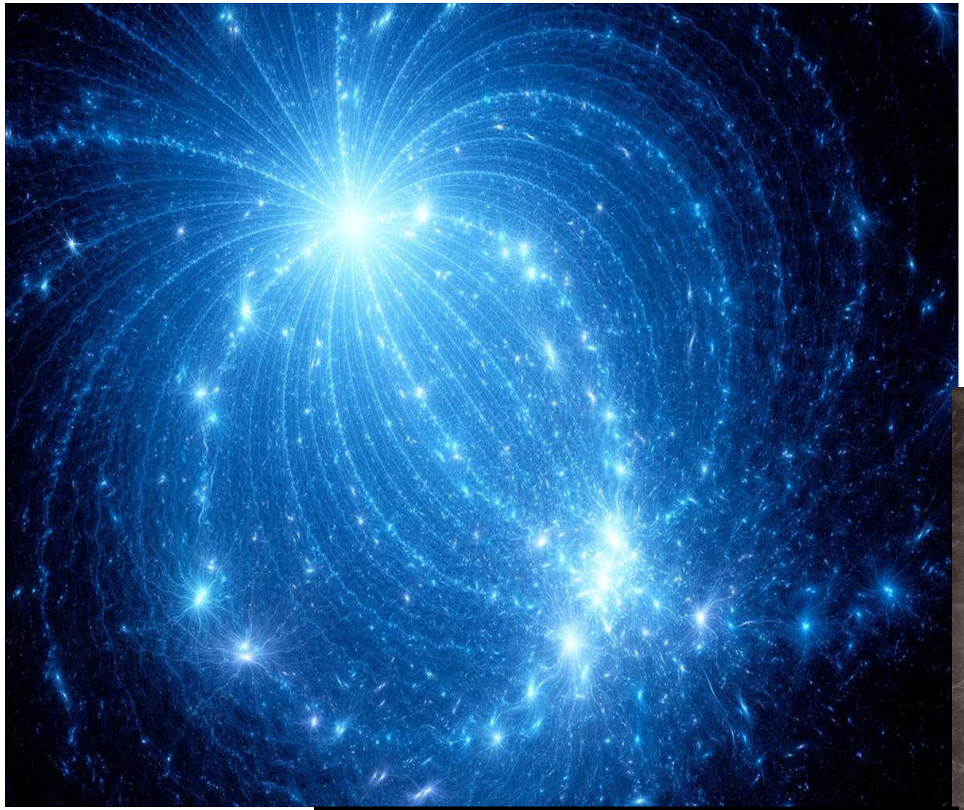
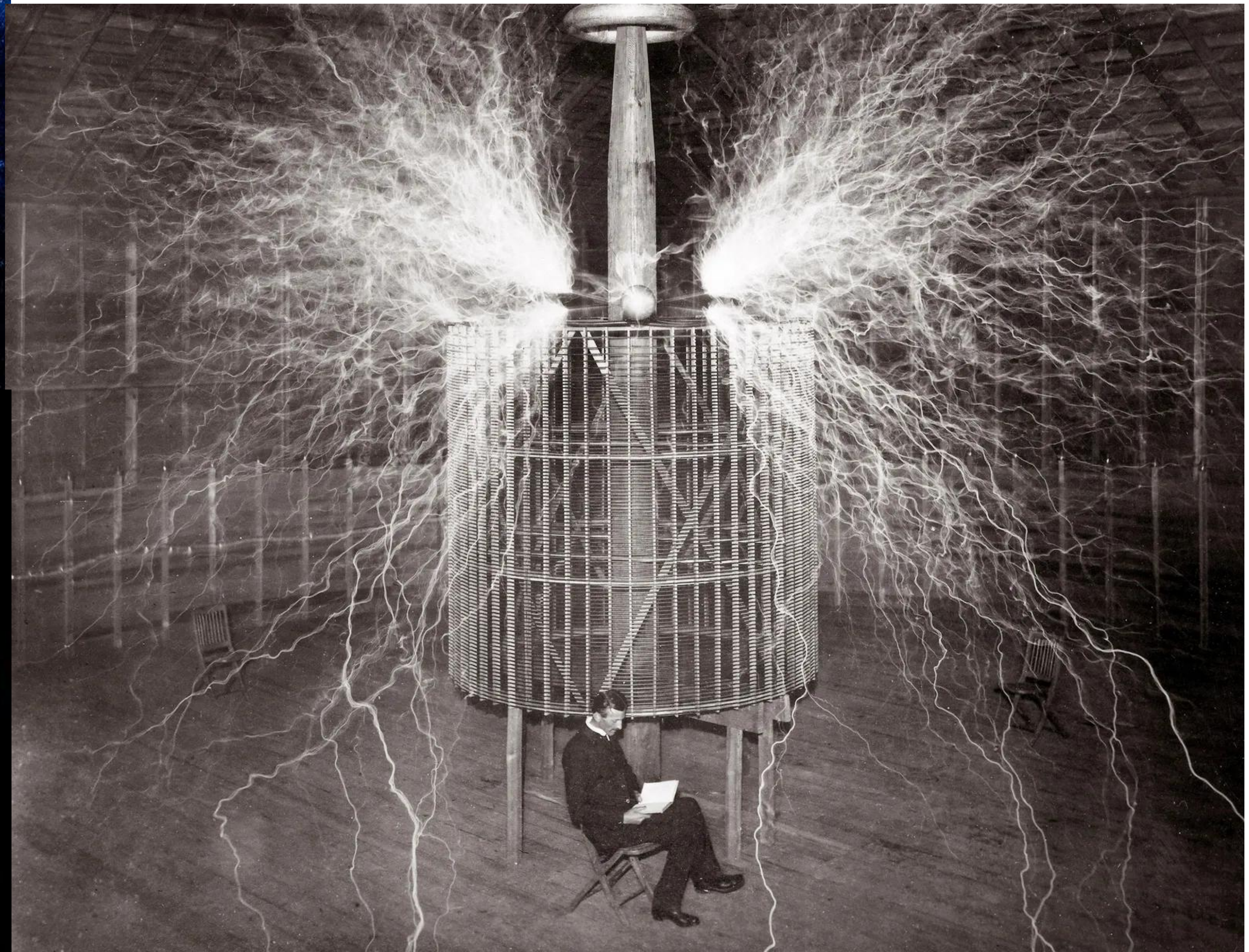


