EPFL

Lecture 2 agenda:

RECAP

Electric Charge.

Review of some things you hopefully learned in high school.

Coulomb’s Law (electrical force between charged particles).

You must be able to calculate the electrical forces between one or more charged particles.

The Electric field.

You must be able to calculate the force on a charged particle in an electric field.

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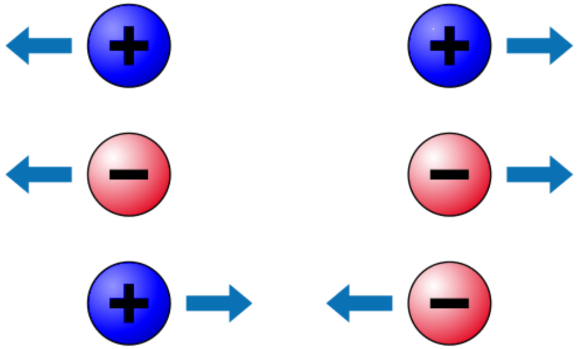
Electric Charge

What is charge?

- basic property of matter (just like, say, mass)
- we can observe charge indirectly via its effects on matter
- customary symbol: q or Q, unit: $[q]$ = C (Coulomb)

Two kinds of charge:

- like charges repel
- unlike charges attract

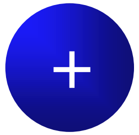


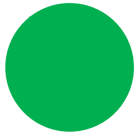
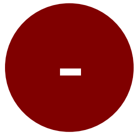
Law of conservation of charge:

- net amount of charge does not change in any process

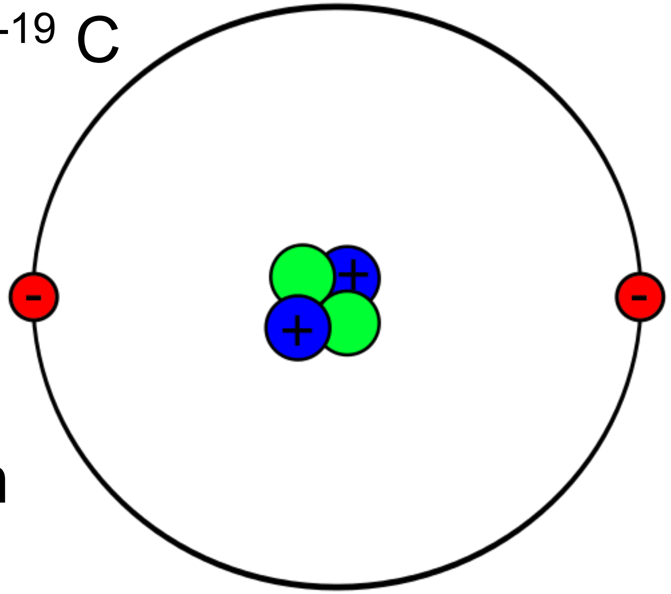
Charges are *quantized* (come in units of e= 1.6×10^{-19} C).

Elementary particles that make up atoms:

- Protons 

charge +e = $+1.6 \times 10^{-19}$ C
- Neutrons  uncharged
- Electrons 

charge –e = -1.6×10^{-19} C



Helium atom

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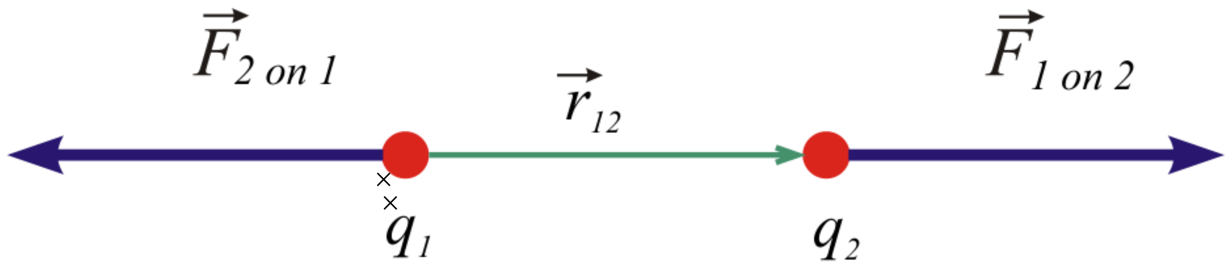
Coulomb's Law

Force between two point charges q_1 and q_2:

- force is **vector,** directed along connecting line



- magnitude:

$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2}$$

r_{12} is the distance between the charges

$$k = 9\times10^9\ \frac{\text{Nm}^2}{\text{C}^2} = \frac{1}{4\pi\varepsilon_0} \quad \text{with} \quad \varepsilon_0 = 8.85\times10^{-12}\ \frac{\text{C}^2}{\text{Nm}^2}.$$

a note on starting equations

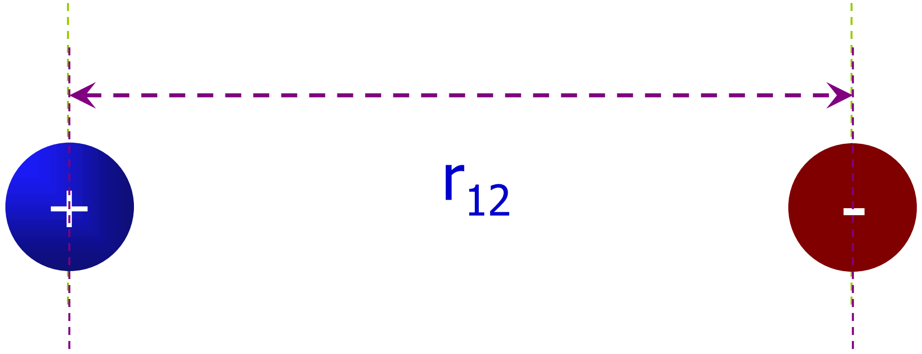
$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2}$$

is an **official starting equation to solve many of the new problems**

- in homework and exam solutions, official starting equations can be used **without deriving them**
- all other steps of a solution need to be derived
- you may formulate the official starting equation in different variables

*"Starting" does not mean that a starting equation has to be the first thing that appears on your paper. It might be several lines before you use a starting equation.

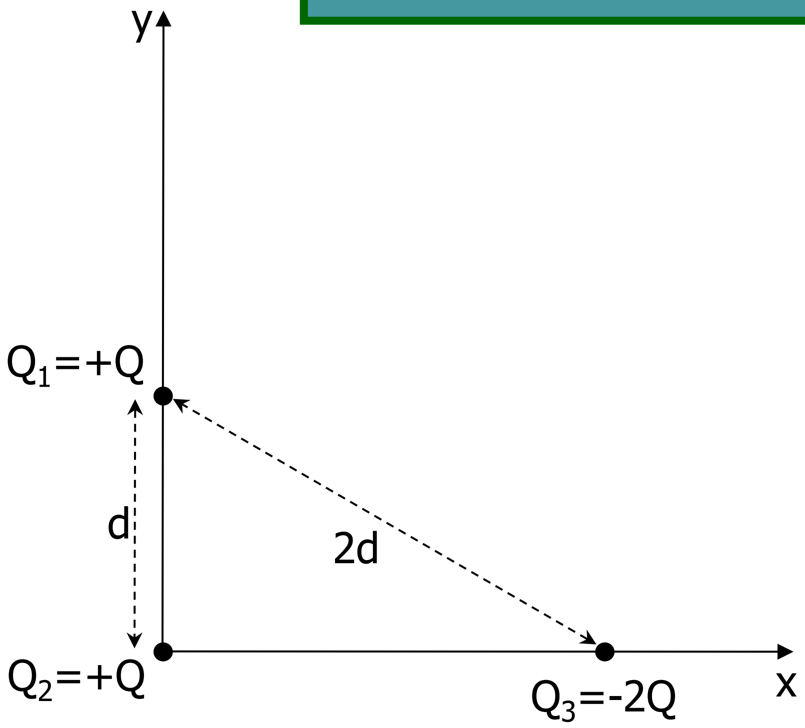
Coulomb's Law is strictly valid for **point charges** only.
It is a good **approximation** for small uniformly charged objects.



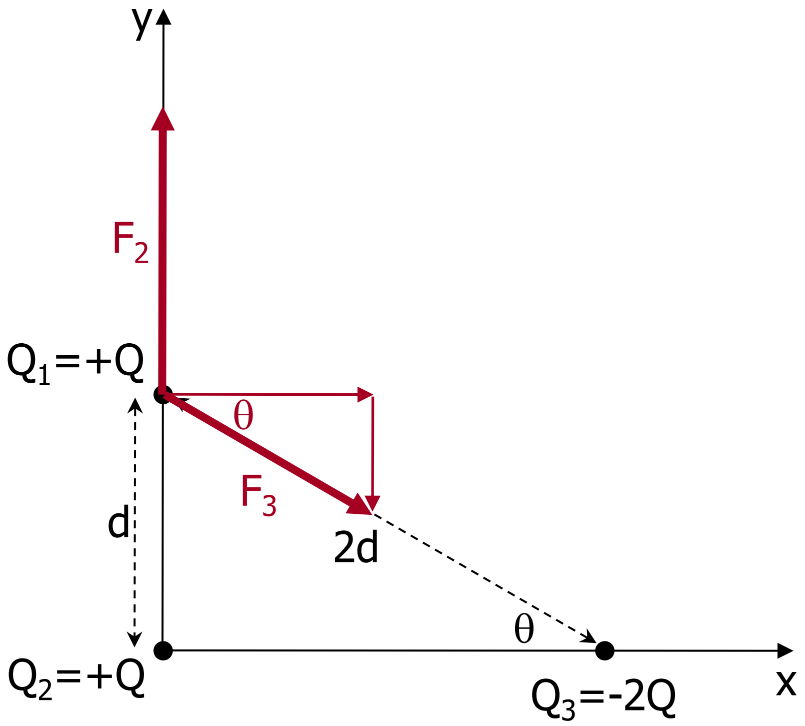
If more than two charges are involved, the net force is the vector sum of all forces (superposition). For objects with complex shapes, you must add up all the forces acting on each separate charge (calculus!!).

Example: a positive charge Q_1 = +Q is located a distance d along the y-axis from the origin. A second positive charge Q_2 = +Q is located at the origin and a negative charge Q_3 = -2Q is located on the x-axis a distance 2d away from Q_1. Calculate the net electrostatic force on Q_1 due to the other two charges.

Example: a positive charge $Q_1 = +Q$ is located a distance d along the y-axis from the origin. A second positive charge $Q_2 = +Q$ is located at the origin and a negative charge $Q_3 = -2Q$ is located on the x-axis a distance 2d away from Q_1. Calculate the net electrostatic force on Q_1 due to the other two charges.



