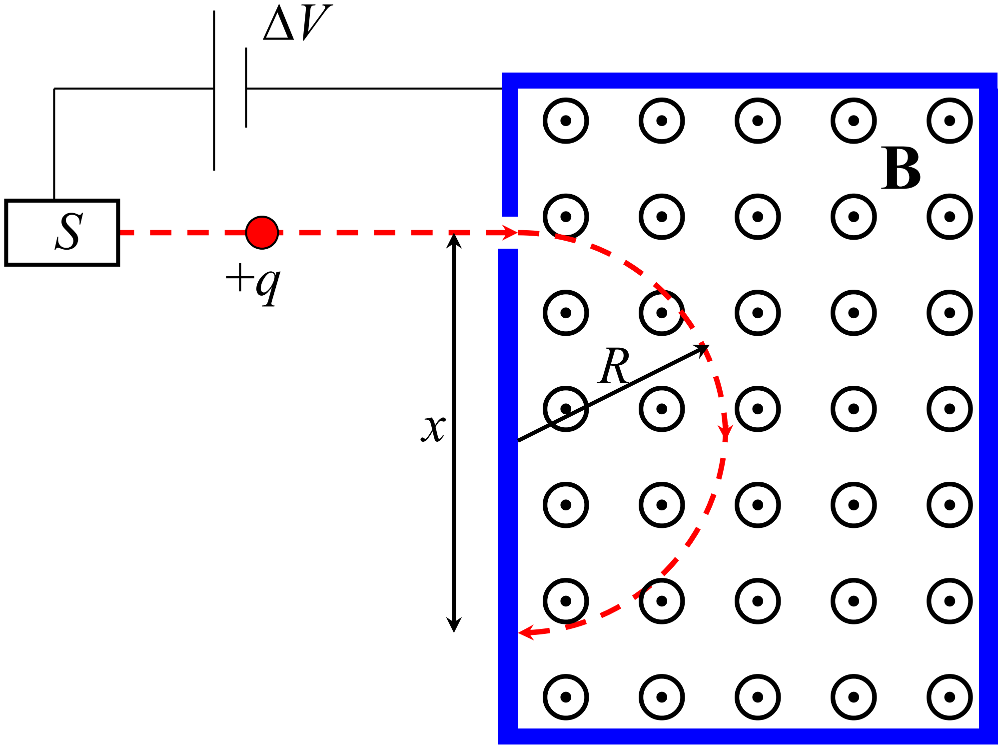
Application: Mass Spectrometer

Mass spectrometers separate charges of different mass.

When ions of fixed energy enter a region of constant magnetic field, they follow a circular path.

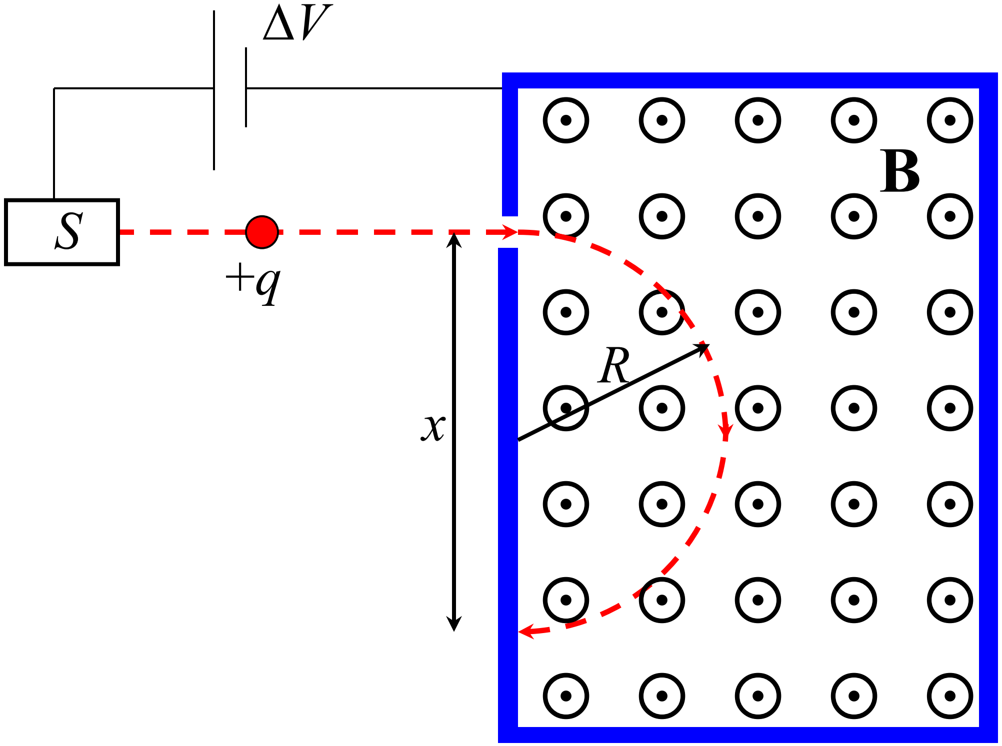
The radius of the path depends on the the mass/charge ratio and speed of the ion, and the magnitude of the magnetic field.

$$x = 2r \quad \text{and} \quad r = \frac{mv}{qB}$$

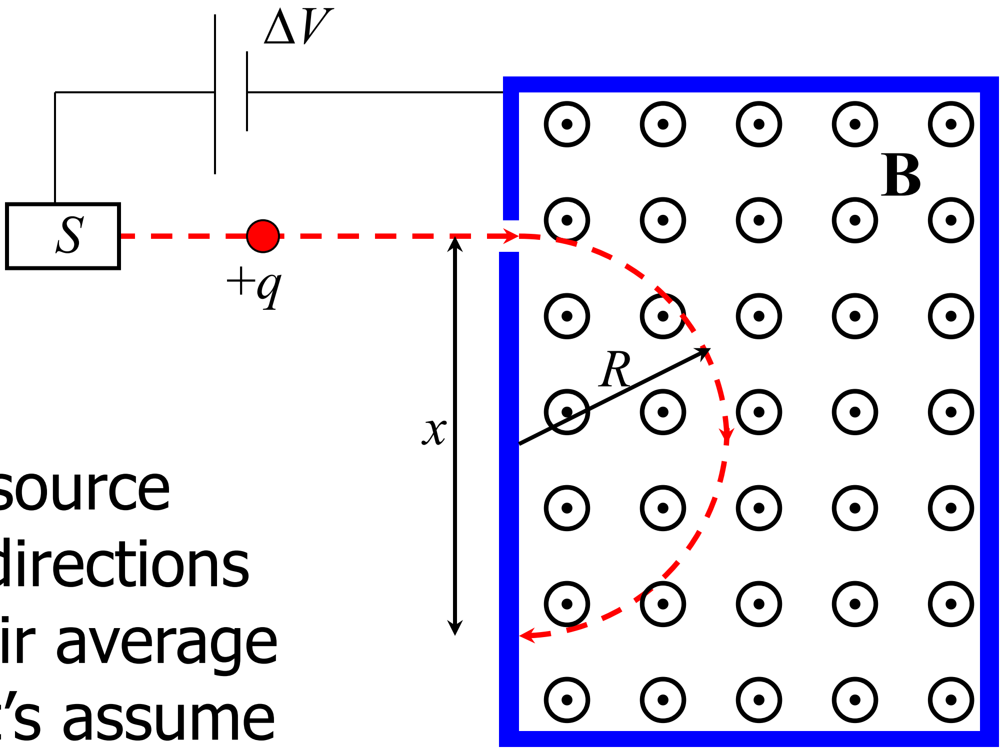


Worked example with numbers to follow.

Example: ions from source S enter a region of uniform magnetic field B that is perpendicular to the ion path. The ions follow a semicircle and strike the detector plate at x = 1.7558 m from the point where they entered the field. If the ions have a charge of 1.6022 x 10^{-19} C, the magnetic field has a magnitude of B = 80.0 mT, and the accelerating potential is ΔV = 1000.0 V, what is the mass of the ion?



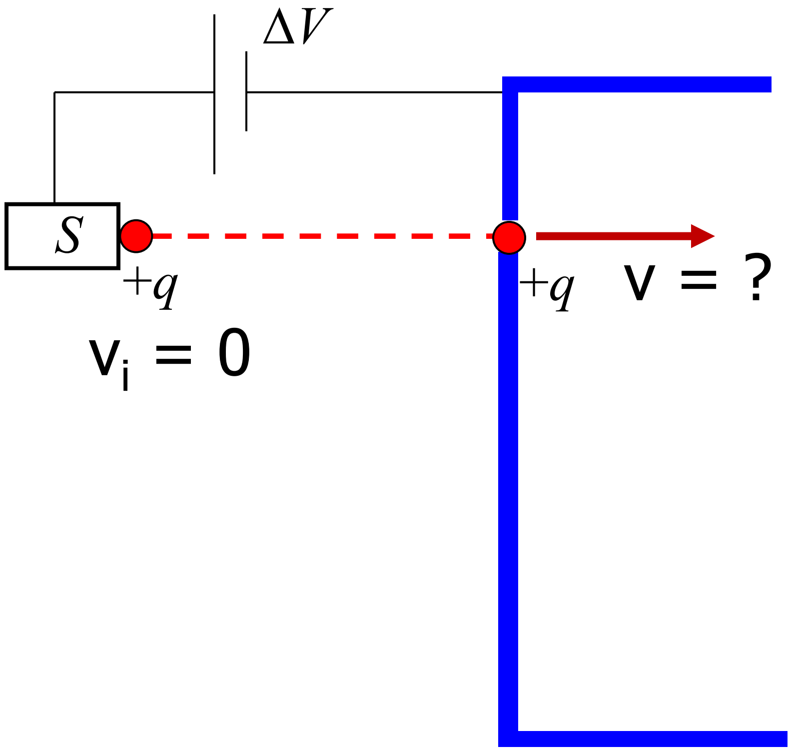
Step 1: find the final speed of the ions if they have a charge of 1.6022×10^{-19} C and the accelerating potential is $\Delta V = 1000.0$ V?