General Physics III: Electromagnetism



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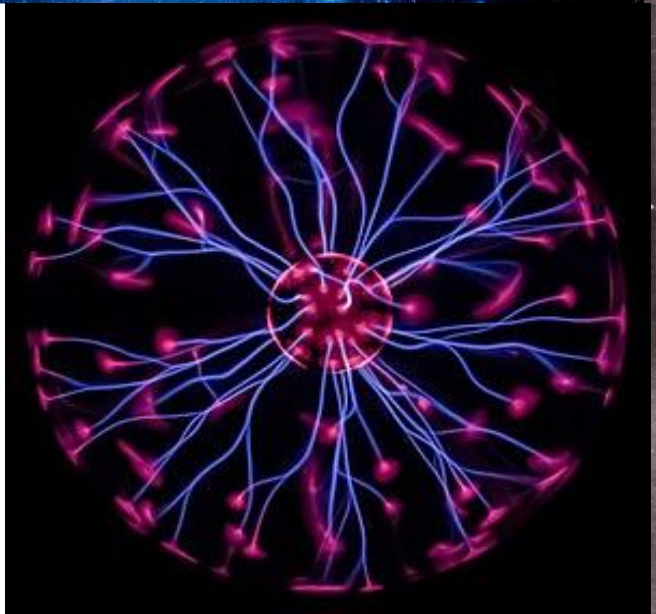
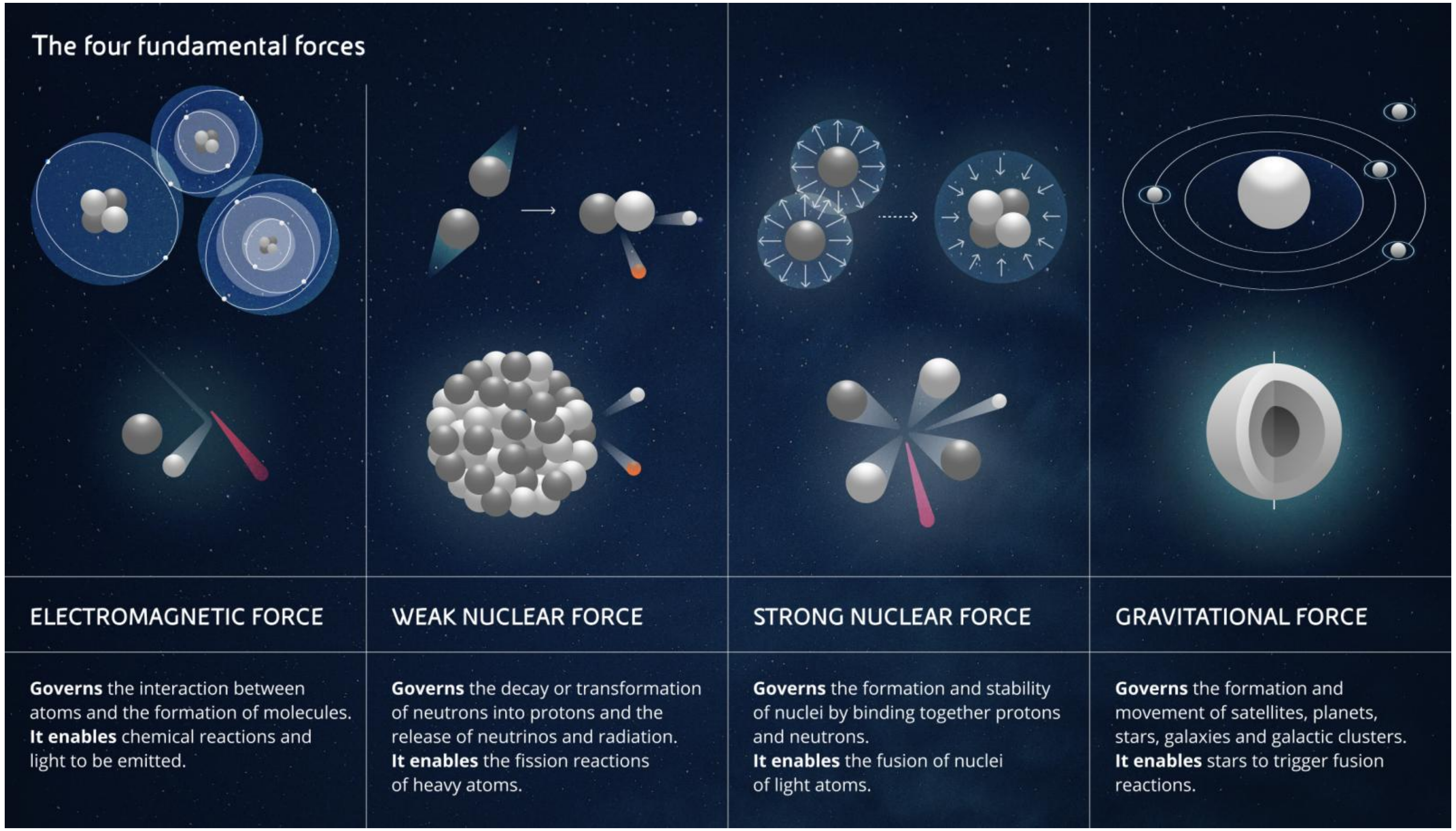


The four interaction forces

All the physical phenomena in our Universe originate from the **four fundamental forces.**



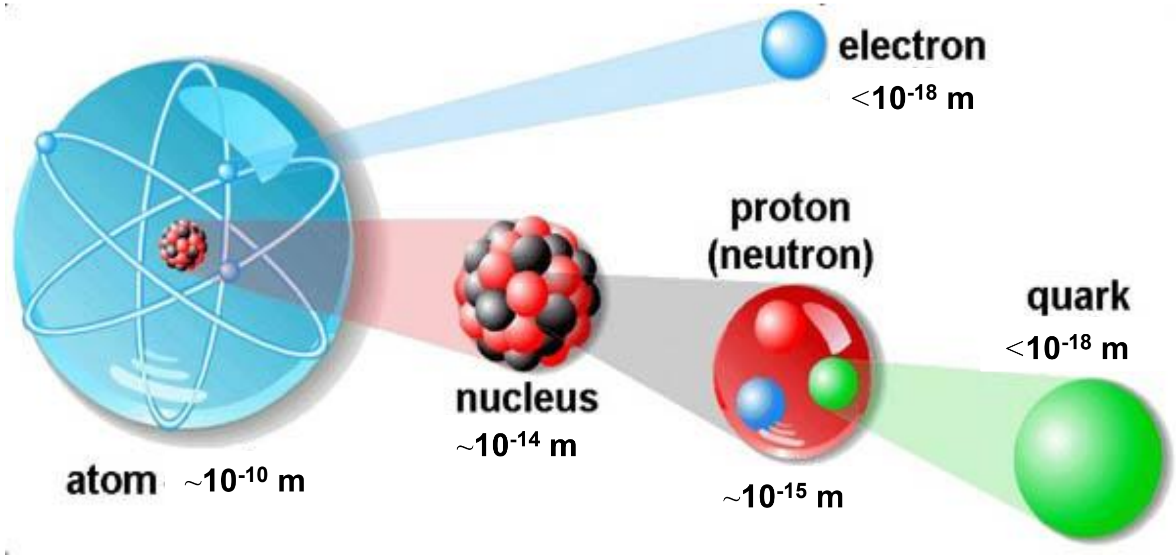
There are, to the best of our knowledge, four forces at play in the Universe. At the very largest scales — those of planets or stars or galaxies — the force of gravity dominates. At the very smallest distances, the two nuclear forces hold sway. For everything in between, it is force of electromagnetism that rules.

The four interaction forces

The revolutionary step forward in the Electromagnetism was to introduce the concept of fields to describe the interactions between two or more charged objects.

Interaction	Mediators	Range of action (m)	Dependence on the distance
Weak Nuclear	Bosons (W, Z)	10^{-18}	$1/r^7$ à $1/r^5$
Strong Nuclear	Gluons	10^{-15}	$1/r^7$
Electromagnetic	Photon	∞	$1/r^2$
Gravitational	Graviton	∞	$1/r^2$

- At the atomic scale, electromagnetism (admittedly in conjunction with some basic quantum effects) governs the interactions between atoms and molecules.
- It is the force that underlies the periodic table of elements, giving rise to all of chemistry and, through this, much of biology.
- It is the force which binds atoms together into solids and liquids. And it is the force which is responsible for the incredible range of properties that different materials exhibit.



Why to study Electromagnetism by non-physicists?

- Mandatory at EPFL, gives 4 credits
- Meet every day outside of school; curiosity
- Need to understand basic principles in many "practical" courses (e.g., Mass spectrometry, Spectroscopy)
- Need to work with basic electronics in any field of science and engineering
- The laws of electricity and magnetism play a central role in the operation of many modern devices.
- The interatomic and intermolecular forces responsible for the formation of solids and liquids are electric in nature.
- Maxwell's equations are valid over a vast length scale from subatomic dimensions to galactic dimensions. Hence, these equations are valid over a vast range of wavelengths, going from static to ultra-violet wavelengths.
- Maxwell's equations are some of the most accurate physical equations that have been validated by experiments. Electromagnetic theory has been validated to one part in a trillion.