Calculate the net electrostatic force on Q_1 due to the other two charges.



$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2}$$

$$\sin\theta = \frac{1}{2} \qquad \cos\theta = \frac{\sqrt{3}}{2}$$

$$\vec{F} = \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_2 = k\frac{|q_1 q_2|}{r_{12}^2}\hat{j} = k\frac{|(+Q)(+Q)|}{d^2}\hat{j} = k\frac{Q^2}{d^2}\hat{j}$$

$$\vec{F}_3 = F_{3x}\hat{i} + F_{3y}\hat{j}$$

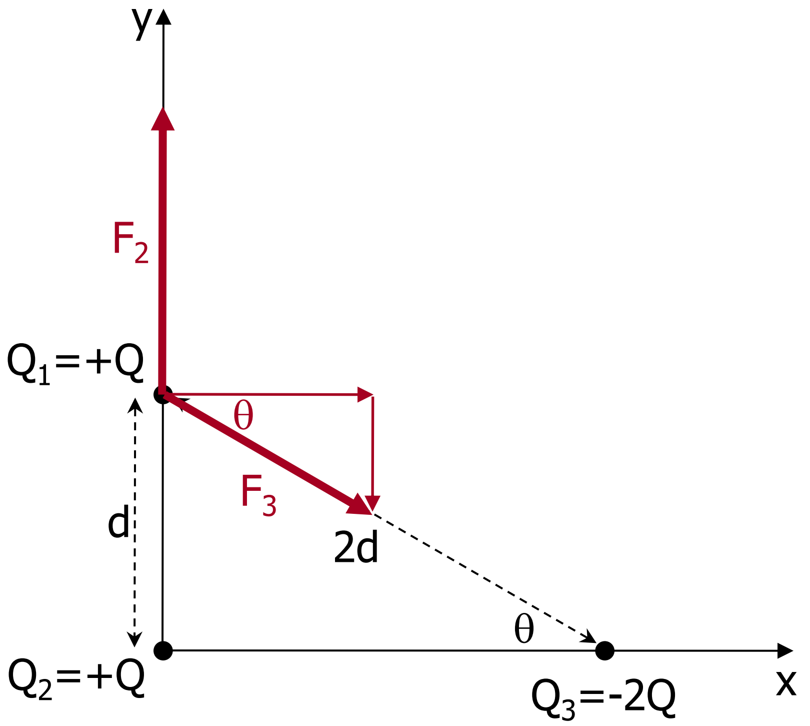
$$\vec{F}_3 = F_3\cos\theta\,\hat{i} - F_3\sin\theta\,\hat{j}$$

$$\vec{F}_3 = k\frac{|q_1 q_3|}{r_{13}^2}\cos\theta\,\hat{i} - k\frac{|q_1 q_3|}{r_{13}^2}\sin\theta\,\hat{j}$$

$$\vec{F}_3 = k\frac{|(+Q)(-2Q)|}{(2d)^2}\frac{\sqrt{3}}{2}\hat{i} - k\frac{|(+Q)(-2Q)|}{(2d)^2}\frac{1}{2}\hat{j}$$

Note: F_2 and F_3 are not drawn to scale (F_3 is “too long”).

Calculate the net electrostatic force on Q_1 due to the other two charges.



$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2}$$

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$$\vec{F}_3 = k\frac{|(+Q)(-2Q)|}{(2d)^2}\frac{\sqrt{3}}{2}\hat{i} - k\frac{|(+Q)(-2Q)|}{(2d)^2}\frac{1}{2}\hat{j}$$

$$\vec{F}_3 = k\frac{2Q^2}{4d^2}\frac{\sqrt{3}}{2}\hat{i} - k\frac{2Q^2}{4d^2}\frac{1}{2}\hat{j}$$

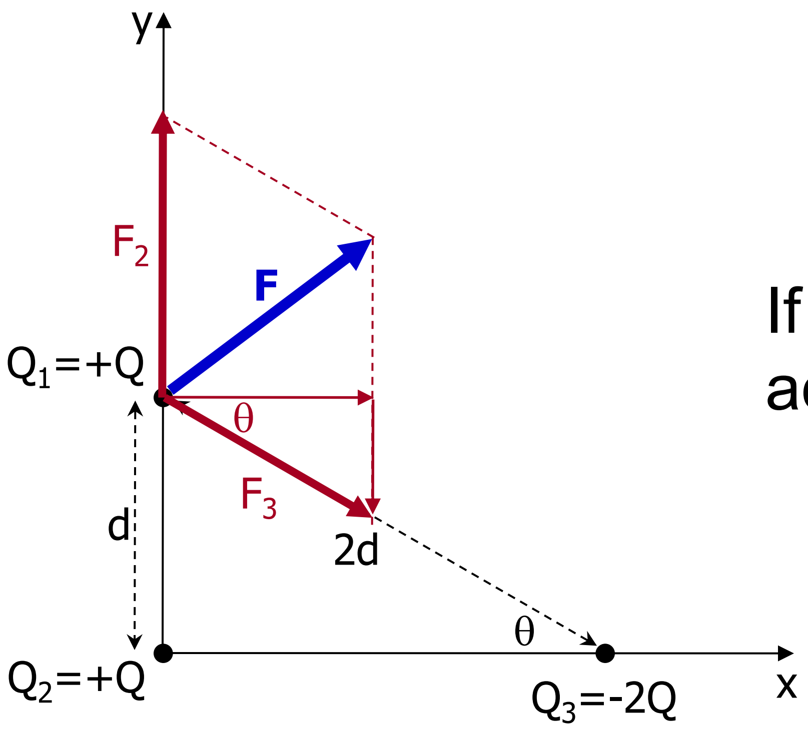
$$\vec{F}_3 = \frac{\sqrt{3}}{4}\frac{kQ^2}{d^2}\hat{i} - \frac{1}{4}\frac{kQ^2}{d^2}\hat{j}$$

$$\vec{F} = \vec{F}_2 + \vec{F}_3 = k\frac{Q^2}{d^2}\hat{j} + \frac{\sqrt{3}}{4}\frac{kQ^2}{d^2}\hat{i} - \frac{1}{4}\frac{kQ^2}{d^2}\hat{j}$$

$$\boxed{\vec{F} = \frac{\sqrt{3}}{4}\frac{kQ^2}{d^2}\hat{i} + \frac{3}{4}\frac{kQ^2}{d^2}\hat{j}}$$

or $F_x = \frac{\sqrt{3}}{4}\frac{kQ^2}{d^2}$ $\qquad F_y = \frac{3}{4}\frac{kQ^2}{d^2}$

Note: F_2 and F_3 are not drawn to scale (F_3 is "too long").



If Q_1 were free to move, what direction would its initial acceleration be? How would I calculate the acceleration?

Would the acceleration remain constant as Q_1 moved? Could I use the equations of kinematics to describe the motion of Q_1?

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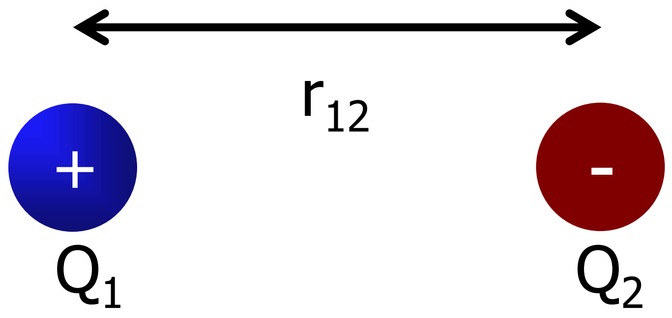
Coulomb's Law: it's just part of a bigger picture

Coulomb's Law:

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r_{12}^2},$$



Charged particles produce forces over great distances.

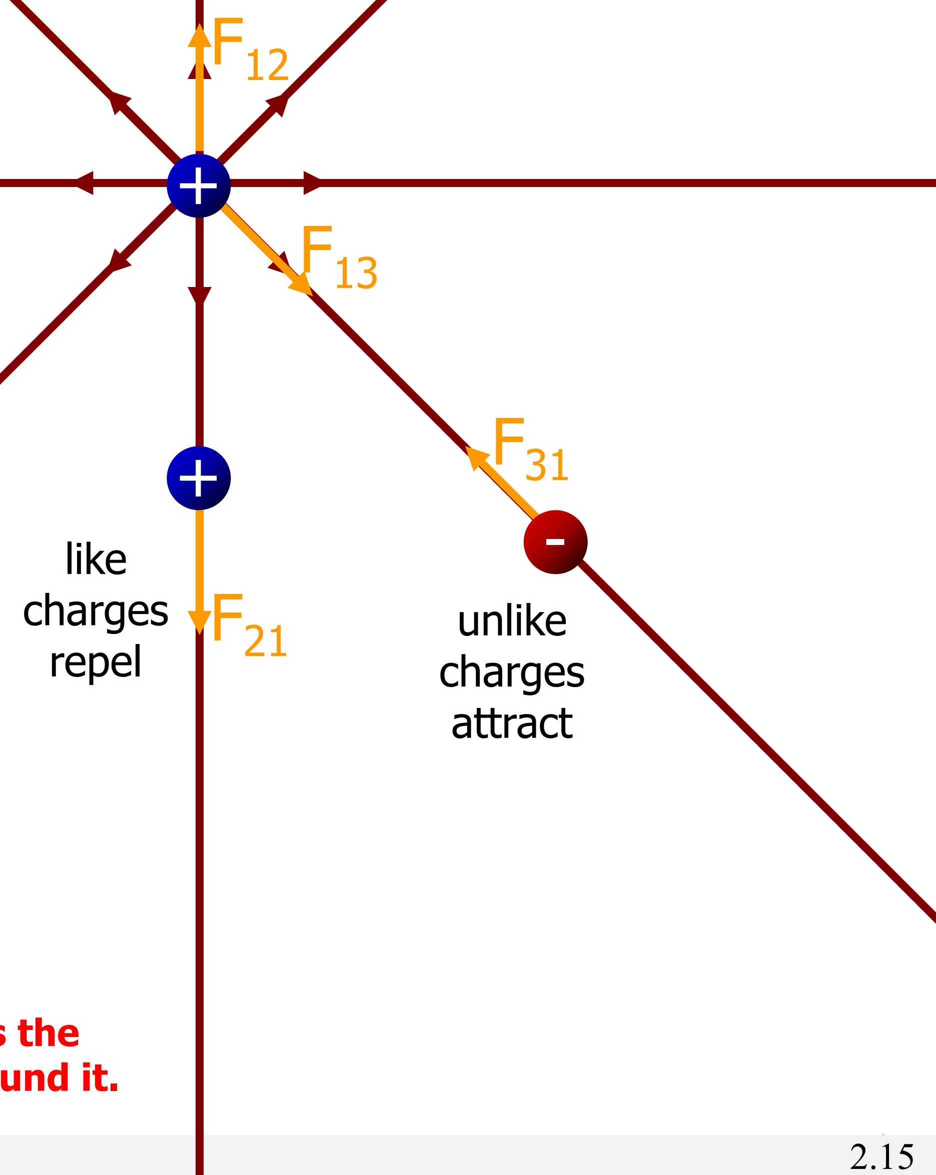
How does a charged particle "know" another one is “there?”