Let's focus on the part of the diagram where the ions from the source are being accelerated.

If the ions are "boiling" off a source filament, their initial velocity directions will be almost random, so their average speed will be almost zero. Let's assume the ions start at rest (corresponding to zero average speed).

Step 1: find the final speed of the ions if they have a charge of 1.6022×10^{-19} C and the accelerating potential is $\Delta V = 1000.0$ V?



$$E_f - E_i = \left[W_{other}\right]_{i\to f}$$

$$K_f + U_f - (K_i + U_i) = \left[W_{other}\right]_{i\to f}$$

(K_i = 0, W_{other} = 0)

$$K_f = -U_f + U_i = -(U_f - U_i) = -\Delta U$$

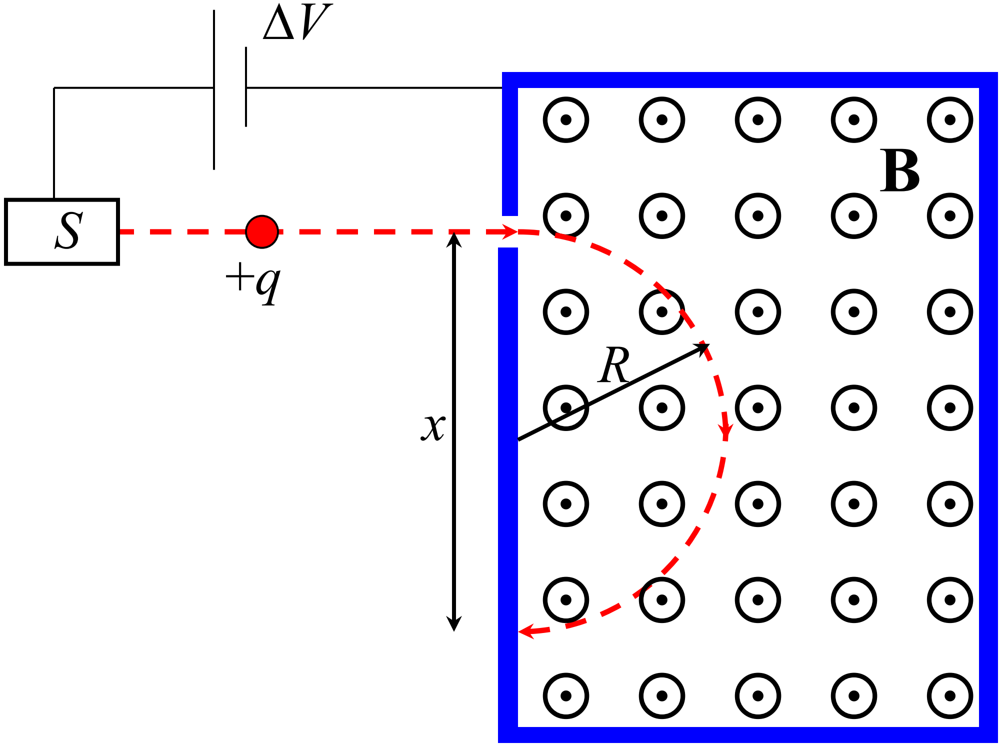
$$\frac{1}{2}mv^2 = -\Delta U = -q\Delta V$$

$$v = \sqrt{\frac{-2q\Delta V}{m}}$$

Notes: we don't know m, so let's keep this symbolic for now. Besides, we want to solve for m later. Also, ΔV must be negative, which makes sense (a + charge would be accelerated away from a higher potential).

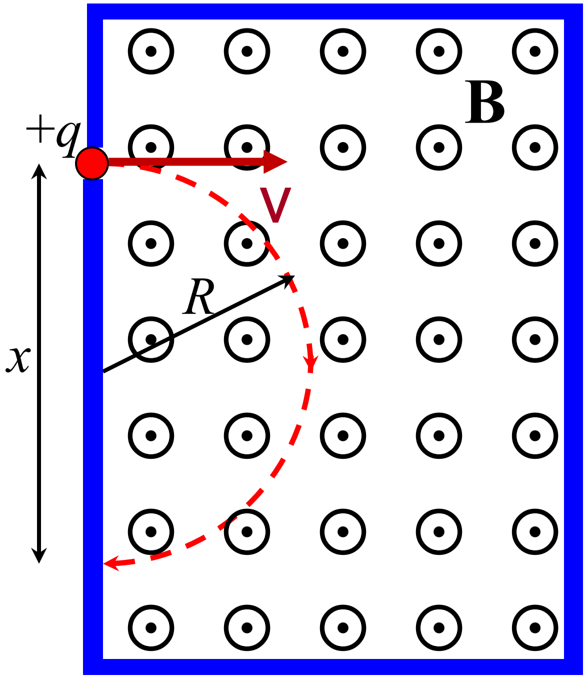
Don't get your v's and V's mixed up! Use a script v for speed!

Step 2: find the radius of the circular path followed by the ions in the region of uniform magnetic field of magnitude B.



Step 2: find the radius of the circular path followed by the ions in the region of uniform magnetic field of magnitude B.

Let's focus on the magnetic field region of the diagram.



Step 2: find the radius of the circular path followed by the ions in the region of uniform magnetic field of magnitude B.

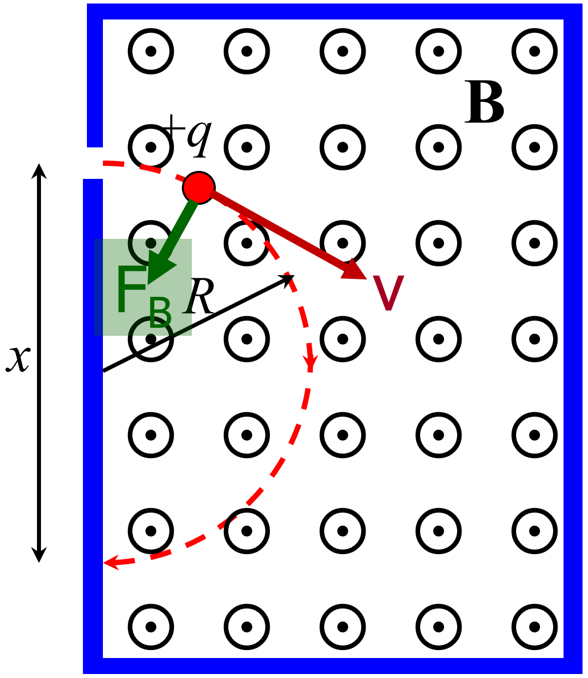
For motion in a circular path...

$$\left|\vec{F}_B\right| = \left|m\vec{a}\right| = \left|m\frac{v^2}{R}\right|$$

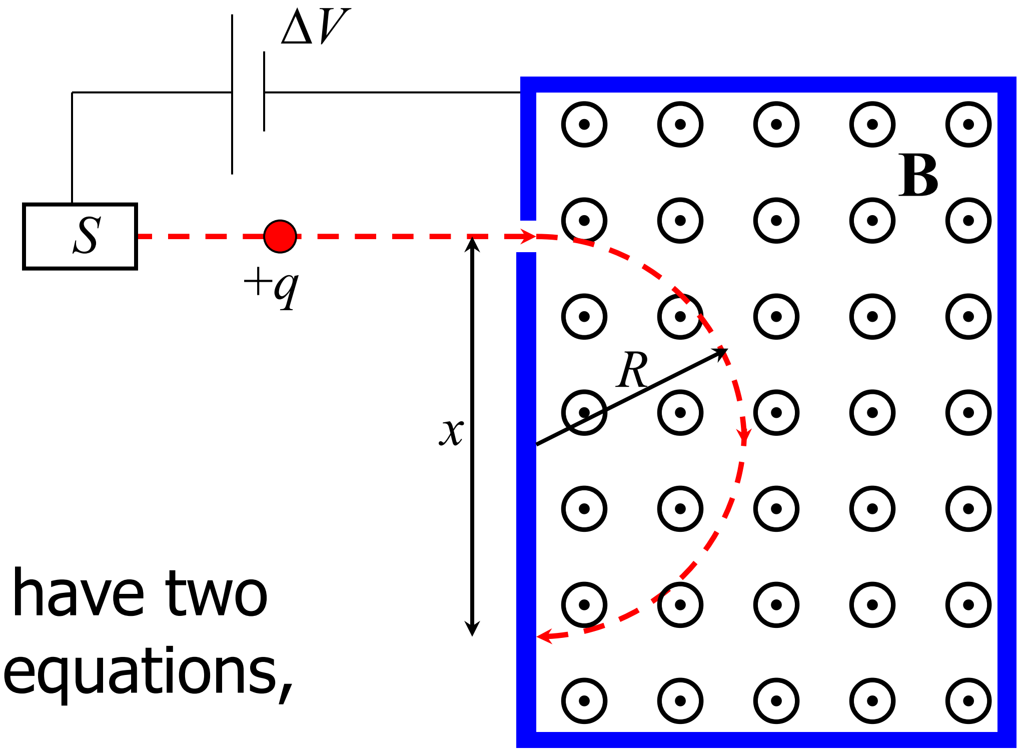
$$\left|\vec{F}_B\right| = \left|q\vec{v}\times\vec{B}\right| = \left|qvB\sin 90°\right|$$

$$\left|q\right| vB = m\frac{v^2}{R}$$

$$R = \frac{mv}{\left|q\right| B}$$



Step 3: solve for m, using $q=1.6022\times10^{-19}$ C, $\Delta V=1000.0$ V, B = 80.0 mT, and R=x/2=1.7558/2=0.8779 m.