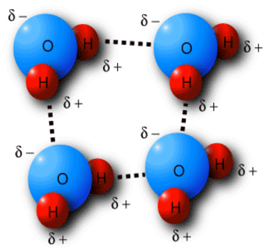
Electromagnétic Force: a Very Important Force

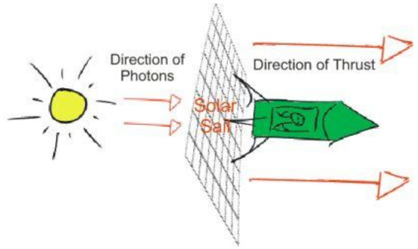
Electromagnetic forces:

The force of friction, elastic force, normal force…
The force that holds electrons in orbit around the atomic nucleus
The force that binds molecules together
The force of impact between two billiard balls…

It is hardly an exaggeration to say that we live in an electromagnetic world!

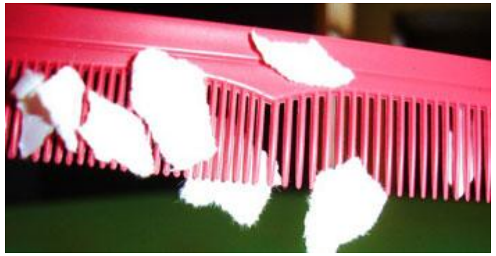
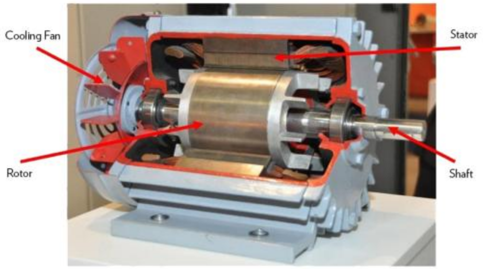
All forces we experience in everyday life, above the nuclear scale and with the exception of gravity, are electromagnetic!!



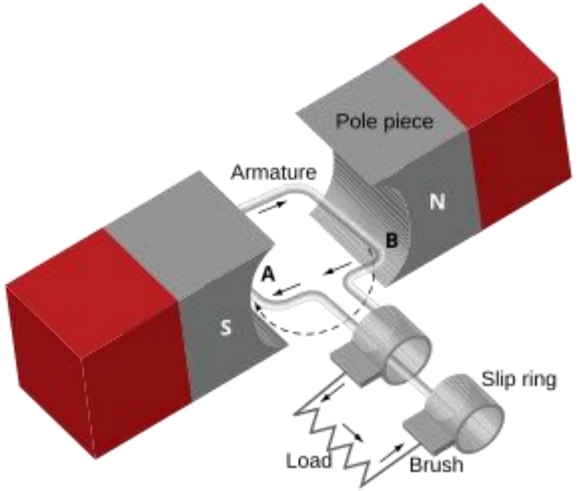




Electromagnetic Force: The Driver of the Power Industry

Electromagnetism is important for enabling the production, transmission and distribution of electricity that powers our society.
It is primarily responsible for converting energy into electrical energy, a process commonly observed in power plants.
Generators play a role in this conversion process by utilizing induction principles.
The electrical energy then channelled into the power grid, for transmission over distances.



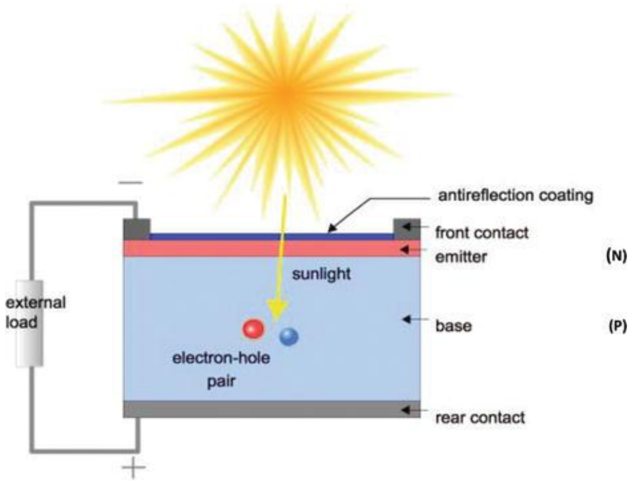
Hydroelectric Power Plants
Hydroelectric power plants utilize the kinetic energy of flowing water, to generate electricity. As water flows through the plant, it turns large turbines.

Thermal Power Plants
Thermal power plants operate by converting heat energy into electrical energy. This heat is generated by burning fossil fuels or through nuclear reactions. The heat produces steam, which drives turbines connected to a generator.

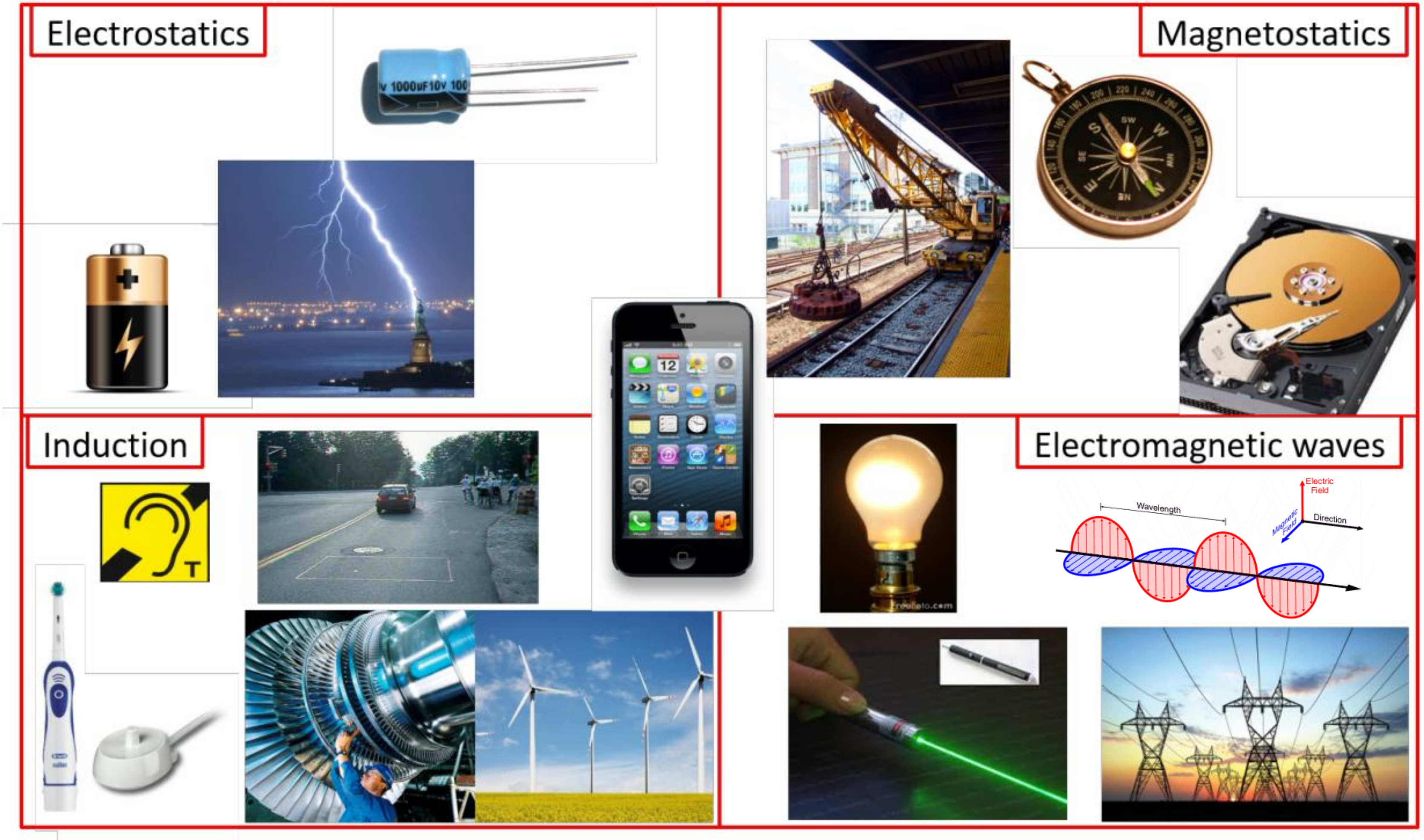
Wind Turbines
Wind turbines capture the kinetic energy of the wind. As the wind blows, it turns the blades of the turbine.