Introduce concept of **electric field**

- new way of thinking about the Coulomb force

The Electric Field

- a charged particle creates a "field" in all space.
- other charged particles sense the field and experience a force in response
- Distinguish **source charges** and **test charges**



A charged particle modifies the properties of the space around it.

Definition of Electric Field:

- one or more **source charges**
- define the electric field $\vec{E}$ via force they exert on a **test charge** q_0:

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

The subscript "0" reminds you the force is on the "test charge".

We introduced electric field ***E*** just a technical auxiliary tool to describe forces between charges.
Spoiler: We will see that in reality ***E*** is much more than that!

$$\boxed{\vec{F} = q\vec{E}}$$

This is your second **starting equation**.

It is a vector equation that tells you magnitude and direction of the force!

The units of electric field are newtons/coulomb.

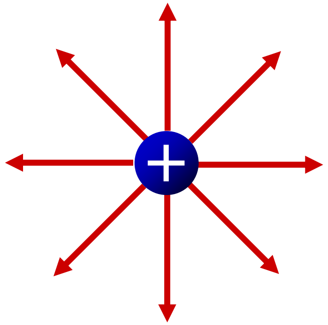
$$\left[\vec{E}\right] = \frac{\left[\vec{F}_0\right]}{\left[q_0\right]} = \frac{N}{C}$$

We will soon learn that the units of electric field can also be expressed as volts/meter:

$$\left[E\right] = \frac{N}{C} = \frac{V}{m}$$

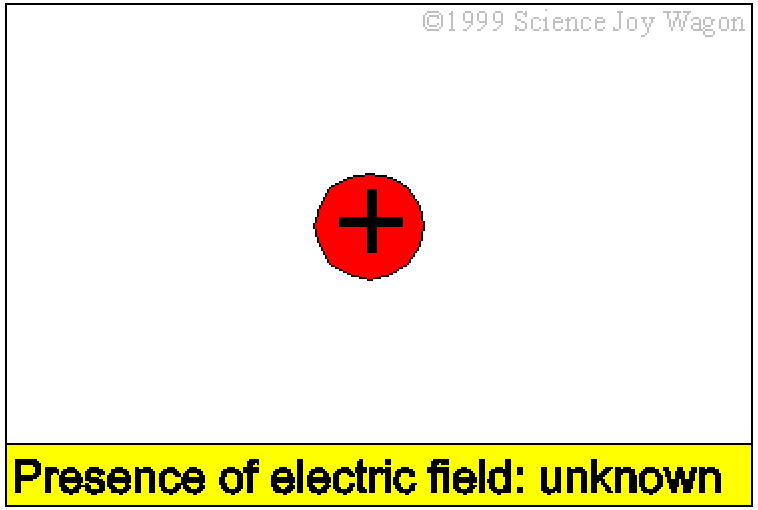
The electric field can exist independent of whether there is a charged particle around to "feel" it.

Remember: the electric field direction is the direction a + charge would feel a force.



A + charge would be repelled by another + charge.

Therefore, the direction of the electric field is away from positive (and towards negative).



http://regentsprep.org/Regents/physics/phys03/afieldint/default.htm

Gravitational Fields

The idea of a field is not new to you. You already experienced fields (gravitational) in Classical Mechanics.

$$\vec{F}_G = G\frac{m_1 m_2}{r_{12}^2}, \text{attractive}$$

$$\vec{g}(\vec{r}) = \frac{\vec{F}_G}{m}$$

Units of g are actually N/kg!

$\vec{g}(\vec{r})$ is the local gravitational field.

On earth, it is about 9.8 N/kg, directed towards the center of the earth.

A particle with mass modifies the properties of the space around it.

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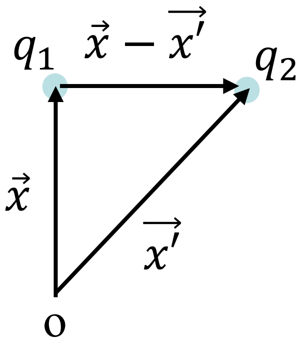
Coulomb law for two charges: Electric Field



$\vec{x} \equiv \boldsymbol{x}$

Force on a charge q_1 due to a charge q_2 at rest at a distance r:

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Coulomb Law (for 2 charges)

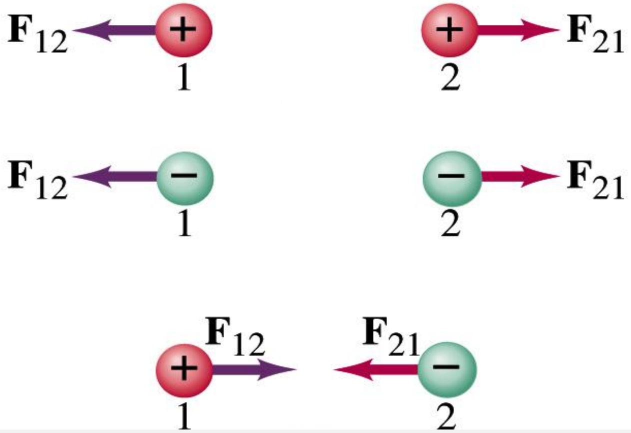
Vacuum Permettivity:
$\varepsilon_0 \cong 8.8 \times 10^{-12}\ \mathrm{Fm}^{-1}$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}$$

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

F_{12} = force on 1 due to 2

F_{21} = force on 2 due to 1



A field is a force waiting to happen !
(R. Shankar-Fundamentals of Physics II)

Electric Field

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r_{12}^2}\hat{\mathbf{r}}_{12} = q_1\mathbf{E}$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Positive Test Charge

Source Charge

To make it easy to calculate the Coulomb force on any charge at position **r**, the **F** expression can be divided by q_1 leaving an expression that only depends on the other charge (the source charge).
The electric field is the electric force of the test charge per unit charge.

The Electric Field Due to a Point Charge

Coulomb's law says

$$F_{12} = k\frac{|q_1 q_2|}{r_{12}^2},$$

treat q_1 as source charge and q_2 as test charge, divide by q_2, the electric field due to point charge q_1 is

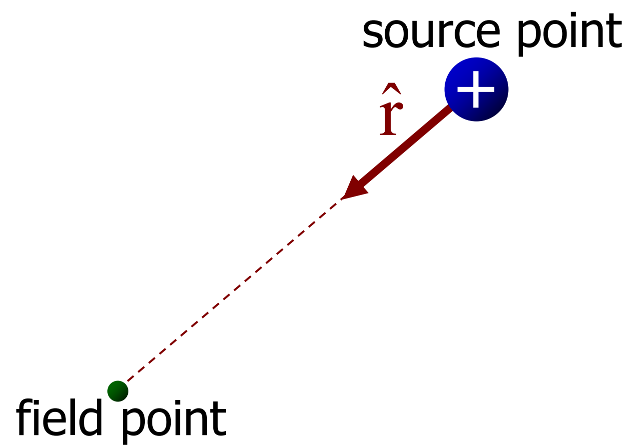
$$|\vec{E}_{q_1}| = k\frac{|q_1|}{{r_{12}}^2}$$

or, generally

$$E = k\frac{|q|}{r^2}$$

This is your third **starting equation.**

If we define $\hat{r}$ as a unit vector from the source point to the field point…



…then the **vector** equation for the electric field of a point charge becomes:

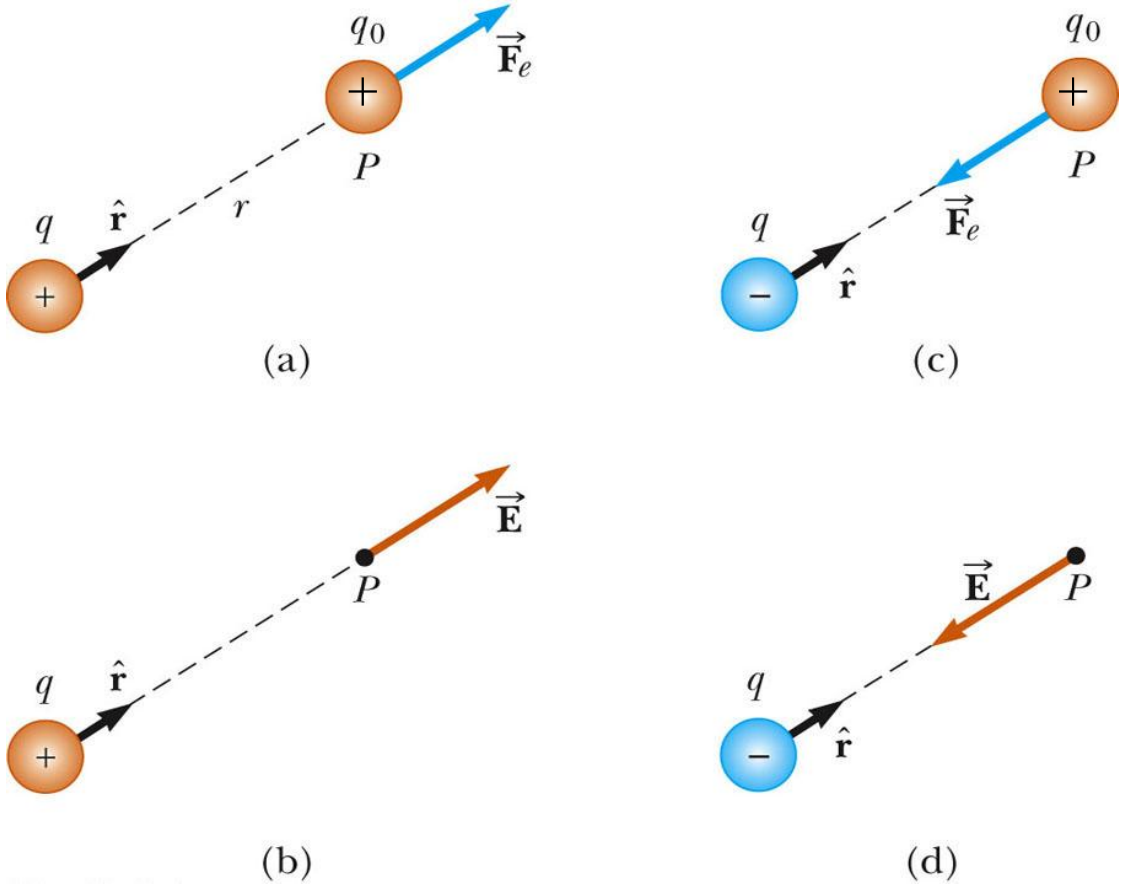
$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

You may start with either equation for the electric field (this one or the one on the previous slide). **But only use this one if you REALLY know what you are doing!**

Force F and field E for two charges

Force **F**

Field **E**



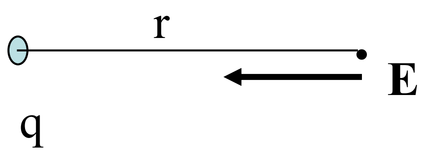
Electric field is directed FROM positive point charge
but TOWARD negative point charge

The existence of the electric field is a property of the source charge. Test charge acts as a detector of the electric force.