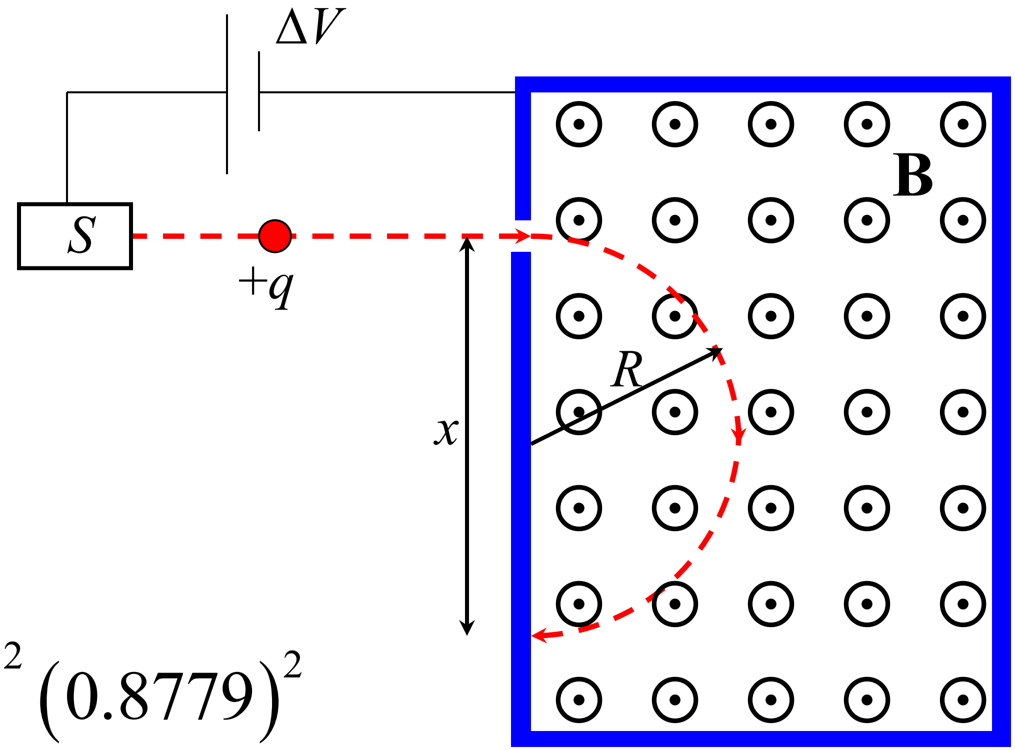
Now we can put it all together.

$$v=\sqrt{\frac{-2q\Delta V}{m}} \qquad R=\frac{mv}{|q|B}$$

We know q, R, B, and ΔV. We have two unknowns, v and m, and two equations, so we can solve for m.

$$R^2=\frac{m^2v^2}{|q|^2B^2}=\frac{m^2\left(\frac{-2q\Delta V}{m}\right)}{|q|^2B^2}=\frac{(-2q\Delta V)m}{|q|^2B^2}=\frac{(-2\Delta V)m}{|q|B^2}$$

Step 3: solve for m, using q=1.6022x10^{-19} C, ΔV=-1000.0 V, B = 80.0 mT, and R=x/2=1.7558/2=0.8779 m.



$$R^2 = \frac{(-2\Delta V)m}{|q|B^2}$$

$$m = \frac{|q|B^2R^2}{(-2\Delta V)}$$

$$m = \frac{(1.6022\times10^{-19})(80\times10^{-3})^2(0.8779)^2}{-(2)(-1000)}$$

$$m = 3.9515\times10^{-25}\ \text{kg}$$

uranium-238!

1 atomic mass unit (u) equals 1.66x10^{-27} kg, so m=238.04 u.

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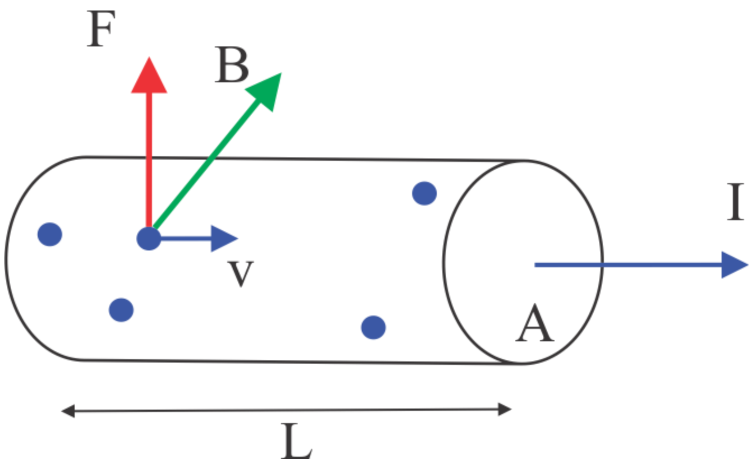
Magnetic Forces on Currents

- magnetic force on single charged particle: $\vec{F} = q\vec{v} \times \vec{B}$

Current-carrying wire:



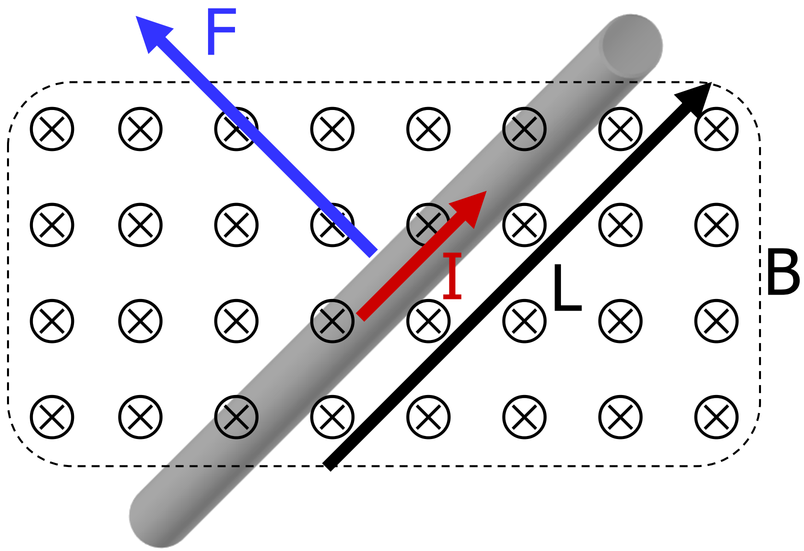
- number of charges: LAn
- total force on all carriers:

$$\vec{F}_{tot} = LA\, n\, q\vec{v} \times \vec{B} = LA\ \vec{j} \times \vec{B} = I\vec{L} \times \vec{B}$$