These turbines are connected to the rotor of a generator. The rotation of the rotor within a magnetic field induces an electric current through electromagnetic induction, thus generating electricity.

Solar Panels
Solar panels, while not typically generators in the traditional sense, operate on the principles of electromagnetic theory, specifically the photovoltaic effect. This effect occurs when sunlight (photons) strikes the surface of a solar cell, exciting electrons and creating an electric current.



Electromagnetism in everyday life



Rather remarkably, a full description of the force of electromagnetism is contained in four simple and elegant equations. These are known as the Maxwell equations. There are few places in physics, or indeed in any other subject, where such a richly diverse set of phenomena flows from so little. The purpose of this course is to introduce the Maxwell equations and to extract some of the many stories they contain.

Electromagnetism is a Field Theory!

A **field theory** describes physical quantities—such as forces, energy, or interactions—as functions that exist at every point in space and time. These quantities are not confined to discrete objects but are continuous and distributed throughout a region.

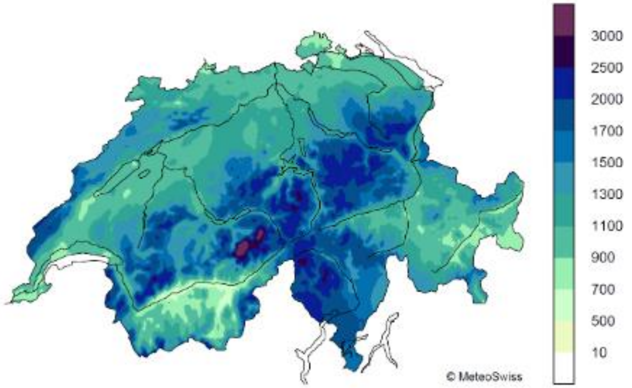
In mathematical terms, a **field** is a function that assigns a value (which can be a scalar, vector, or tensor) to every point in space and time.

Examples of fields include:

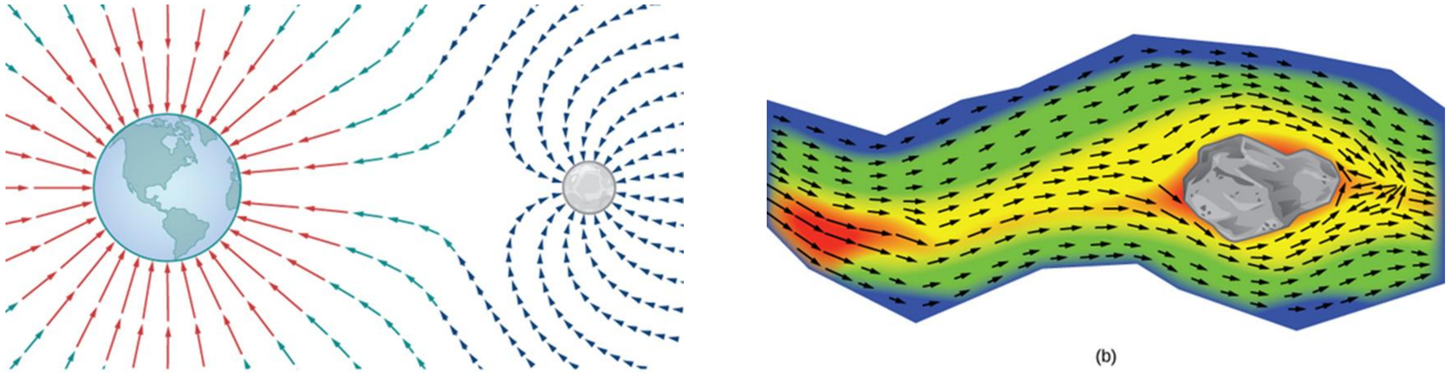
•**Scalar fields**, like temperature, which assign a single value (temperature) to every point in space.

•**Vector fields**, like the velocity of a fluid or the electric field, which assign a vector (with direction and magnitude) to every point in space.

In a **field theory**, the behavior of fields is governed by differential equations, which describe how the fields change in space and time.



Scalar Field: Annual total precipitation (mm) for the period 1991-2020



Vector Fields: (a) The gravitational field exerted by two astronomical bodies on a small object. (b) The vector velocity field of water on the surface of a river shows the varied speeds of water.

EPFL

Why is Electromagnetism a Field Theory?

Electromagnetism is a field theory because it describes electric and magnetic interactions in terms of **fields** that exist continuously throughout space and time. The behavior of these fields is governed by **Maxwell's equations**, which are a set of four differential equations that describe how electric and magnetic fields are generated and how they evolve over time.

1. The **electric field E**(**r**,t) assigns a vector (force per unit charge) to every point **r** in space and at every time t. It describes the force that would be exerted on a positive test charge placed at that point.
2. The **magnetic field B**(**r**,t) assigns a vector (force per unit current element) to every point **r** in space and time t, describing how a moving charge or current experiences a force due to magnetism.

These fields exist everywhere, even in regions where no charges or currents are present. For instance, a point in space where no particles exist can still have non-zero electric or magnetic fields, meaning the fields themselves are fundamental objects in the theory.

In earlier theories, such as Newtonian gravity, forces were described as **action-at-a-distance**, meaning objects influenced each other directly, regardless of the space between them.

However, in electromagnetism, charges and currents do not directly exert forces on distant charges and currents. Instead, they generate **electric and magnetic fields** in their surroundings, and these fields interact with other charges and currents.

Syllabus of the Course

1. Electrostatics

Coulomb's law. The electric field E and potential due to a point charge and systems of point charges, including the electric dipole. The couple and force on, and the energy of, a dipole in an external electric field. Energy of a system of point charges; energy stored in an electric field. Gauss' Law; the E field and potential due to surface and volume distributions of charge (including simple examples of the method of images), no field inside a closed conductor. Force on a conductor. The capacitance of parallel-plate, cylindrical and spherical capacitors, energy stored in capacitors.

2. Magnetostatics

The forces between wires carrying steady currents. The magnetic field B, Ampere's law, Gauss' Law ("no magnetic monopoles"), the Biot-Savart Law. The B field due to currents in a long straight wire, in a circular loop (on axis only) and in straight and toroidal solenoids. The magnetic dipole; its B field. The force and couple on, and the energy of, a dipole in an external B field. Energy stored in a B field. The force on a charged particle in E and B fields.

3. Induction

Electromagnetic induction, the laws of Faraday and Lenz. EMFs generated by an external, changing magnetic field threading a circuit and due to the motion of a circuit in an external magnetic field, the flux rule. Self and mutual inductance: calculation for simple circuits, energy stored in inductors. The transformer.

4. Electromagnetic waves

Charge conservation, Ampere's law applied to a charging capacitor, Maxwell's addition to Ampere's law ("displacement current"). Maxwell's equations for fields in a vacuum (rectangular coordinates only). Plane electromagnetic waves in empty space: their speed; the relationships between **E**, **B** and the direction of propagation.

Structure of the Course

1. Electrostatics

Charges create "electric fields" which represent the resulting force experienced by a small test charge.

$$\oint_S \mathbf{E}\,.\,\mathbf{da} = \frac{Q}{\varepsilon_0}$$

(closed surface)

GAUSS LAW

2. Magnetostatics

Electrical currents create "magnetic fields" which create forces on moving test charges. There are no magnetic monopoles.