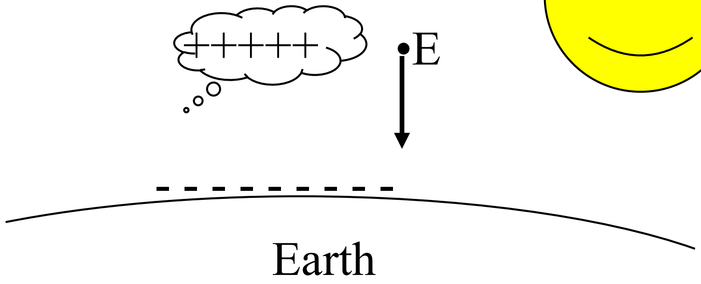
Typical Electric Fields (SI Units)

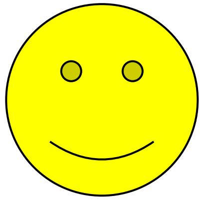
1N / C = Volt/meter

- 1 cm away from 1 nC of negative charge
  - $E = kq/r^2 = 10^{10} * 10^{-9} / 10^{-4} = 10^5$ N /C
  - $N.m^2/C^2 \;\; C / m^2 = N/C$



- Fair weather atmospheric electricity = **100 N/C** downward
  100 km high in the ionosphere





- Field due to a proton at the location of the electron in the H atom. The radius of the electron orbit is $0.5*10^{-10}$ m.
  - $E = kq/r^2 = 10^{10} * 1.6*10^{-19} / (0.5 * 10^{-10})^2 = 4*10^{11}$ N /C

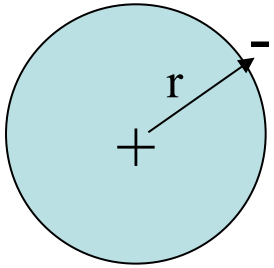
For comparison, air begins to break down and conduct electricity at about

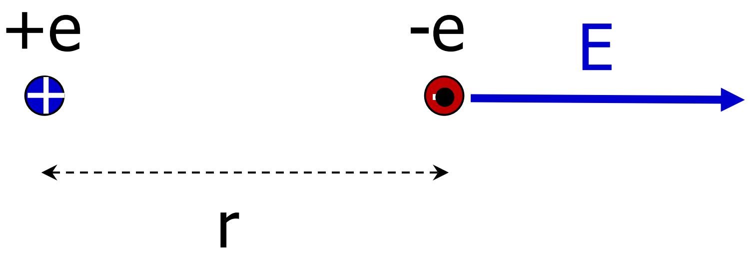
30 kV/cm,

or

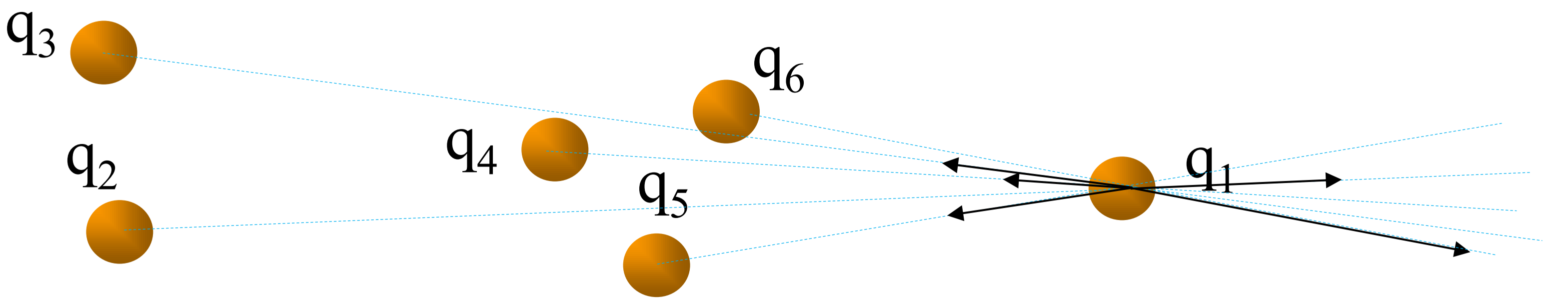
$3x10^6$ V/m.

Hydrogen atom





Force and Electric Field from multiple charges: superposition



$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r_{12}^2}\hat{\mathbf{r}}_{12} = q_1\mathbf{E}$$

Force on q_1 due to the field $\mathbf{E}$ generated by q_2

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Electric field $\mathbf{E}$ on q_1 generated by q_2

$\Rightarrow$

$$\mathbf{F} = \sum_{i=2\ldots n}\frac{1}{4\pi\varepsilon_0}\frac{q_i q_1}{r_{1i}^2}\hat{\mathbf{r}}_{1i} = q_1\sum_{i=2..n}\mathbf{E}_i = q_1\mathbf{E}$$

Force on q_1 due to the field $\mathbf{E}$ generated by $q_2, q_3, q_4,.. q_n$

$$\mathbf{E} = \sum_{i=2\ldots n}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i} = \sum_{i=2..n}\mathbf{E}_i$$

Total electric field $\mathbf{E}$ on q_1 generated by the fields **Ei** produced by the charges $q_2, q_3, q_4,.. q_n$

To calculate the electric field at a point P due to a small number of point charges, we:

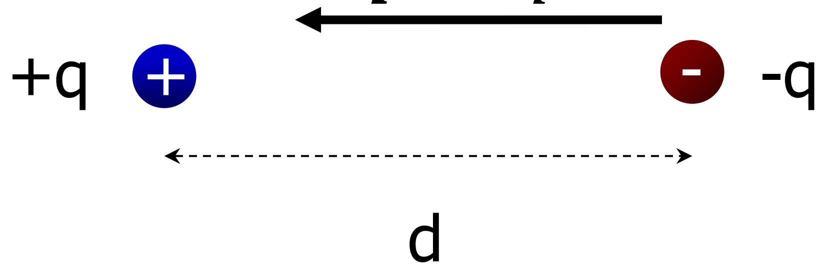
1) calculate the electric field vectors at P individually;

2) then add them vectorially.

A Dipole

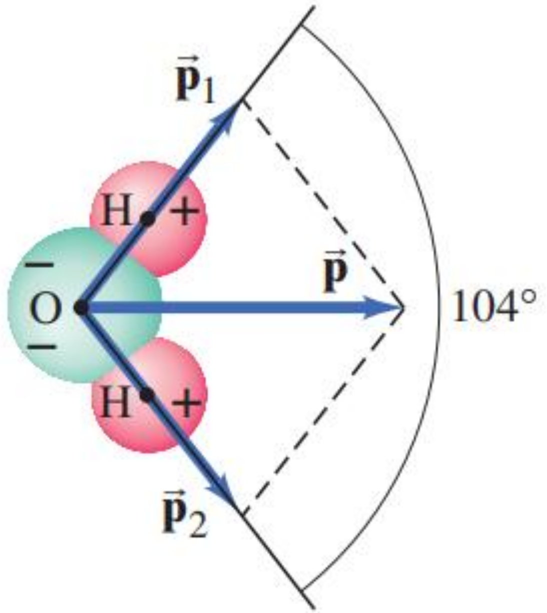
A combination of two electric charges with equal magnitude and opposite sign, separated by a fixed distance, is called a dipole.

$$\vec{\boldsymbol{p}} = q\vec{\boldsymbol{d}}$$



A dipole consists of equal but opposite charges, and separated by a distance d.
The dipole moment is $\vec{\boldsymbol{p}} = q\vec{\boldsymbol{d}}$ and points from the negative to the positive charge.

Dipoles are "everywhere" in nature.

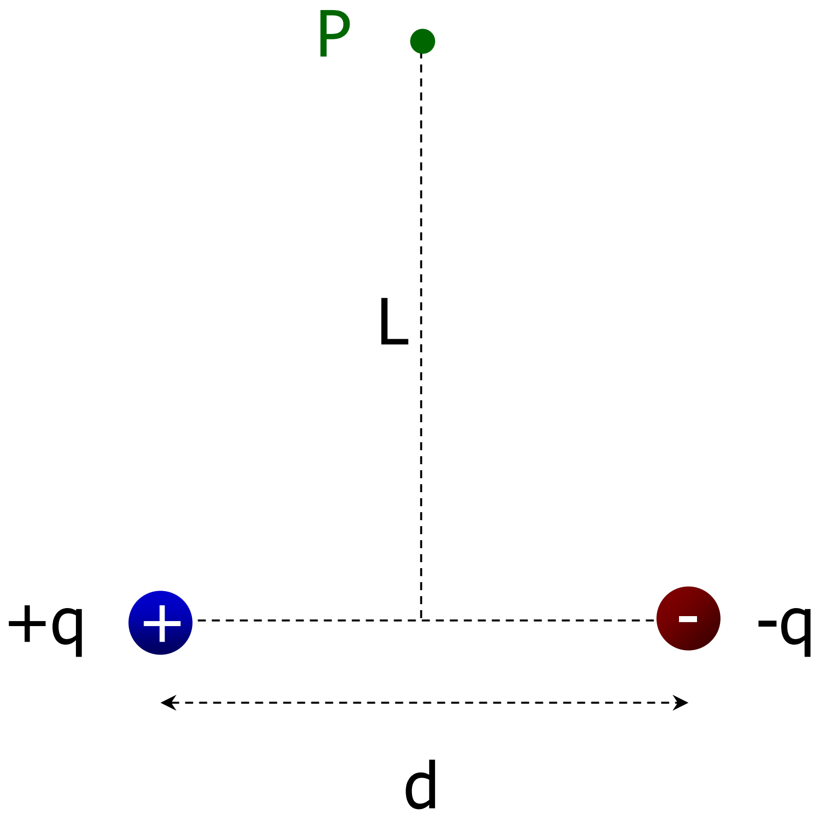


- A molecule has a permanent dipole moment that points in the direction from the center of negative charge to the center of positive charge.
- In the water molecule the electrons spend more time around the oxygen atom than around the two hydrogen atoms.