Magnetic force on current-carrying wire:

$$\boxed{\vec{F} = I\vec{L} \times \vec{B}}$$

vector $\vec{L}$ is in direction of current

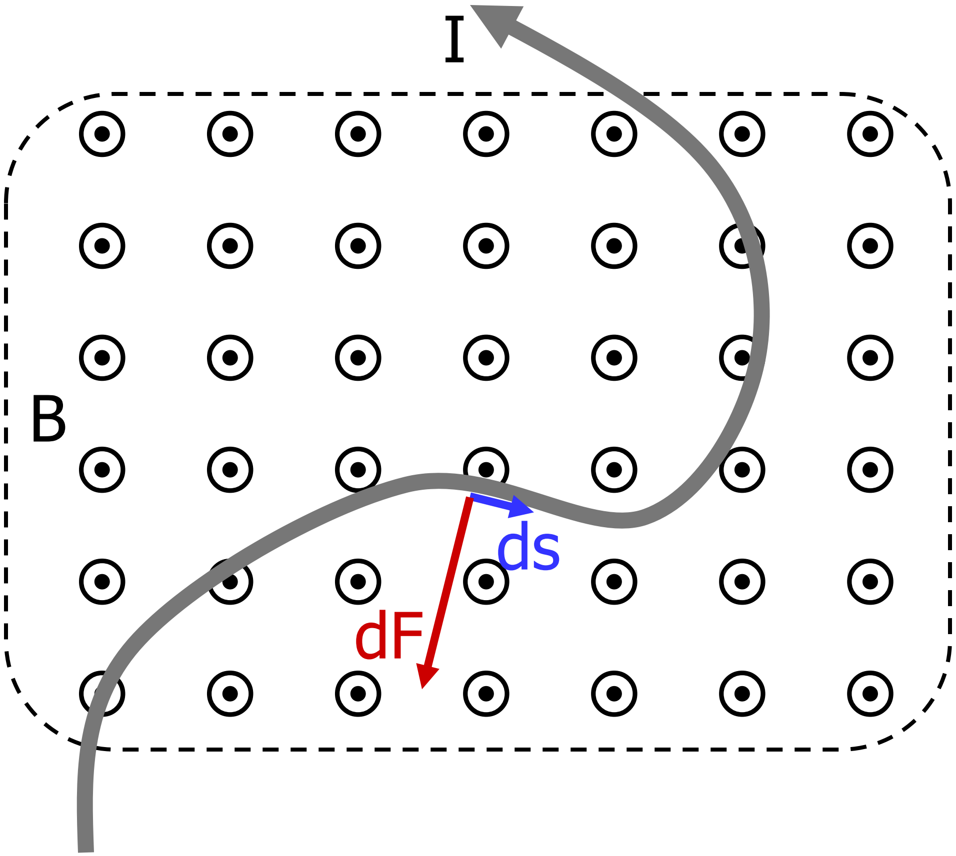


$$\vec{F} = I\vec{L} \times \vec{B}$$

Valid for straight wire, length L inside region of magnetic field, constant magnetic field, constant current I

You could apply this equation to a beam of charged particles moving through space, even if the charged particles are not confined to a wire.

What if the wire is not straight?



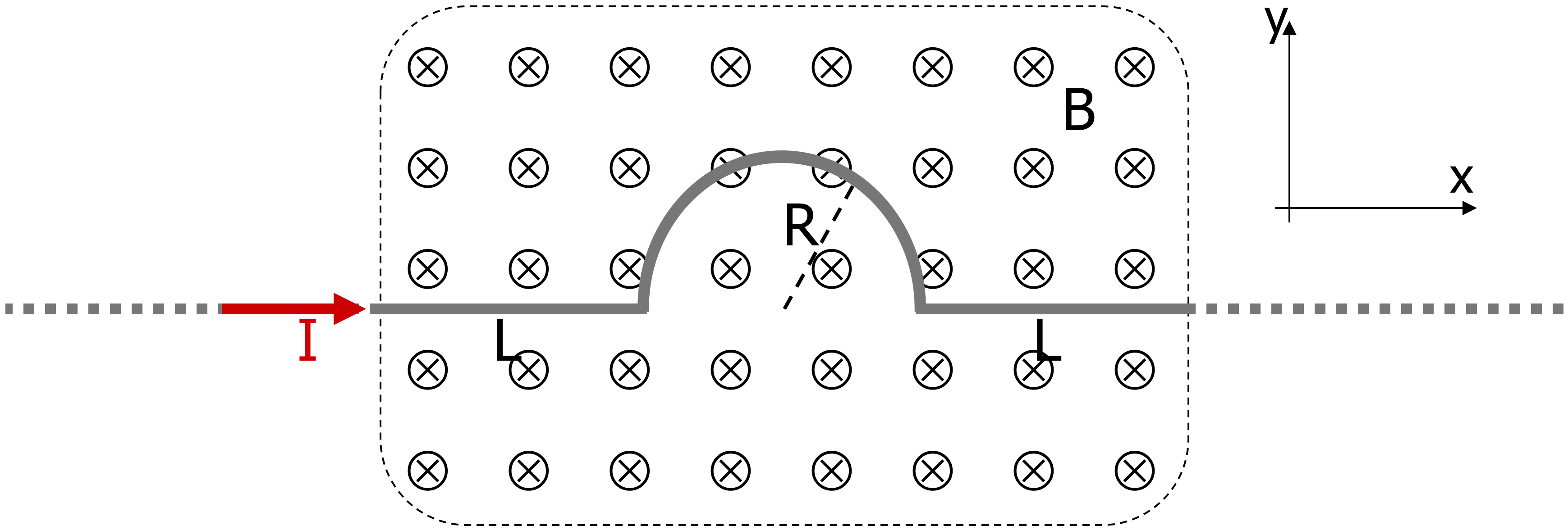
$$d\vec{F} = I\, d\vec{s} \times \vec{B}$$

$$\vec{F} = \int d\vec{F}$$

$$\vec{F} = I \int \left(d\vec{s} \times \vec{B} \right)$$

Integrate over the part of the wire that is in the magnetic field region.

Example: a wire carrying current I consists of a semicircle of radius R and two horizontal straight portions each of length L. It is in a region of constant magnetic field as shown. What is the net magnetic force on the wire?



There is no magnetic force on the portions of the wire outside the magnetic field region.

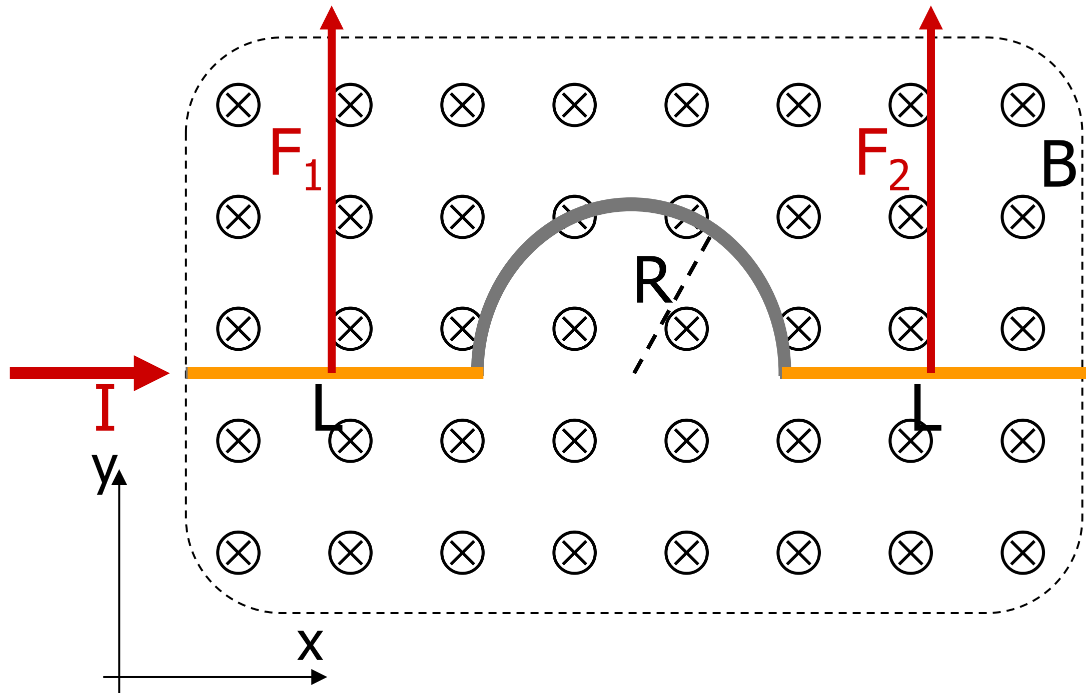
Straight sections:

$$\vec{F} = I\vec{L} \times \vec{B}$$

$\vec{L} \perp \vec{B}$, so

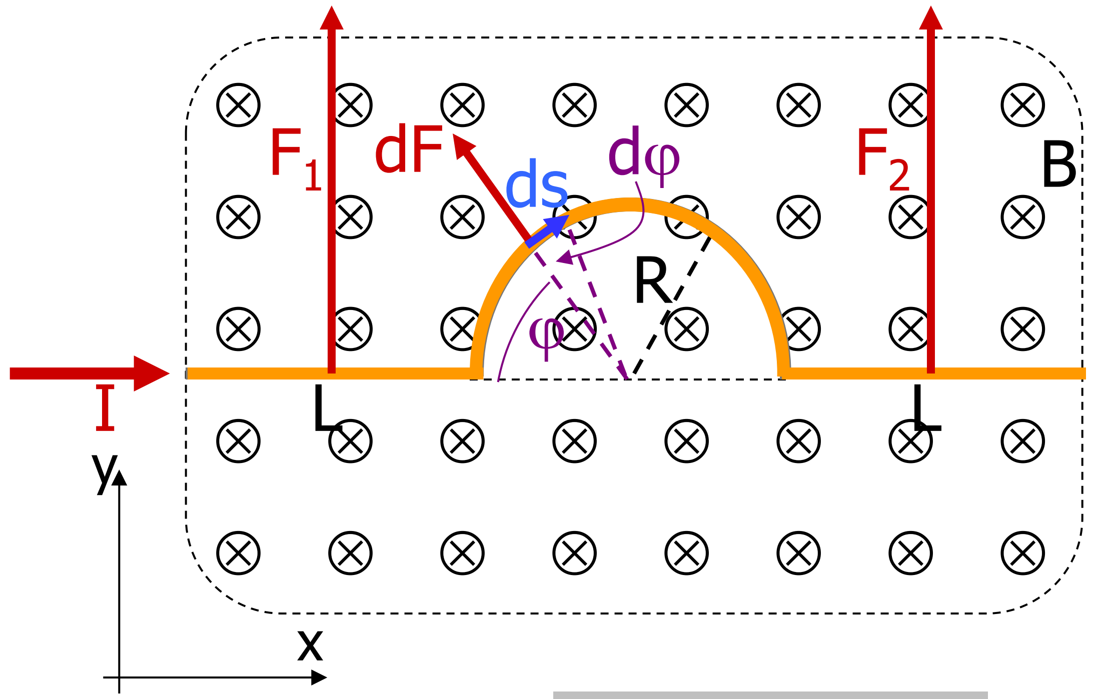
$$F_1 = F_2 = ILB$$

in positive y-direction



Semicircular section:

infinitesimal force $\vec{dF}$
on an infinitesimal $\vec{ds}$
of current-carrying wire



$\vec{ds}$ subtends angle from φ to $\varphi+d\varphi$.

infinitesimal force is $d\vec{F} = I\, d\vec{s} \times \vec{B}$.