AMPERE'S CURRENT LAW

$$\frac{1}{\mu_0}\oint \boldsymbol{B}\,.\,\boldsymbol{dl} = I$$

$$\oint_S \boldsymbol{B}\,.\,\boldsymbol{da} = 0$$

(closed surface)

3. Induction

A time-varying magnetic flux through an area creates an electromotive force along the area's rim.

$$\oint \mathbf{E}\,.\,\boldsymbol{dl} = -\frac{\mathrm{d}}{\mathrm{dt}}\int_S \mathbf{B}\,.\,\mathbf{da}$$

FARADAY / LENS LAW

4. Electromagnetic waves

A time-varying electric flux through an area creates an magnetic field along the area's rim.

$$\frac{1}{\mu_0}\oint \mathbf{B}\,.\,\boldsymbol{dl} = I + \varepsilon_0\frac{\mathrm{d}}{\mathrm{dt}}\int_S \mathbf{E}\,.\,\mathbf{da}$$

ELECTROMAGNETIC WAVE PROPAGATION

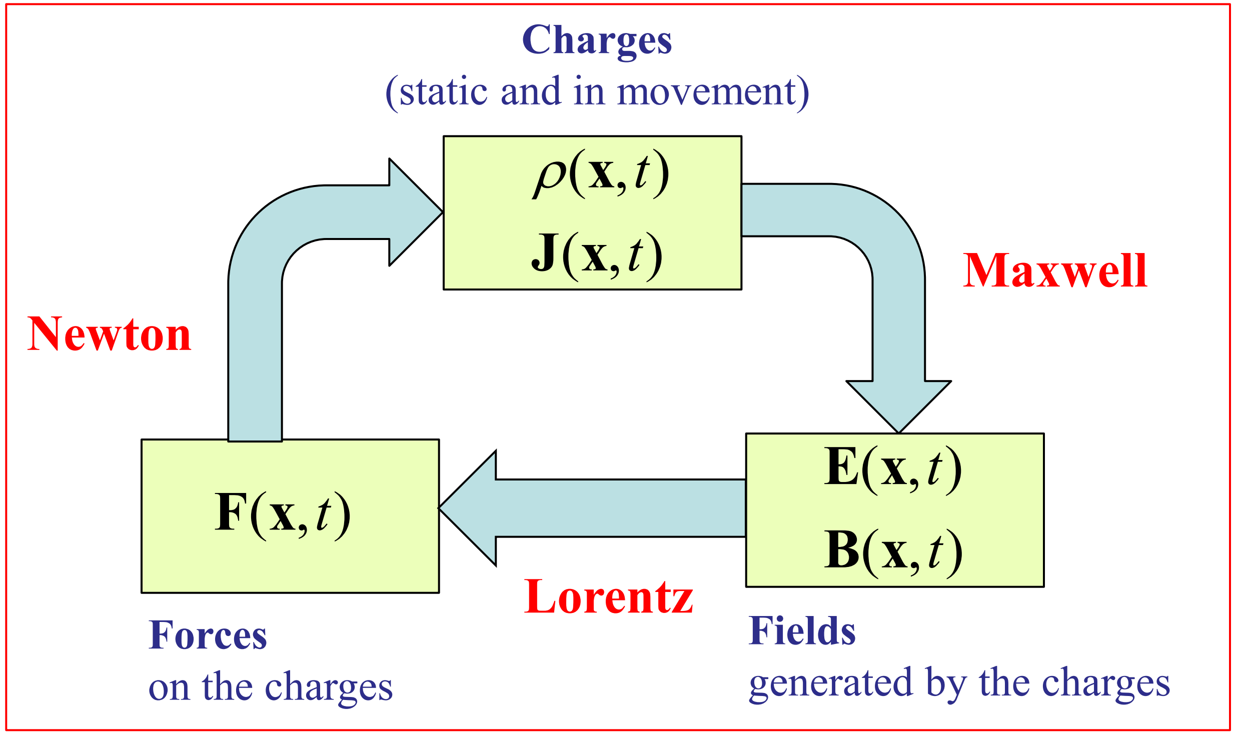
Complete description of the classical dynamics of interactions between charged particles and electromagnetic fields (classical electrodynamics)

Electro-Magnetism

Differential form	Integral form
$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$	$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{1}{\varepsilon_0} \int_V \rho \, dV$
$\nabla \cdot \vec{\mathbf{B}} = 0$	$\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0$
$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$	$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{S}}$
$\nabla \times \vec{\mathbf{B}} = \mu_0 \left(\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right)$	$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \int_S \left(\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right) \cdot d\vec{\mathbf{S}}$

We will try to minimize the use of differential and integral equations

«Problems in Electromagnetism»



Electricity and Magnetism – Forces

The concept of force links the study of electromagnetism to previous study.

The electromagnetic force between charged particles is one of the fundamental forces of nature.

Newton

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Lorentz

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

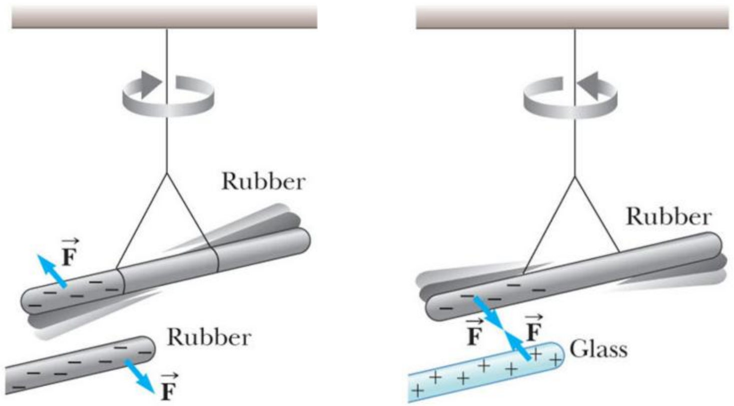
Maxwell

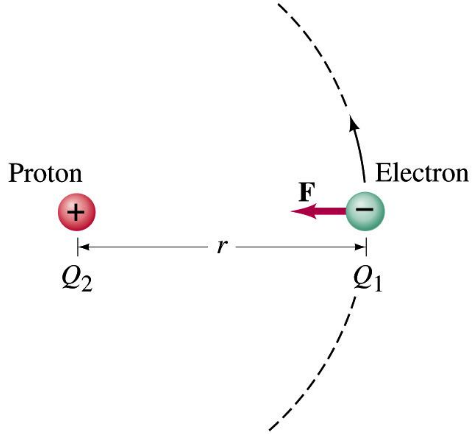
1. $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$
2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$
3. $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$
4. $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$

A new force: Electromagnetic Force (Lorentz Force)

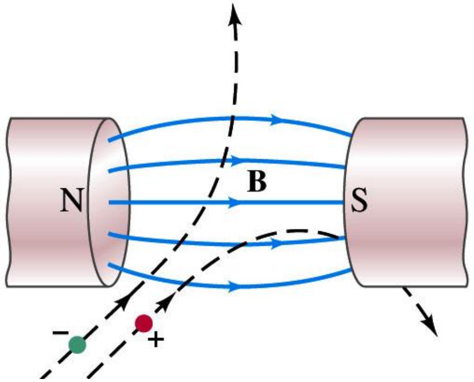
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

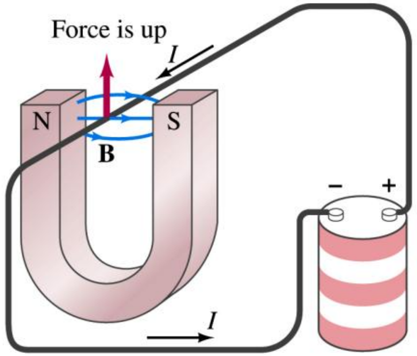
"electric" Interactions (static and moving charges)

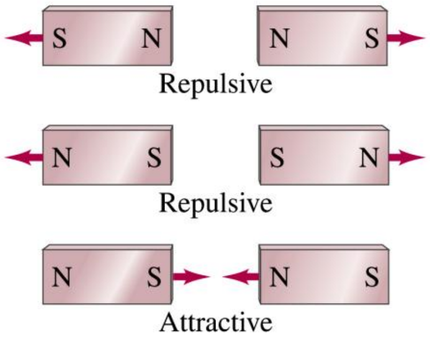




"magnetic" Interactions (moving charges)