This is an **electric** dipole. Later in the course we'll study magnetic dipoles.

The Electric Field of a Dipole

Example: calculate the electric field at point P, which lies on the perpendicular bisector a distance L from a dipole of charge q.



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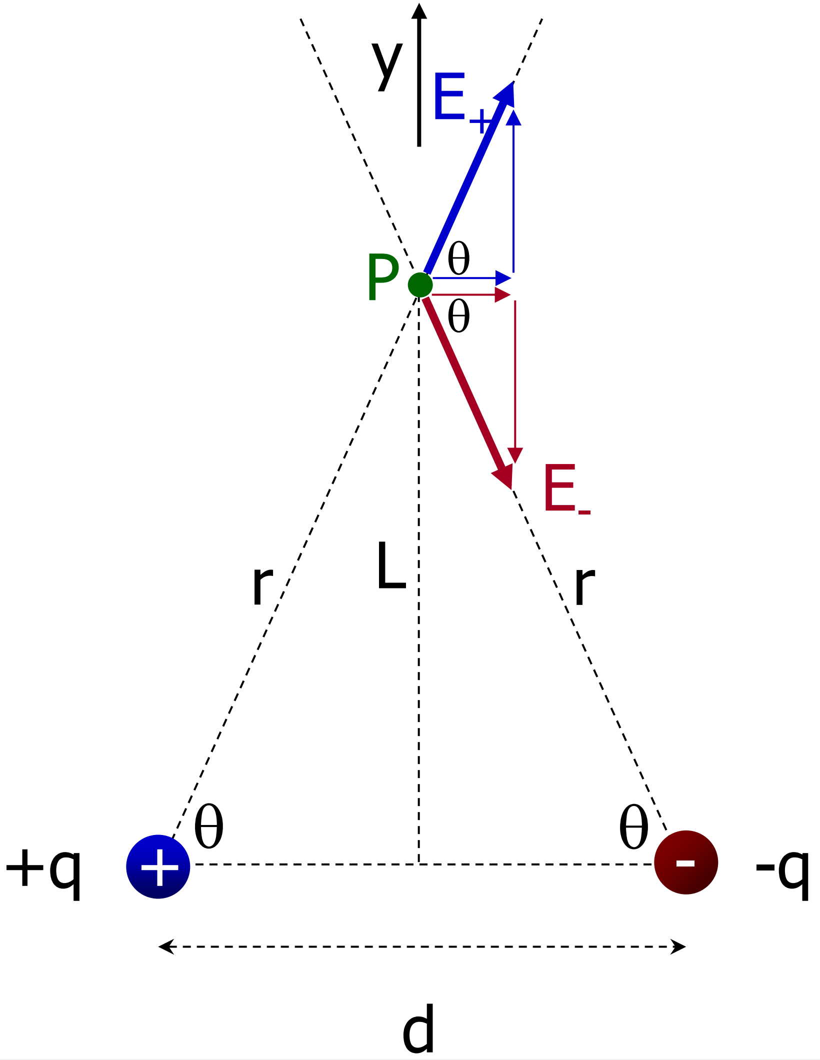
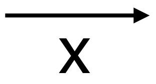


$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$E_y = 0 \quad \text{(symmetry)}$$

$$E_x = 2E_{+,x} \quad \text{(symmetry)}$$

$$E_x = +2E_+ \cos\theta$$



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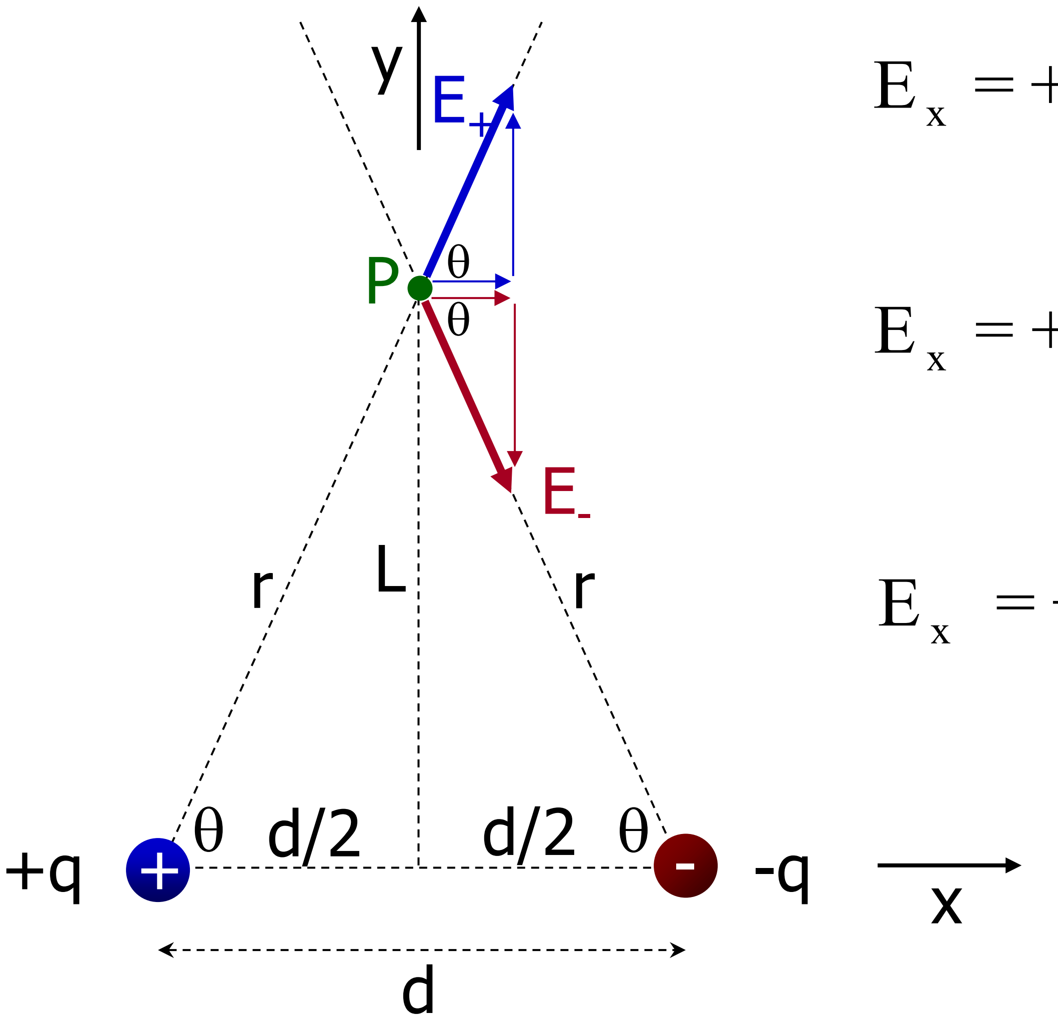


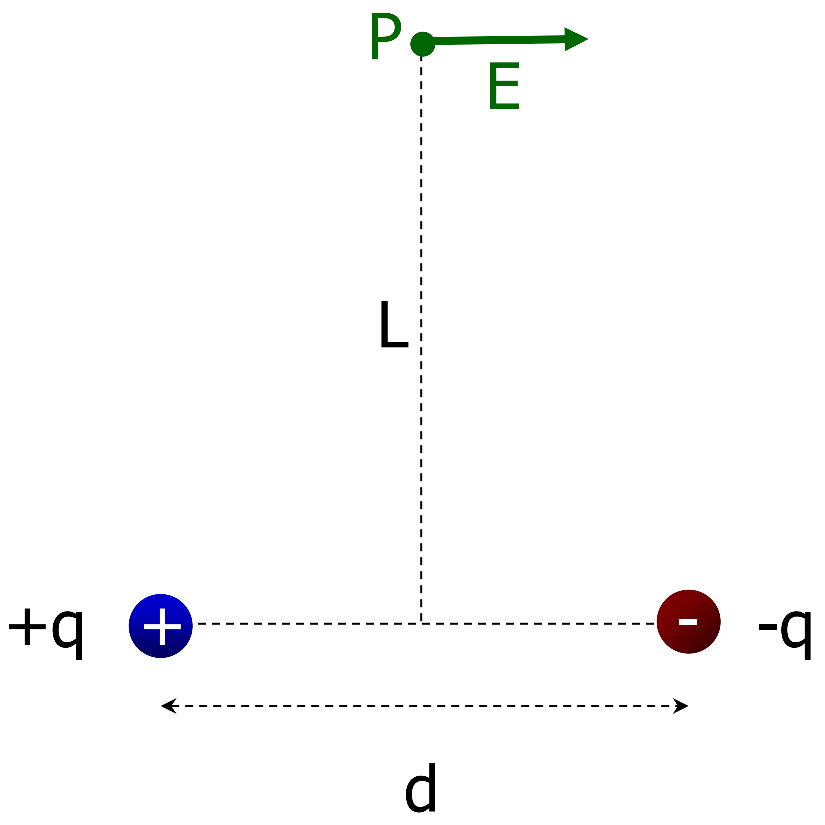
$$E_x = +2E_+ \cos\theta$$

$$E_x = +2E_+ \frac{d/2}{r} = +E_+ \frac{d}{r}$$

$$E_x = +\frac{k|+q|}{r^2}\frac{d}{r} = \frac{kqd}{r^3}$$

$$\vec{E} = \frac{qd}{4\pi\varepsilon_0 r^3}\hat{i}$$

"Charge on dipole" is positive by convention, so no absolute value signs needed around q.