$\vec{ds} \perp \vec{B}$, so $dF = I\, ds\, B$.

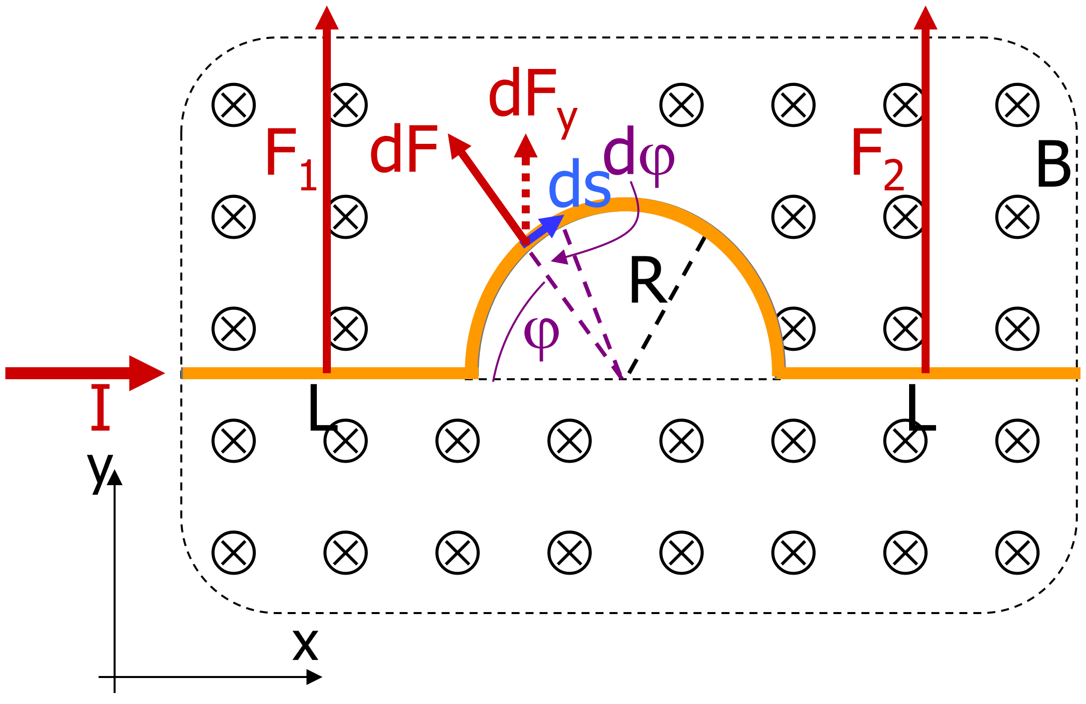
Arc length $ds = R\, d\varphi$.

Finally, $dF = I\, R\, d\varphi\, B$.

Why did I call that angle φ instead of θ?

Because we usually use θ for the angle in the cross product.

y-component of F:



$$dF_y = I\,R\,d\varphi\,B\,\sin\varphi$$

$$F_y = \int_0^{\pi} dF_y$$

$$F_y = \int_0^{\pi} I\,R\,d\varphi\,B\,\sin\varphi$$

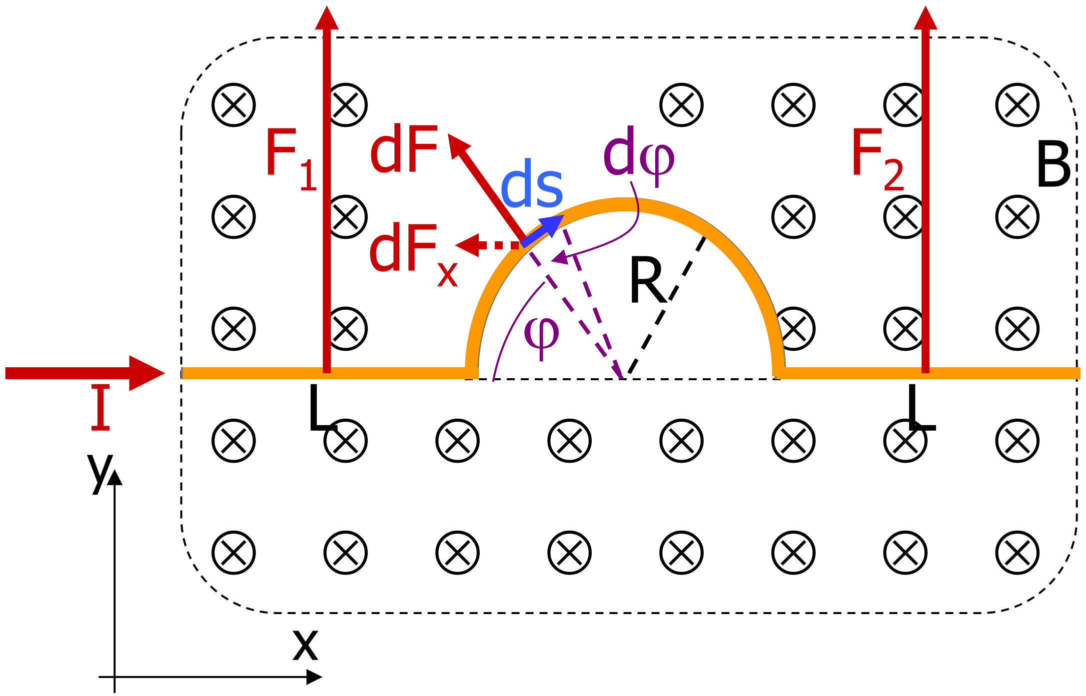
$$F_y = I\,R\,B\int_0^{\pi} \sin\varphi\,d\varphi$$

$$F_y = \left(-I\,R\,B\cos\varphi\right)\Big|_0^{\pi}$$

$$F_y = 2\,I\,R\,B$$

Interesting—just the force on a straight horizontal wire of length 2R.

Does symmetry give you F_x immediately?



x-component of F:

$$dF_x = -I\,R\,d\varphi\,B\cos\varphi$$

$$F_x = -\int_0^{\pi} I\,R\,d\varphi\,B\cos\varphi$$

$$F_x = -I\,R\,B\int_0^{\pi}\cos\varphi\,d\varphi$$

$$F_x = -\left(I\,R\,B\sin\varphi\right)\Big|_0^{\pi}$$

$$F_x = 0$$

Total force:

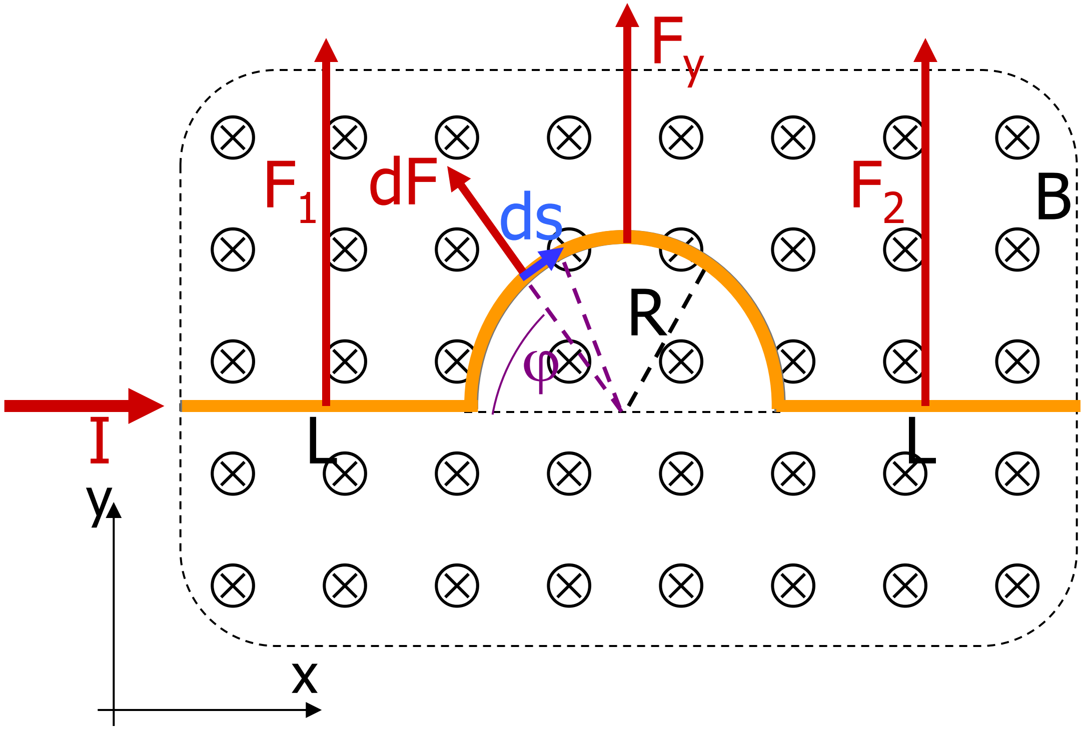
$$F_{y,tot} = F_1 + F_2 + F_{y,arc}$$

$$F_{y,tot} = ILB + ILB + 2IRB$$

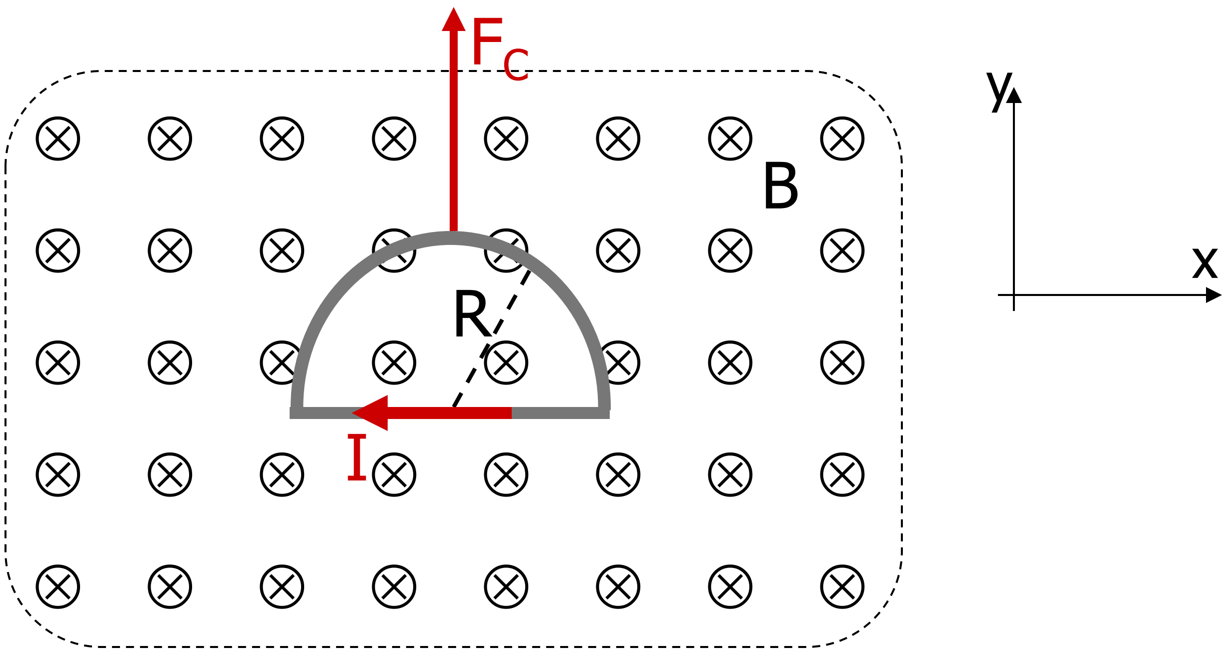
$$F_{y,tot} = 2IB\left(L + R\right)$$

$$\vec{F} = 2IB\left(L + R\right)\hat{j}$$

We probably should write the force in vector form.



Example: a semicircular closed loop of radius R carries current I. It is in a region of constant magnetic field as shown. What is the net magnetic force on the loop of wire?

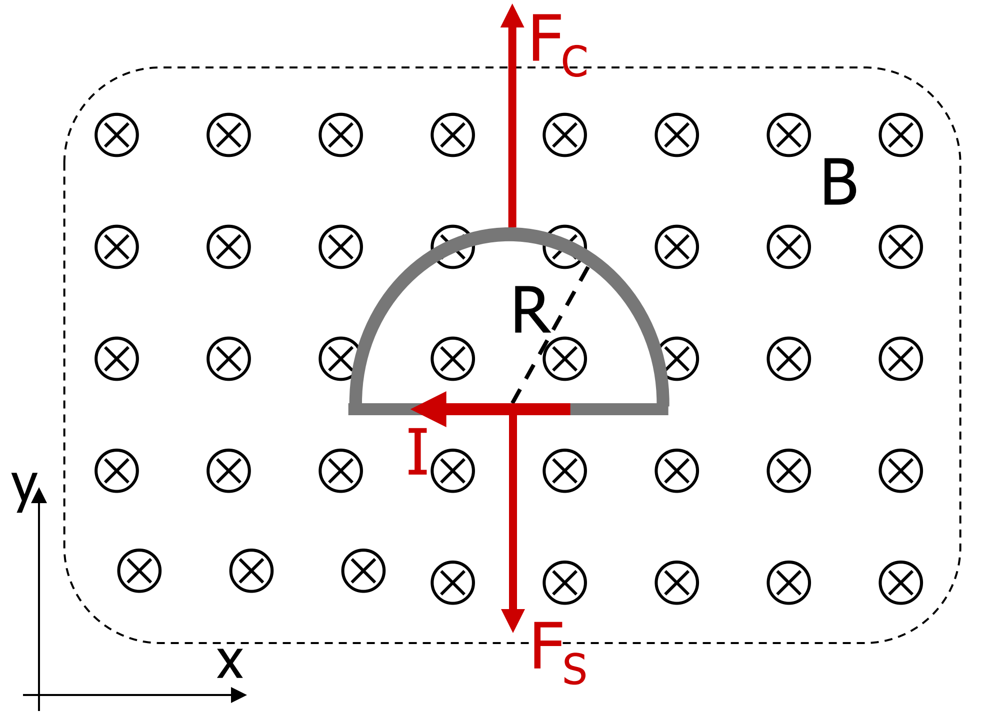


We calculated the force on the semicircular part in the previous example (current is flowing in the same direction there as before).

$$F_C = 2\,I\,R\,B$$

Next look at the straight section.

$$\vec{F}_S = I\vec{L} \times \vec{B}$$



$\vec{L} \perp \vec{B}$, and L=2R so

$$F_S = 2IRB$$

$\vec{F}_s$ is directed in the –y direction (right hand rule).

$$\vec{F}_{net} = \vec{F}_S + \vec{F}_c = -2IRB\,\hat{j} + 2IRB\,\hat{j} = 0$$

The net force on the closed loop is zero!

This is true in general for closed loops in a uniform magnetic field.

Today's agenda:

Magnetic Fields.

You must understand the similarities and differences between electric fields and field lines, and magnetic fields and field lines.

Magnetic Force on Moving Charged Particles.

You must be able to calculate the magnetic force on moving charged particles.

Motion of a Charged Particle in a Uniform Magnetic Field.

You must be able to calculate the trajectory and energy of a charged particle moving in a uniform magnetic field.

Magnetic forces on currents and current-carrying wires.

You must be able to calculate the magnetic force on currents.

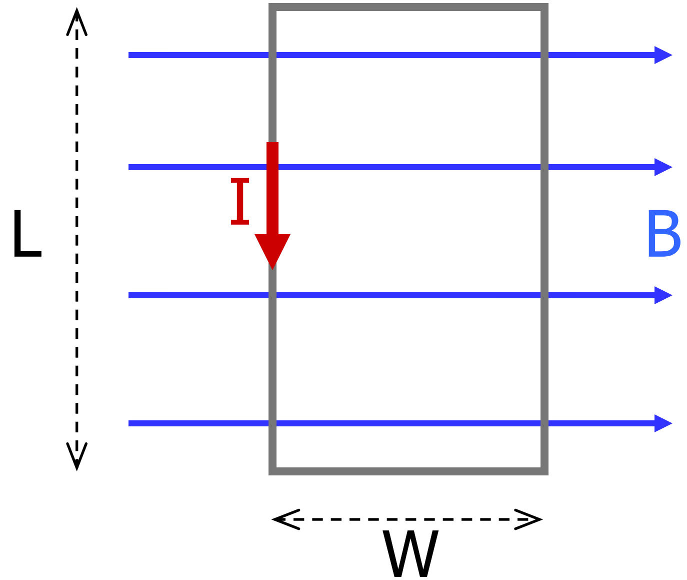
Magnetic forces and torques on current loops.

You must be able to calculate the torque and magnetic moment for a current-carrying wire in a uniform magnetic field.