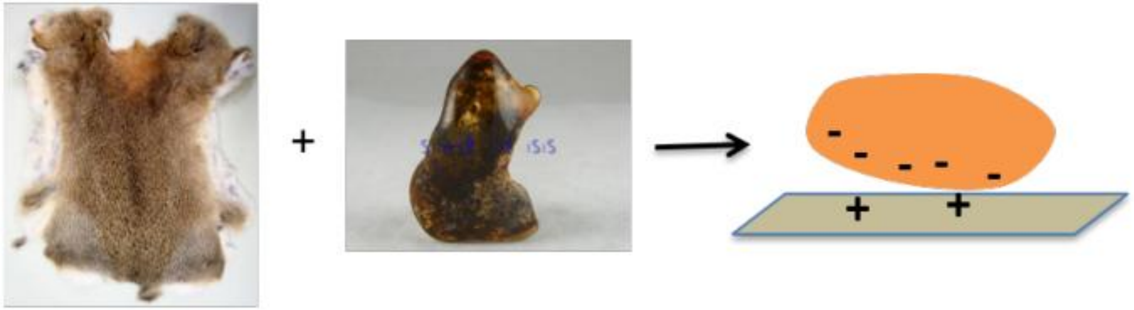




Electricity and Magnetism, Some History

Electrostatics

Ancient Greece: rubbing amber against fur allows it to attract other light substances such as dust or papyrus



Greek word for "amber":
ἤλεκτρον (*elektron*)

Magnetostatics

Magnesia (ancient Greek city in Ionia, today in Turkey):
Naturally occurring minerals were found to attract
metal objects (first references ~600BC).

Crystals are referred to as: Iron ore, Lodestone, Magnetite, Fe_3O_4

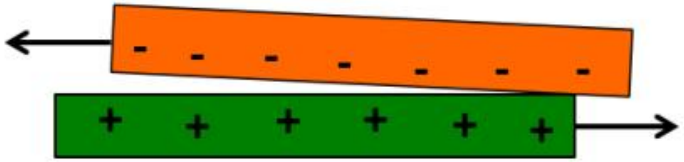
Use of Lodestone compass for navigation in medieval China

EPFL CURIOSITY

Electricity and Magnetism, Some History

17th century AD to mid 18th century:

Dominated by "frictional electrostatics" :

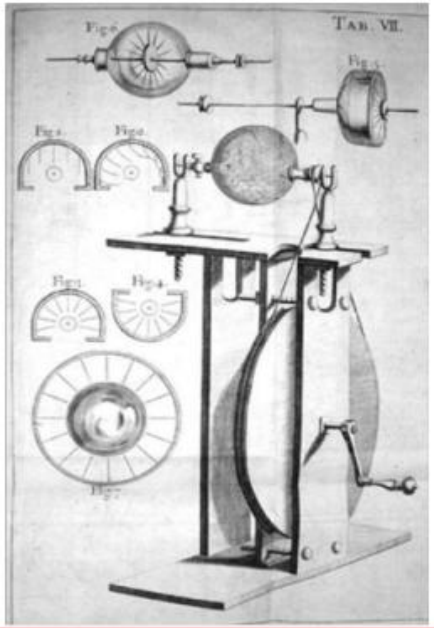


- When two different materials are brought into contact, charge flows to equalize their electro-chemical potentials, bonds form across surface
- Separating them may lead to charge remaining unequally distributed when bonds are broken
- Rubbing enhances effect through repeat contact

N.B.: Till now Electricity and Magnetism are disconnected

Focus on "electrostatic generators" – today's van de Graaff Generators:

Machines involved frictional passage of "positive" materials such as hair, silk, fur, leather against "negative" materials such as amber, sulfur



Electricity and Magnetism, Some History

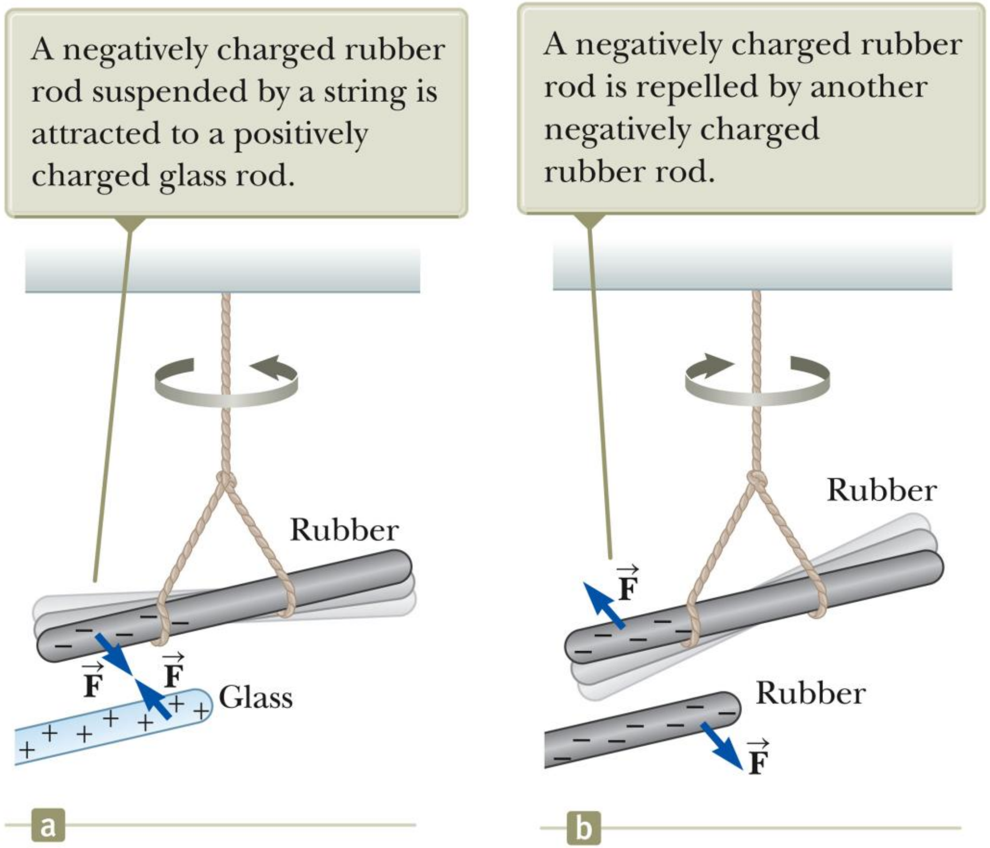
From late 18th century: *Rapid progress on both fundamental science and technology:*

N.B.: at the end of this journey, Electricity and Magnetism will be strongly interconnected and self-sustain each other (when changing with time)

- 1784: Charles-Augustin de Coulomb uses "torsion balance" to show that forces between two charged spheres vary with the square of the inverse distance between them.
- 1800: Alessandro Volta constructs the first electrochemical battery (zinc/copper/sulfuric acid) allowing high-density electrical energy storage
- 1821: André-Marie Ampère investigates attractive and repulsive forces between current-carrying wires
- 1831-55: Michael Faraday discovers electromagnetic induction by experimenting with two co-axial coils of wire, wound around the same bobbin.
- 1830ies: Heinrich Lenz shows that induced currents have a direction that opposes the motions that produce them
- 1831: first commercial telegraph line, from Paddington Station to West Drayton
- 1864: James Clerk Maxwell introduces unified theory of electromagnetism, including a link to light waves
- 1887: Heinrich Hertz demonstrates the existence of electromagnetic waves in space
- Late 19th century: development of "wireless telegraphy" – radio!

Electric Charges (static): Electrostatic

- There are two kinds of **electric charges**
  - Called **positive** and **negative**
    - Negative charges are the type possessed by electrons.
    - Positive charges are the type possessed by protons.
- Charges of the same sign repel one another and charges with opposite signs attract one another.



- The rubber rod is negatively charged.
- The glass rod is positively charged.
- The two rods will attract.