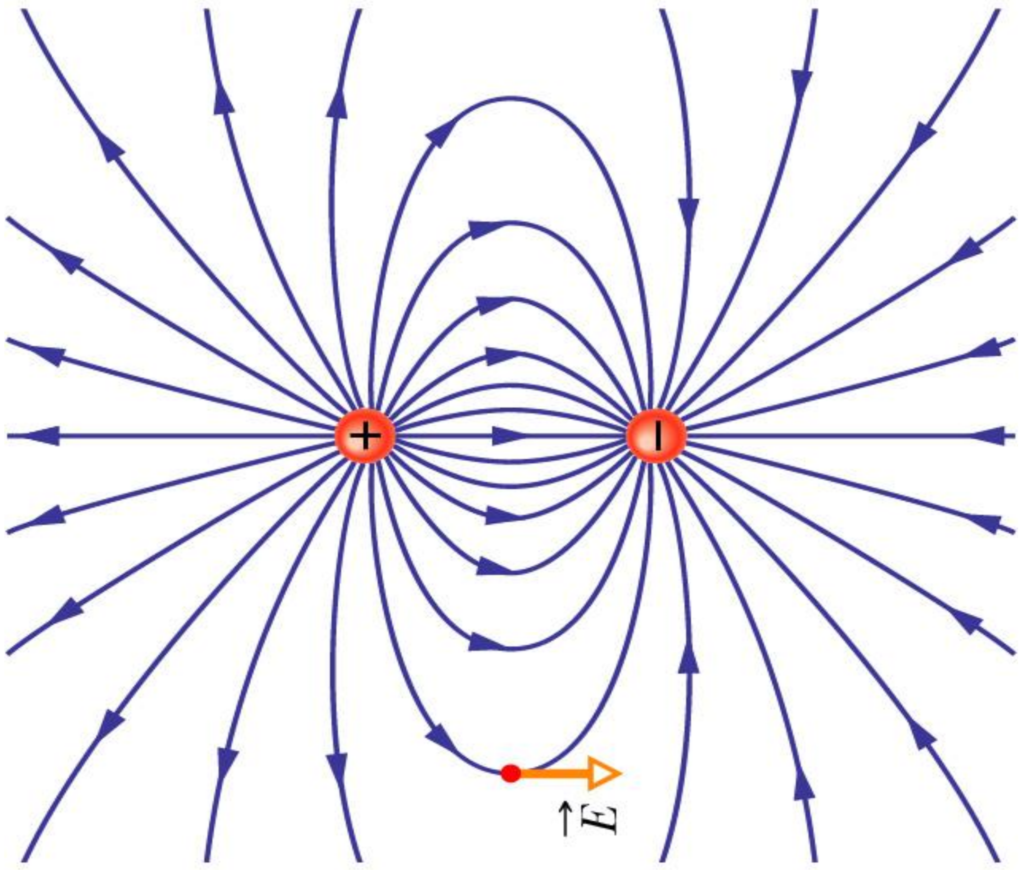


$$E = \frac{qd}{4\pi\varepsilon_o r^3}$$

Caution!

The above equation for E applies only to points along the perpendicular bisector of the dipole.

Electric field lines of an electric dipole.

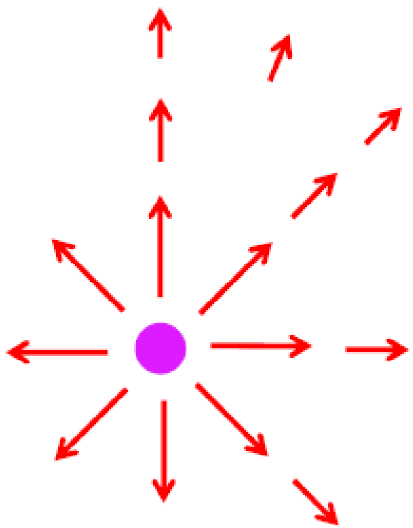


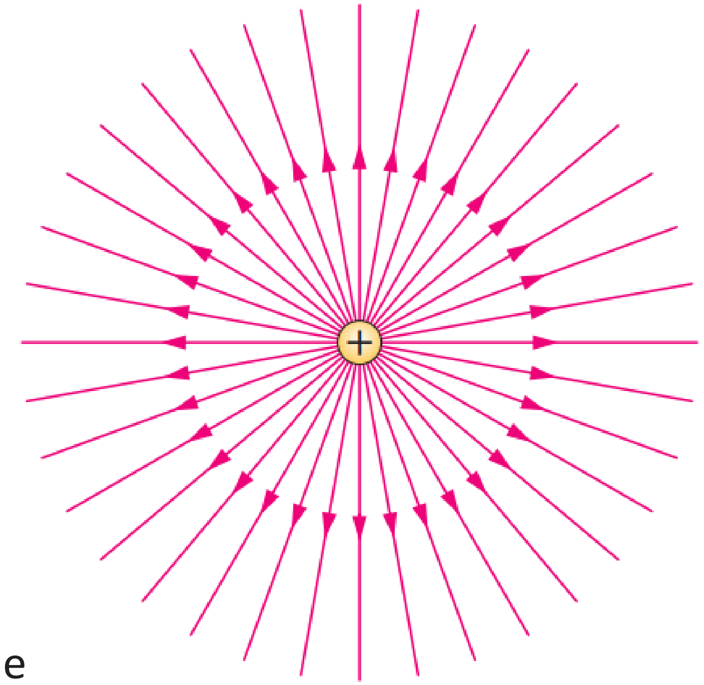
The field decreases more rapidly for a dipole than for a single point charge ($1/r^3$ instead $1/r^2$).

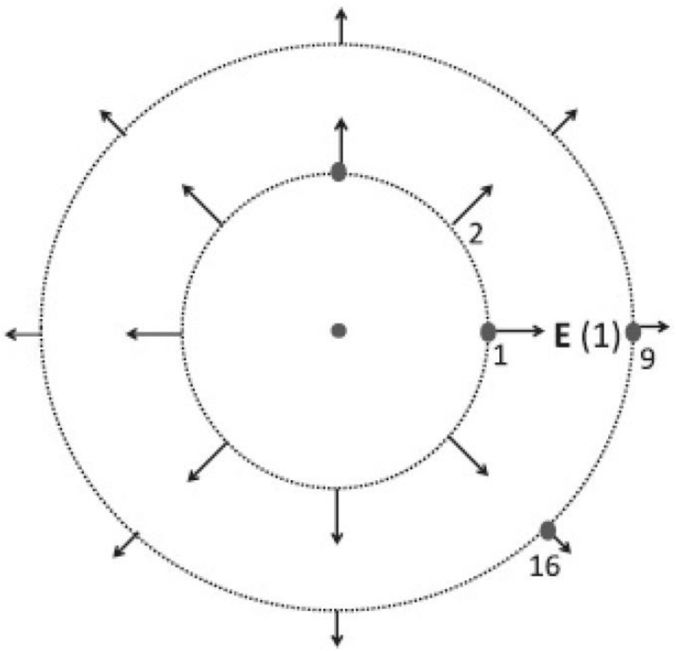
Visualizing the Electric Field

"field lines" are a way to depict electromagnetic and other vector fields

For a single point charge, we can easily draw vectors at various points indicating the strength of the field there:





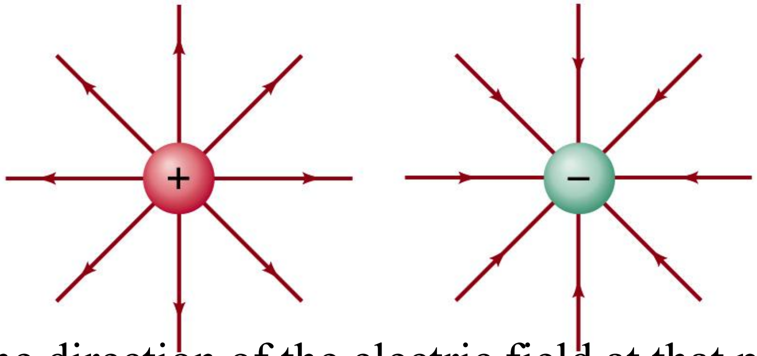


A **vector field** defines a direction and magnitude at each point in space. A **field line** for that vector field may be constructed by starting at a point and tracing a line through space that follows the direction of the vector field, by making the field line **tangent** to the field vector at each point.

They are a graphic concept used to draw pictures as an aid to develop intuition about its behavior.

The "field lines" (also "lines of force") of the E field

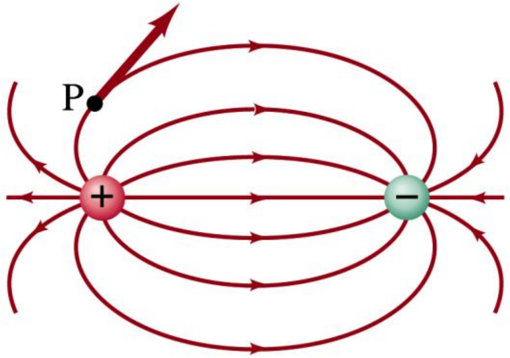
Electric field lines are **not path of particles**!
Electric field lines represent the field at various locations. Except in very special cases, they do not represent the path of a charged particle moving in an electric field.

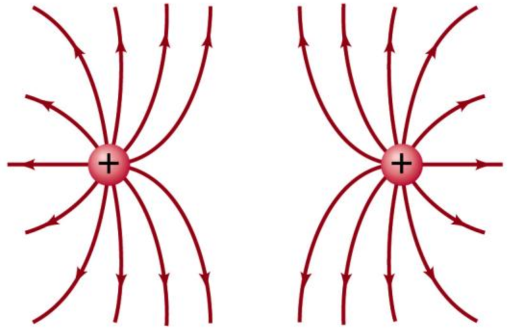


1. They are tangent at each point to the field vector **E;**

The tangent to the electric field line passing through any point in space gives the direction of the electric field at that point

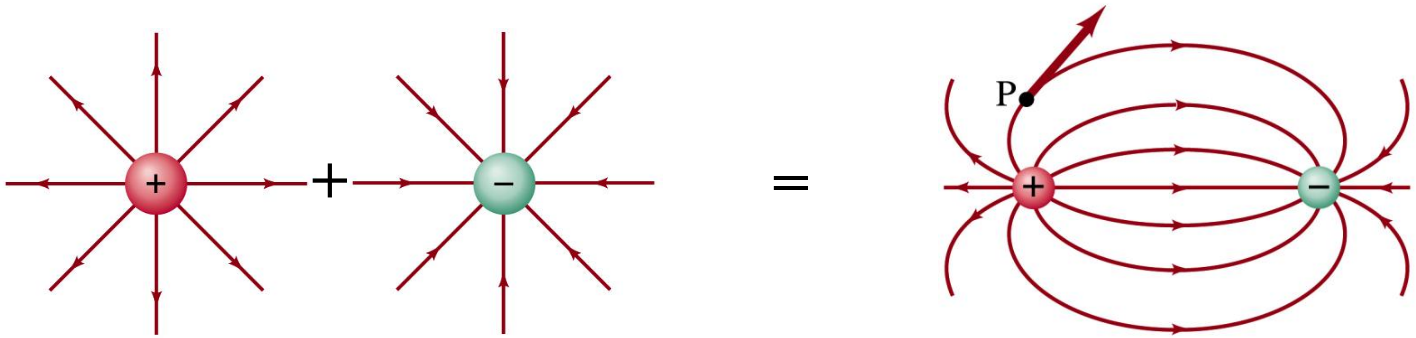
2. The lines must begin on a positive charge and terminate on a negative charge.
3. In the case of excess of one type of charge, some lines will begine or end infinitely far away.
4. They can never meet; No two field lines can cross
5. Line density (number of lines that cross a unit area) is proportional to field strength **E**;
6. The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge.
7. At large distances from a system of charges, the lines become isotropic and radial as from a single point charge equal to the net charge of the system.