Magnetic Forces and Torques on Current Loops

Rectangular loop:

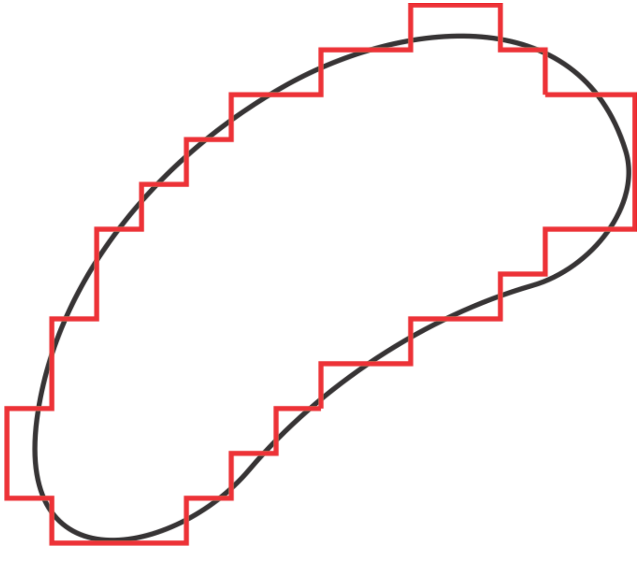
- currents in left and right wires are equal and opposite
- magnetic forces on left and right wires are equal and opposite
- same for top and bottom wires
- **net magnetic force on loop is zero**



General loop:

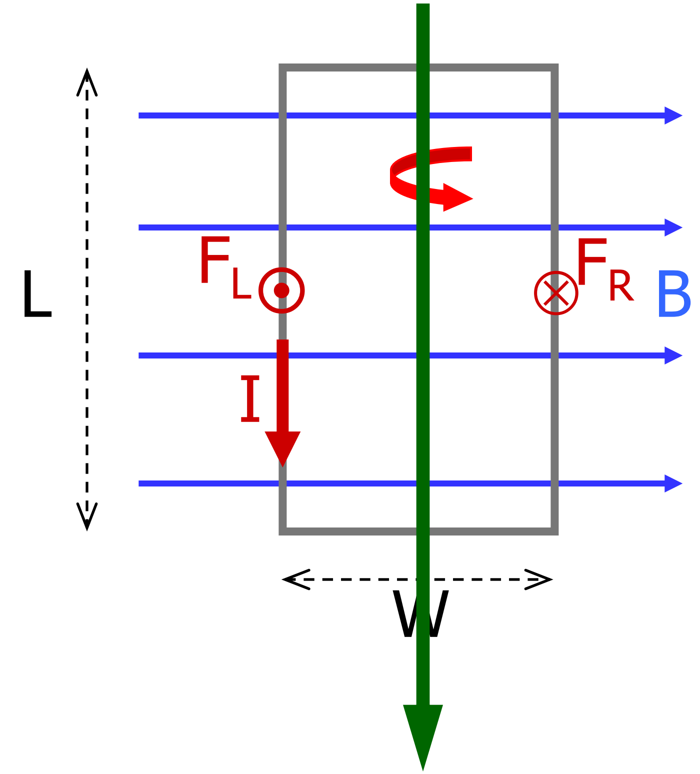
- can be decomposed into rectangular pieces

Net force on any closed current loop is zero



Torque on current loop:

- no force on top and bottom segments because current and magnetic field are parallel
- left vertical segment feels force "out of the page"
- right vertical segment feels force "into the page"



The two forces have the same magnitude: $F_L = F_R = I L B$.

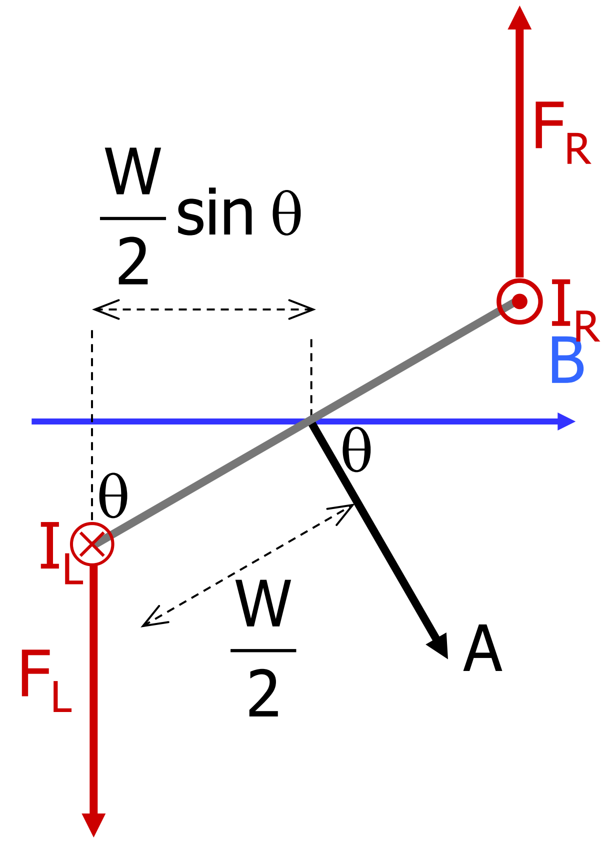
Because $\vec{F}_L$ and $\vec{F}_R$ are in opposite directions, there is no net force on the current loop, but there is a net torque.

Top view:

$$\tau_R = \frac{W}{2} F_R \sin\theta = \frac{1}{2} WILB \sin\theta$$

$$\tau_L = \frac{W}{2} F_L \sin\theta = \frac{1}{2} WILB \sin\theta$$

$$\tau_{net} = \tau_R + \tau_L = WILB \sin\theta = IAB \sin\theta$$



In vector form:

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Direction of $\vec{A}$: (right-hand rule)

- curl your fingers around the loop in the direction of the current; thumb points in direction of $\vec{A}$.

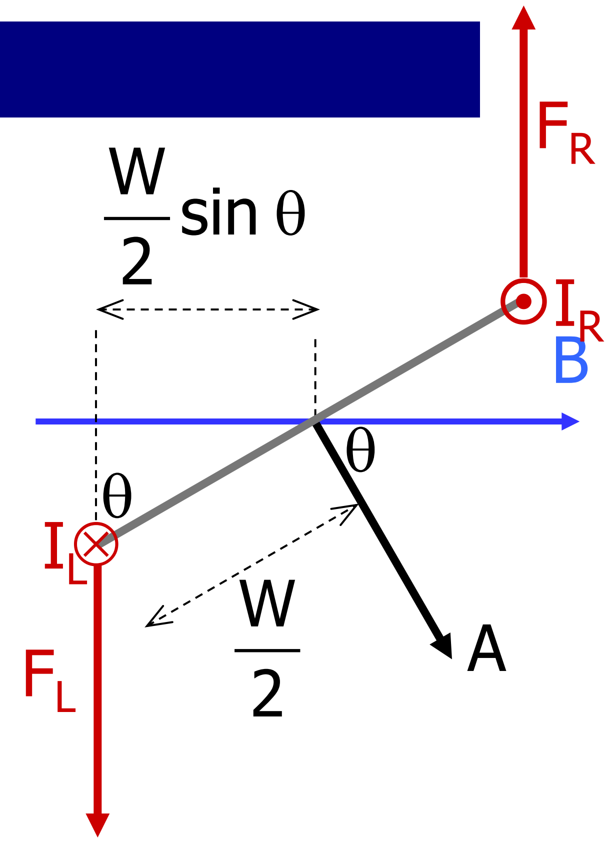
Magnetic Moment of a Current Loop

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

magnetic moment of loop

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



Your starting equation sheet has:

$$\vec{\mu} = N I \vec{A} \quad (N = 1 \text{ for a single loop})$$

Magnetic dipoles

- current loop acts as magnetic dipole similar to bar magnet

You already know this:

Electric Dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Today:

Magnetic Dipole

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

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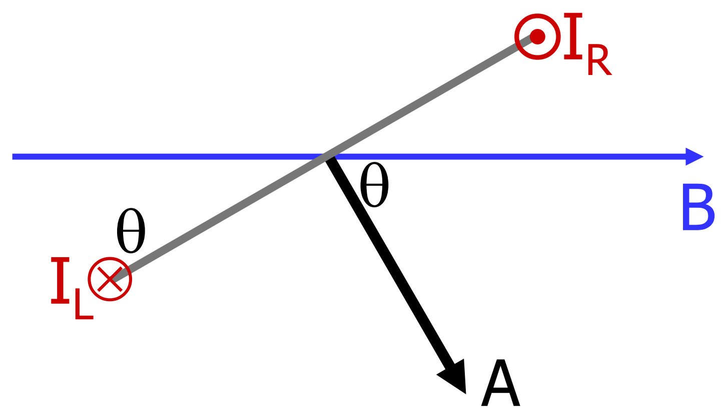
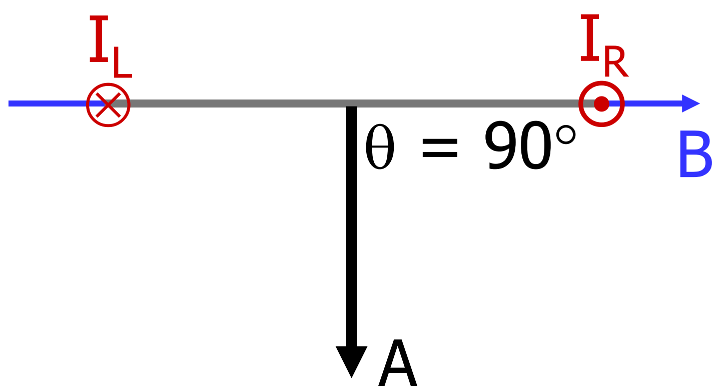
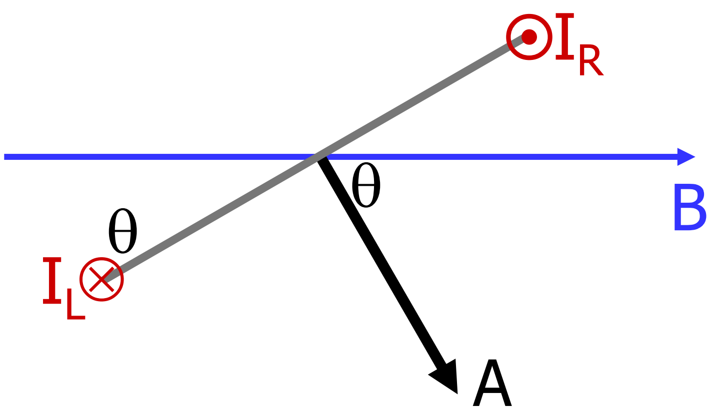


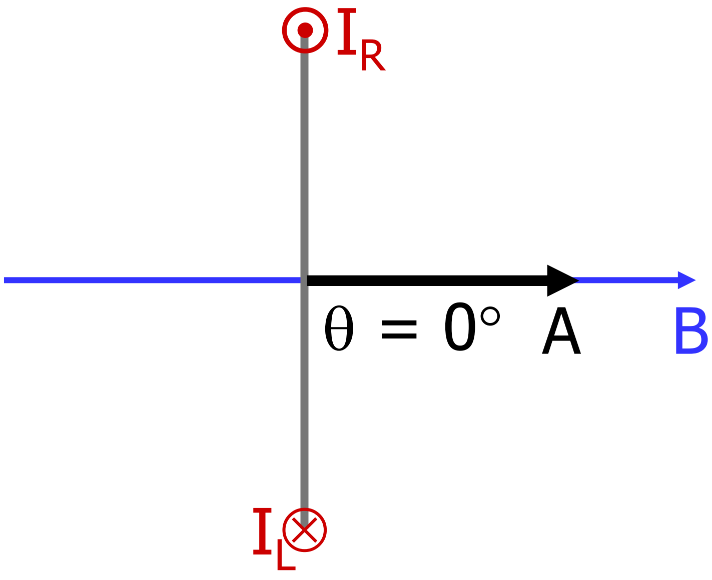


$$U = -I\vec{A}\cdot\vec{B} = -IAB\cos\theta$$

Potential energy is zero when $\cos\theta = 0$, or when $\theta = 90^\circ$ or 270°.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

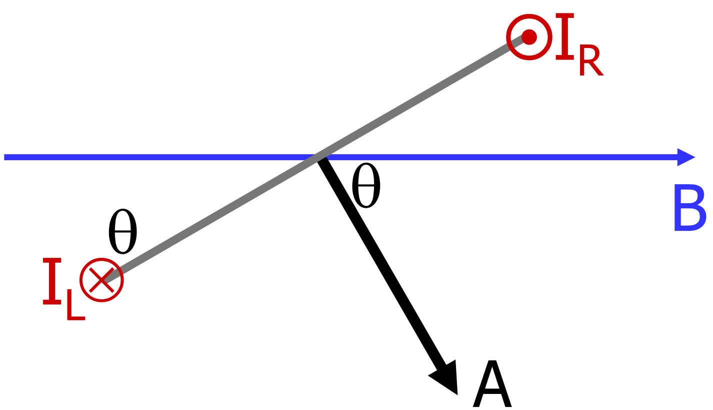


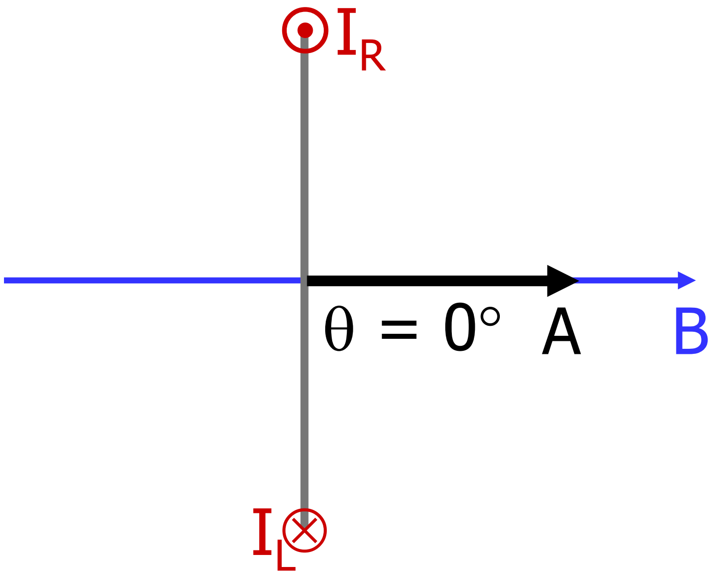


$$U = -I\vec{A}\cdot\vec{B} = -IAB\cos\theta$$

Potential energy is minimum when $\cos\theta = 1$, or when $\theta = 0°$ or 180°.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

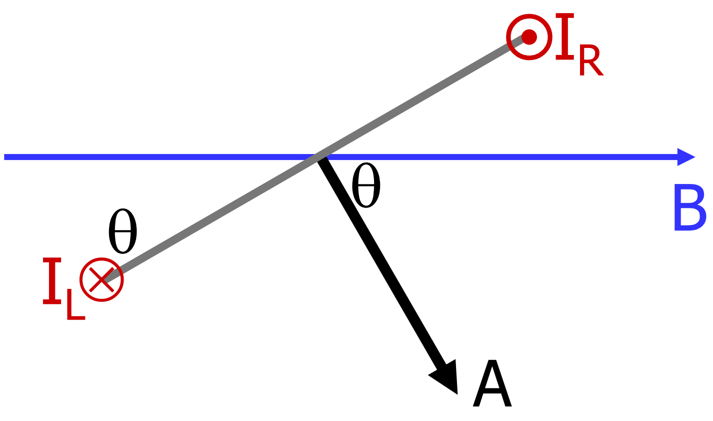


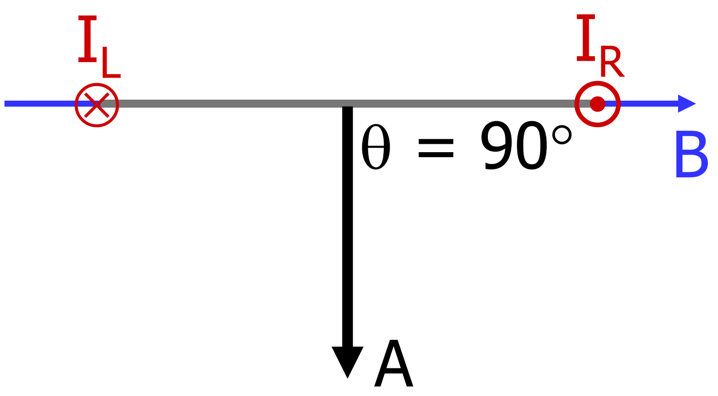


$$|\vec{\tau}| = \left|I\vec{A} \times \vec{B}\right| = IAB|\sin\theta|$$

Torque magnitude is minimum when $\sin\theta = 0$, or when $\theta = 0°$ or $180°$.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

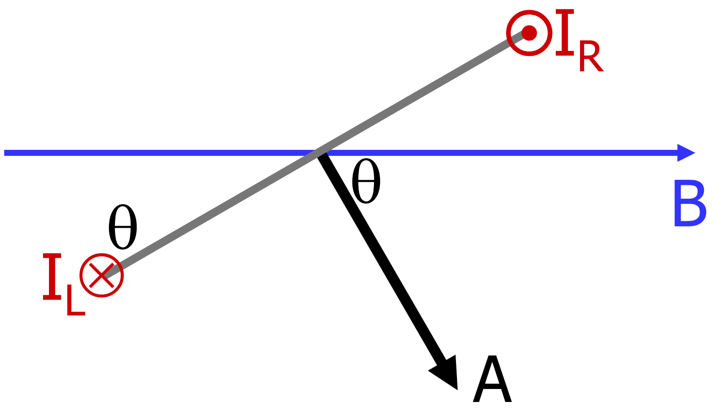




$$\left|\vec{\tau}\right| = \left|I\vec{A}\times\vec{B}\right| = IAB\left|\sin\theta\right|$$

Torque magnitude is maximum when $|\sin\theta| = 1$, or when $\theta = 90^\circ$ or 270°.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy maximum?



$U = -I\vec{A}\cdot\vec{B} = -IAB\cos\theta$

I left this for you to figure out.