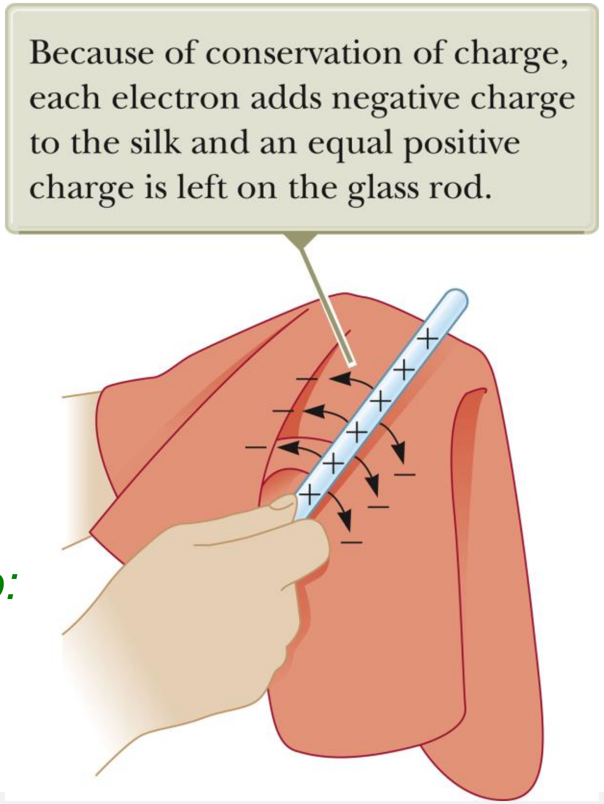
- The rubber rod is negatively charged.
- The second rubber rod is also negatively charged.
- The two rods will repel.

More About Electric Charges

- Electric charge is always **conserved** in an isolated system.
  - For example, charge is not created in the process of rubbing two objects together.
  - The electrification is due to a transfer of charge from one object to another.
    - A glass rod is rubbed with silk.
    - Electrons are transferred from the glass to the silk.
    - Each electron adds a negative charge to the silk.
    - An equal positive charge is left on the rod.

DEMO

https://auditoires-physique.epfl.ch/experiment/431/electrostatique-peau-de-chat-loi-de-coulomb



- The electric charge, q, is said to be **quantized**.
  - q is the standard symbol used for charge as a variable.
  - Electric charge exists as discrete packets.
  - $q = \pm Ne$
    - N is an integer
    - e is the fundamental unit of charge
    - $|e| = 1.6 \times 10^{-19}$ C
    - Electron: $q = -e$
    - Proton: $q = +e$

Charges produced by rubbing are typically around a microcoulomb:

$1\ \mu C = 10^{-6}\ C.$

The Electric Charge

The electric charge is:

1) **Positive or Negative**
2) **Quantized**
3) **Conserved** (locally)

All known microscopic particles and macroscopic objects have an electric charge which is an integer multiple of the charge of the electron:

$$q = ne \quad n \in Z \qquad\qquad e = 1.602176634 \times 10^{-19}\ \text{C}$$

The total charge of the Universe is fixed at all times.
A positive charge can "annihilate" an equal negative charge, but a positive or negative charge cannot disappear on its own.

Value (C)	Item
1.602×10^{-19} C	*e:* proton (positive), electron (negative)
1.473×10^{-17} C	92*e:* uranium nucleus
1×10^{-15} C	Charge on a typical dust particle
1×10^{-12} C	Charge in typical microwave frequency capacitors
1×10^{-6} C	Charge in typical audio frequency capacitors
1×10^{-6} C	Charge from rubbing materials together
5×10^{3} C	Charge in alkaline AA battery
1.8×10^{5} C	Charge in a car battery
4.5×10^{5} C	Charge of the Earth (without the atmosphere)(negative)
5.9×10^{8} C	Charge in world's largest battery bank

Note: from 20 mai 2019, the elementary charge, indicated as e, is *exactly* equal to:

e = 1,602 176 634 $\times 10^{-19}$ C

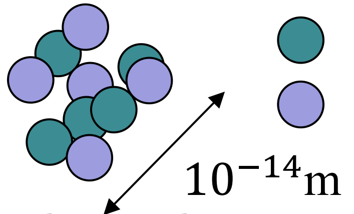
Until that date, the value of the elementary charge was :

e = 1,602 176 620 8(98) $\times 10^{-19}$ C

where the two digits in parentheses represent the absolute error on this value.

Nuclei and Atoms

Nucleus (N$_p$ protons + N$_n$ neutrons)



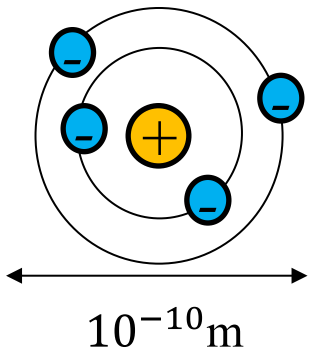
Proton (charge: e)

Neutron (charge: 0)

$e \cong 1.6\times10^{-19}$ C

The nucleus is positively charged (*total* charge $= N_p e$) («***bounded***» **charge**)

Atom (N$_p$ protons + N$_n$ neutrons + N$_e$ électrons)



Atomic nucleus (total charge: $N_p e$)

Electron is «bounded» on the nucleus(charge: -e)

The atom can be positively charged("positive ion")",
"negatively charged" ("negative ion"), or be neutral ("neutral atom")
(total charge = $(N_p - N_e)e$) («***bounded***» **charge**)

Conductors and Insulators

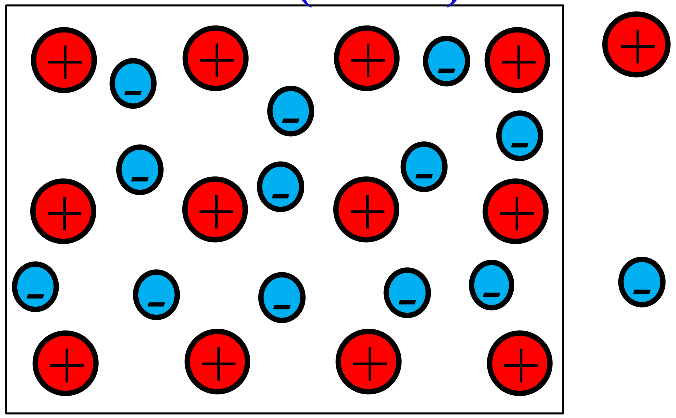
"Free" and "bound" charges

It is convenient to classify materials in terms of the <u>ability of electrons to move through the material</u>

- Some solids (e.g. aluminum) have electrons free to move from atom to atom. These are **conductors**. They're usually **shiny**, because the freely moving electrons reflect light very well.
- In solids like NaCl, or glass, etc., the electrons are tied to atoms, no charge can flow: these solids are **insulators**.
- Semiconductors exist.

The power is carried by aluminum wires, suspended by glass, polymer, or porcelain insulators.

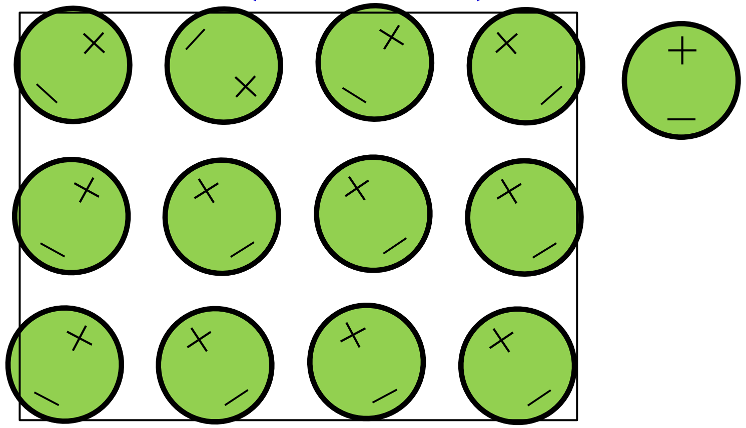
Conducator (Metal)



Fixed atom with lack of electrons (positive ion) **("bound" charge)**

Electron "free" to move ("free" charge)

Insulator (Dielectric)



Fixed or free to move "neutral" molecule or atom with "dipolar" charge distribution (permanent or induced electric dipole) **("bound" charge)**

Conductors (metals), Insulators (dielectrics)

Conductors

- Electrical conductors are materials in which some of the electrons are free electrons.
  - Free electrons are not bound to the atoms.
  - These electrons can move relatively freely through the material.
  - Examples of good conductors include copper, aluminum and silver.
  - When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material.