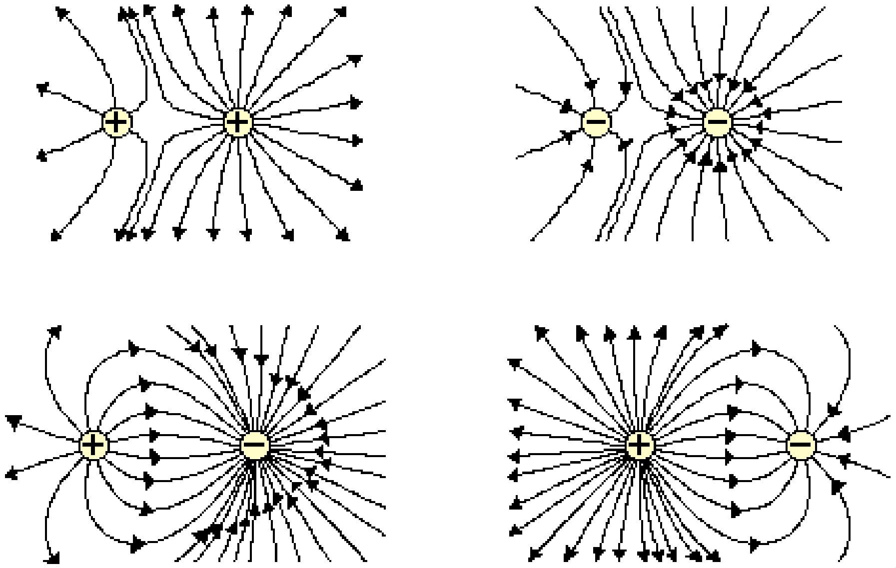


The "field lines" (also "lines of force") of the E field

Note 1: The principle of superposition is not "obvious" when using the representation of field lines.
The field lines do not overlap. One must calculate the total field which will be tangential to the field lines at each point.



Note 2: Electric field lines for objects with unequal amount of charge.

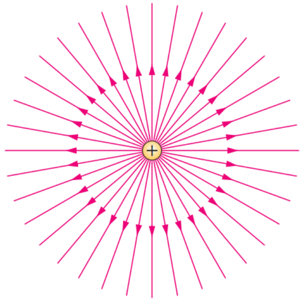


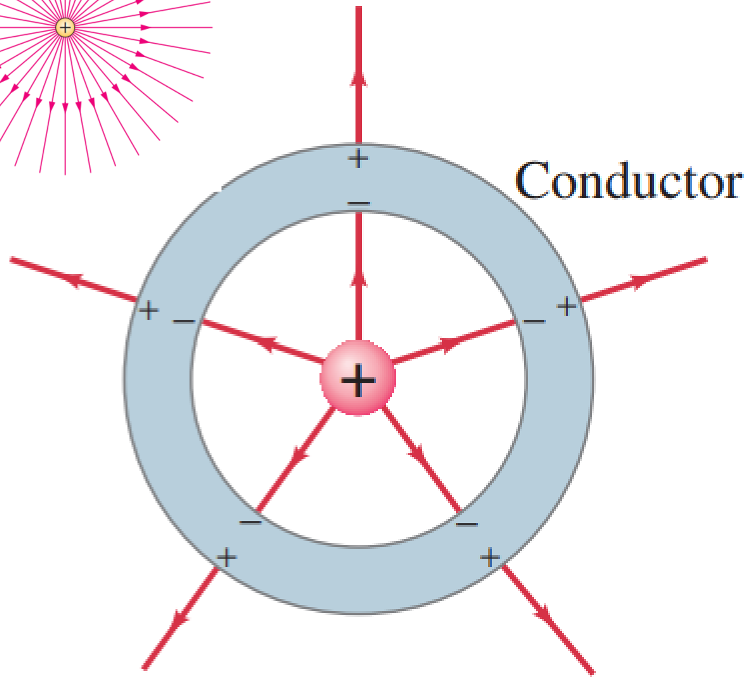
DEMO

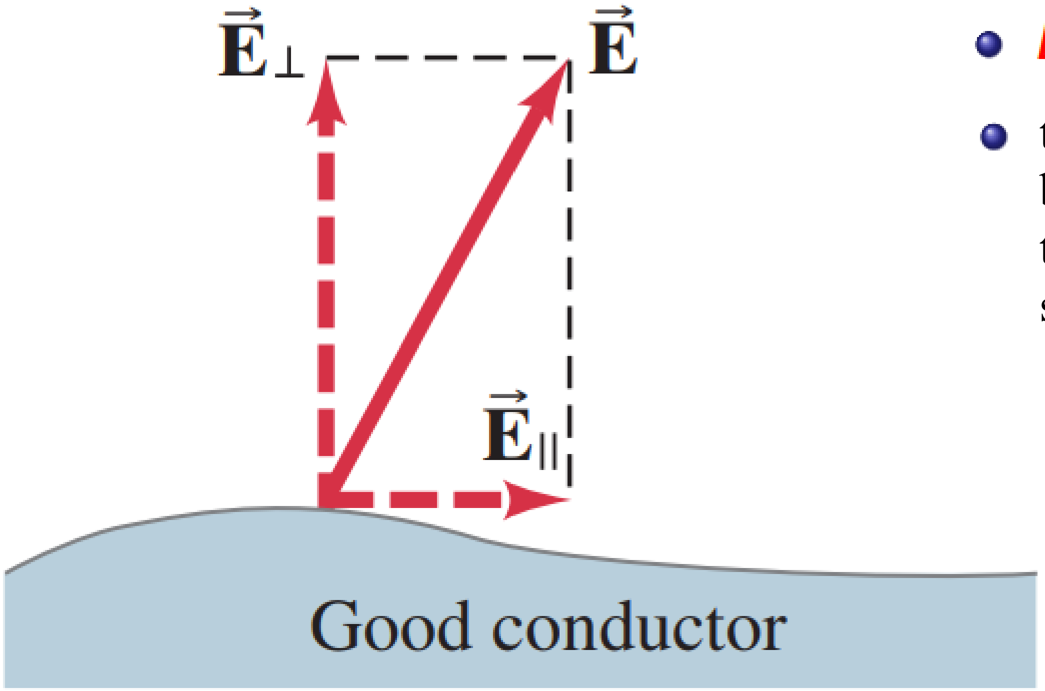
https://auditoires-physique.epfl.ch/experiment/409/lignes-de-champ-electrostatiques-semoule

Electric Field of a Conductor

The electric field inside a conductor is zero in the static situation: otherwise, the free electrons would move and arrange so that there is no field and hence no force acting on them







In the static case:

- $\vec{E}_\parallel = 0$
- the electric field must be perpendicular to the conductor's surface: $\vec{E} = \vec{E}_\perp$

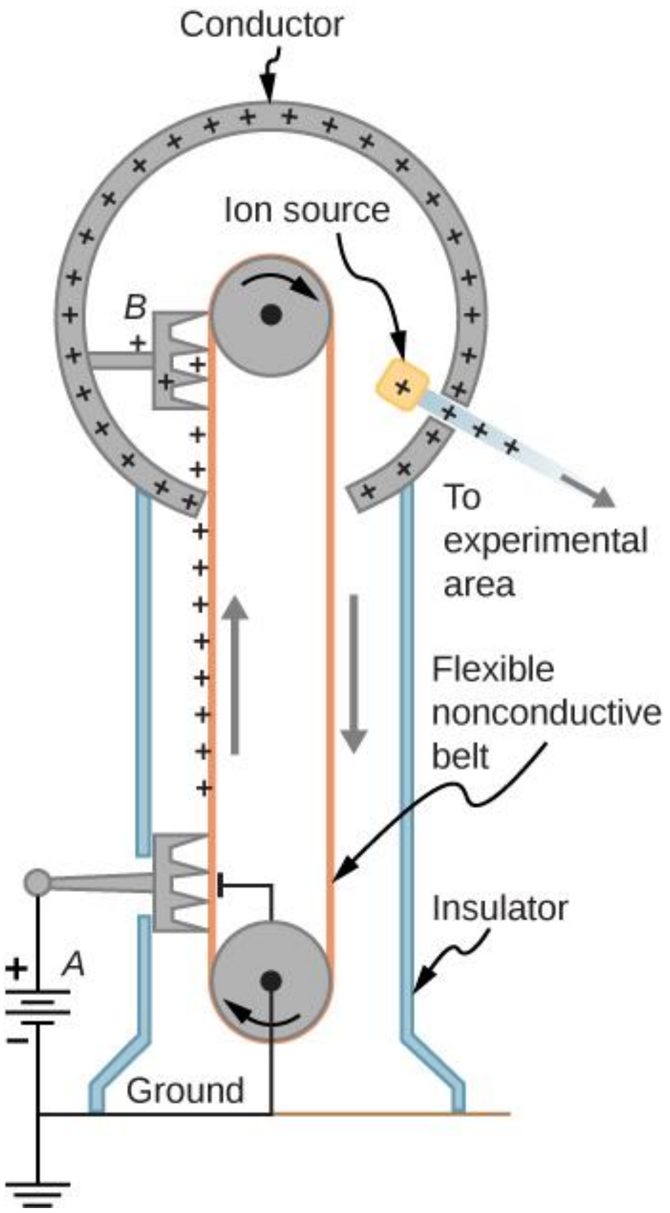
Consequence 1: any net charge on a conductor distributes itself on the surface

Consequence 2: the electric field is always perpendicular to the surface outside of a conductor

DEMO

https://auditoires-physique.epfl.ch/experiment/417/sphere-chargee-avec-fils

Van der Graaf Generator



Lecture 2 agenda:

RECAP

Electric Charge.
Review of some things you hopefully learned in high school.

Coulomb’s Law (electrical force between charged particles).
You must be able to calculate the electrical forces between one or more charged particles.

The Electric field.
You must be able to calculate the force on a charged particle in an electric field.

Electric field due to point charges.
You must be able to calculate electric field of one or more point charges.

Electric field due to a continuous charge distribution.
You must be able to calculate electric field of a charge distribution with some symmetries.

Electric Field of Continuous Charge Distributions

Often we will consider problems where the charges are **continuous**. The **superposition principle** allows for the calculation of the electric field due to a continuous distribution of charge

A continuous distribution of charge may be treated as a sum of infinitesimal (point) charges.

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

The total field is then the integral of the infinitesimal fields due to each bit of charge:

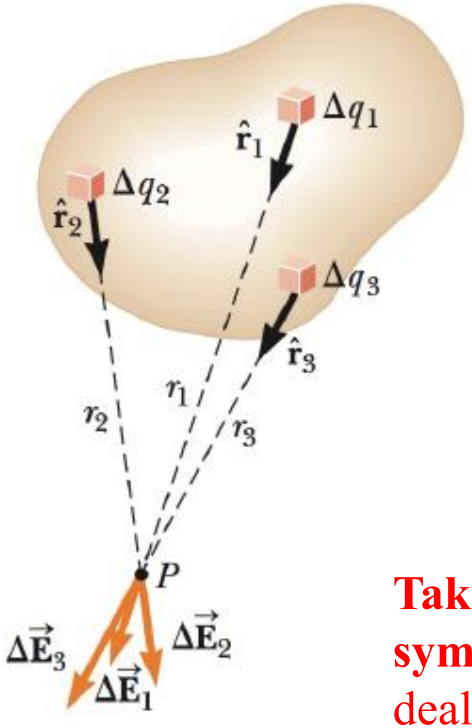
$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charges can be in volume, on surface or over length:

$$\rho \equiv \frac{Q}{V} \qquad \sigma \equiv \frac{Q}{A} \qquad \lambda \equiv \frac{Q}{\ell}$$

The respective differentials of charge:

$$dq = \rho\, dV \quad dq = \sigma\, dA \quad dq = \lambda\, d\ell$$

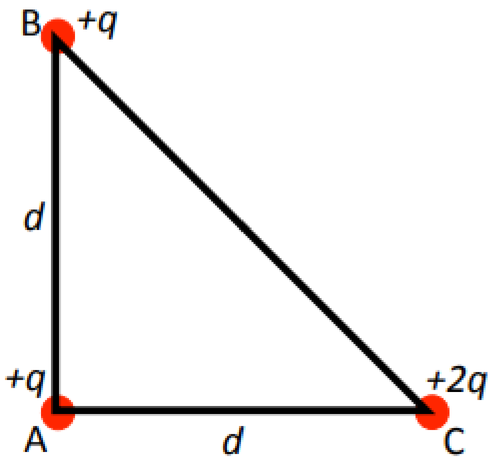


Take advantage of any symmetry when dealing with either a distribution of point charges or a continuous charge distribution.

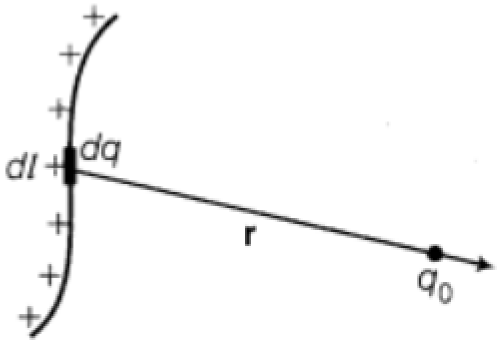
Different Charge Distributions

Point charge

- is a hypothetical charge located at a single point in space,
- it's a mathematic concept to simplify discussion,
- in practice, any charge is a point charge if its spatial distribution size is much smaller than the distance at which the charge is considered



Charge density (for a Continuous Charge Distributions) is the amount of electric charge per unit length, surface area, or volume.

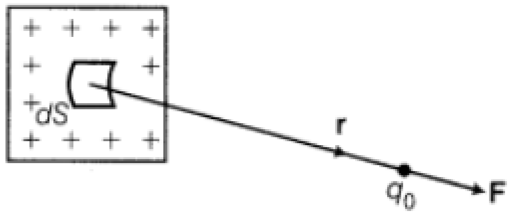


line charge density λ: used when only one coordinate is important (wires);

$$dq = \lambda dl$$

$$Q = \int_x \lambda dx = \int_L \lambda dl$$

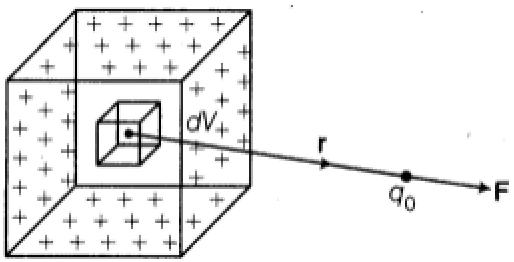
Different Charge Distributions



surface charge density σ: used when two coordinates are important (plane, sphere, cylinder)

$$\mathrm{d}q = \sigma \mathrm{d}S$$

$$Q = \int_x \int_y \sigma \, \mathrm{d}y \, \mathrm{d}x = \int_S \sigma \mathrm{d}S$$



volume charge density ρ: most realistic

- depends on three coordinates
- used for charged volumes
- or when no details are specified (general laws)

$$\mathrm{d}q = \rho \mathrm{d}V$$

$$Q = \int_x \int_y \int_z \rho(x, y, z) \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = \int_V \rho \, \mathrm{d}V$$

Problem-Solving Strategies

We have discussed how electric field can be calculated for both the discrete and continuous charge distributions.
For the former, we apply the superposition principle:

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \ .$$

For the latter, we must evaluate the vector integral

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \hat{\mathbf{r}} \ ,$$

where r is the distance from dq to the field point P and $\hat{\boldsymbol{r}}$ is the corresponding unit vector. To complete the integration, we shall follow the procedures outlined below:

(1) Start with $d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{\mathbf{r}}$.

(2) Rewrite the charge element dq as

$$dq = \begin{cases} \lambda \, d\ell & \text{(length)} \\ \sigma \, dA & \text{(area)} \\ \rho \, dV & \text{(volume)} \end{cases} ,$$

depending on whether the charge is distributed over a length, an area, or a volume.

Problem-Solving Strategies

(3) Substitute dq into the expression for $d\vec{\mathbf{E}}$.

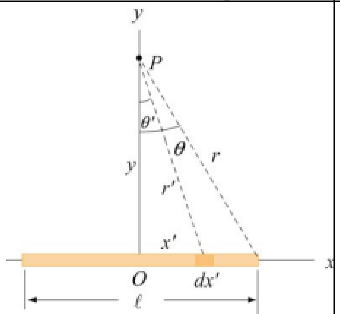
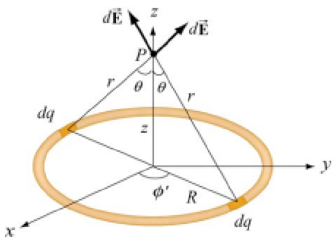
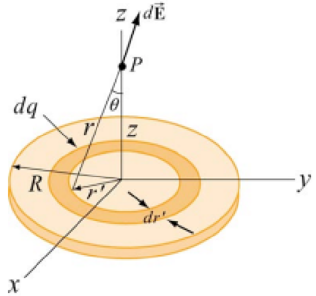
(4) Specify an appropriate coordinate system (Cartesian, cylindrical or spherical) and express the differential element ($d\ell$, dA, or dV) and r in terms of the coordinates (see Table 2.1 below for summary.)

	Cartesian (x, y, z)	Cylindrical (ρ, ϕ, z)	Spherical (r, θ, ϕ)
dl	$dx,\ dy,\ dz$	$d\rho,\ \rho d\phi,\ dz$	$dr,\ r\,d\theta,\ r\sin\theta\,d\phi$
dA	$dx\,dy,\ dy\,dz,\ dz\,dx$	$d\rho\,dz,\ \rho\,d\phi\,dz,\ \rho\,d\phi\,d\rho$	$r\,dr\,d\theta,\ r\sin\theta\,dr\,d\phi,\ r^2\sin\theta\,d\theta\,d\phi$
dV	$dx\,dy\,dz$	$\rho\,d\rho\,d\phi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\phi$

(5) Rewrite $d\vec{\mathbf{E}}$ in terms of the integration variable(s), and apply symmetry argument to identify non-vanishing component(s) of the electric field.

(6) Complete the integration to obtain $\vec{\mathbf{E}}$.

In the Table below we illustrate how the above methodologies can be utilized to compute the electric field for an infinite line charge, a ring of charge and a uniformly charged disk.

	Line charge	Ring of charge	Uniformly charged disk
Figure			
(2) Express dq in terms of charge density	$dq = \lambda\, dx'$	$dq = \lambda\, d\ell$	$dq = \sigma\, dA$
(3) Write down dE	$dE = k_e \dfrac{\lambda\, dx'}{r'^2}$	$dE = k_e \dfrac{\lambda\, dl}{r^2}$	$dE = k_e \dfrac{\sigma\, dA}{r^2}$
(4) Rewrite r and the differential element in terms of the appropriate coordinates	dx' $\cos\theta = \dfrac{y}{r'}$ $r' = \sqrt{x'^2 + y^2}$	$d\ell = R\, d\phi'$ $\cos\theta = \dfrac{z}{r}$ $r = \sqrt{R^2 + z^2}$	$dA = 2\pi r' dr'$ $\cos\theta = \dfrac{z}{r}$ $r = \sqrt{r'^2 + z^2}$
(5) Apply symmetry argument to identify non-vanishing component(s) of dE	$dE_y = dE\cos\theta$ $= k_e \dfrac{\lambda y dx'}{(x'^2 + y^2)^{3/2}}$	$dE_z = dE\cos\theta$ $= k_e \dfrac{\lambda R z\, d\phi'}{(R^2 + z^2)^{3/2}}$	$dE_z = dE\cos\theta$ $= k_e \dfrac{2\pi\sigma z r'\, dr'}{(r'^2 + z^2)^{3/2}}$
(6) Integrate to get E	$E_y = k_e \lambda y \int_{-\ell/2}^{+\ell/2} \dfrac{dx}{(x^2 + y^2)^{3/2}}$ $= \dfrac{2k_e\lambda}{y} \dfrac{\ell/2}{\sqrt{(\ell/2)^2 + y^2}}$	$E_z = k_e \dfrac{R\lambda z}{(R^2 + z^2)^{3/2}} \int_0^{2\pi} d\phi'$ $= k_e \dfrac{(2\pi R\lambda) z}{(R^2 + z^2)^{3/2}}$ $= k_e \dfrac{Qz}{(R^2 + z^2)^{3/2}}$	$E_z = 2\pi\sigma k_e z \int_0^R \dfrac{r'\, dr'}{(r'^2 + z^2)^{3/2}}$ $= 2\pi\sigma k_e \left(\dfrac{z}{\lvert z \rvert} - \dfrac{z}{\sqrt{z^2 + R^2}} \right)$