Insulators

- Electrical insulators are materials in which all of the electrons are bound to atoms.
  - These electrons can not move relatively freely through the material.
  - Examples of good insulators include glass, rubber and wood.
  - When a good insulator is charged in a small region, the charge is unable to move to other regions of the material.

Semicon-ductors

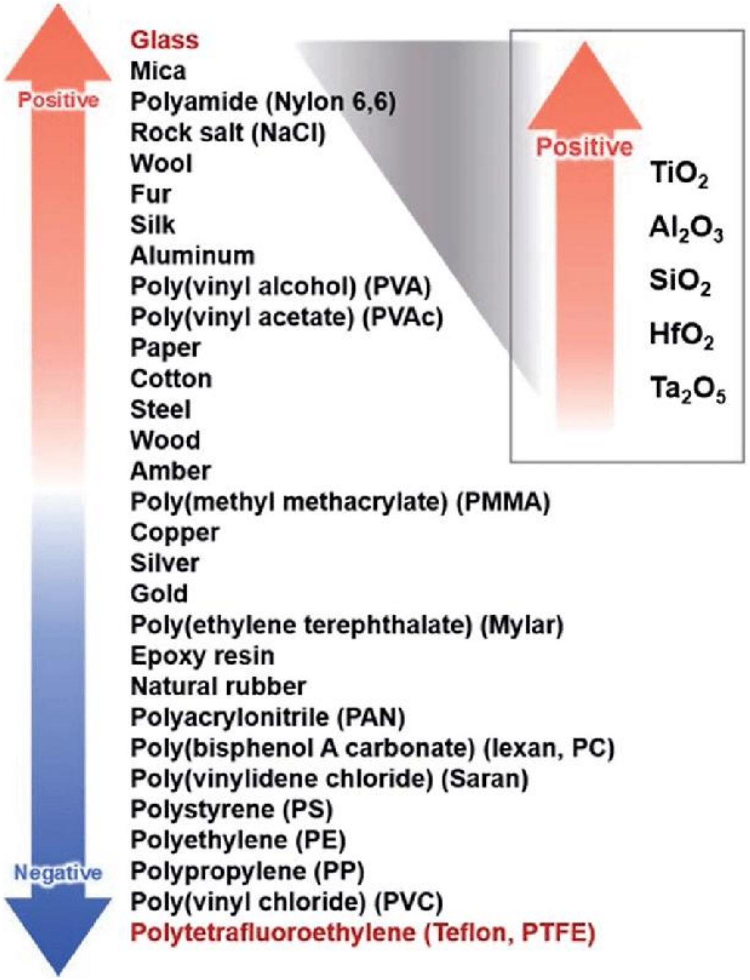
- The electrical properties of semiconductors are somewhere between those of insulators and conductors.
- Examples of semiconductor materials include silicon and germanium.
- The electrical properties of semiconductors can be changed.

Contact Electrification (for insulators)

- If two dissimilar surfaces are placed in close contact, usually some bonding occurs between atoms, and therefore some electron transfer takes place. Hence, when the surfaces are moved out of contact, one will have an excess of electrons, and the other one a deficiency.
- True contact at the atomic scale is best achieved by rubbing the surfaces together.

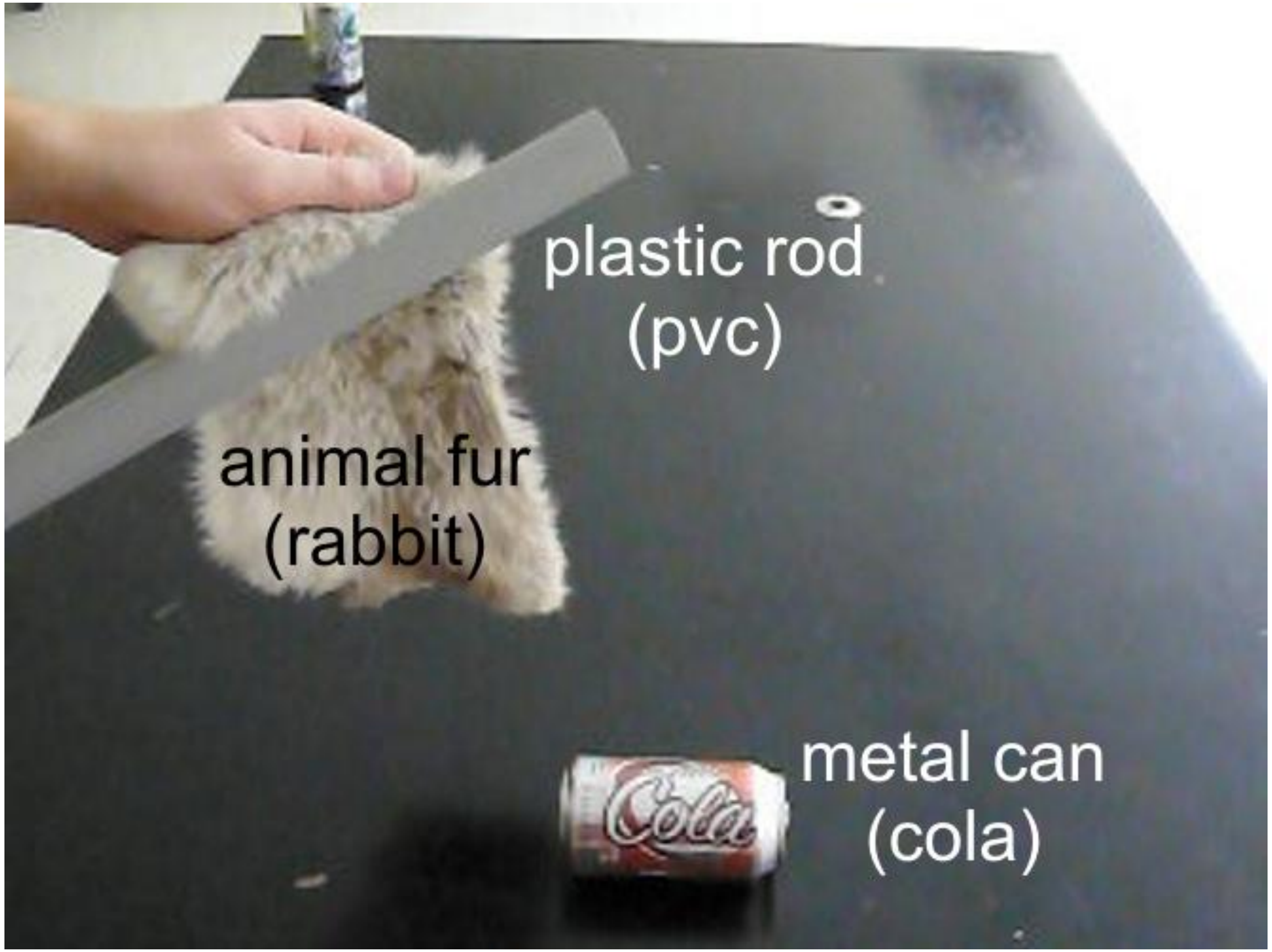
Triboelectric Series

- Countless experiments in rubbing things together—cats, balloons, etc.—have established the table on the right.
- For two different surfaces, the one higher in the table will lose electrons and become positively charged.
- Note: tribo is from the Greek for rub.



Observe that in all cases, it is only the electrons that do the moving

Charge in action?



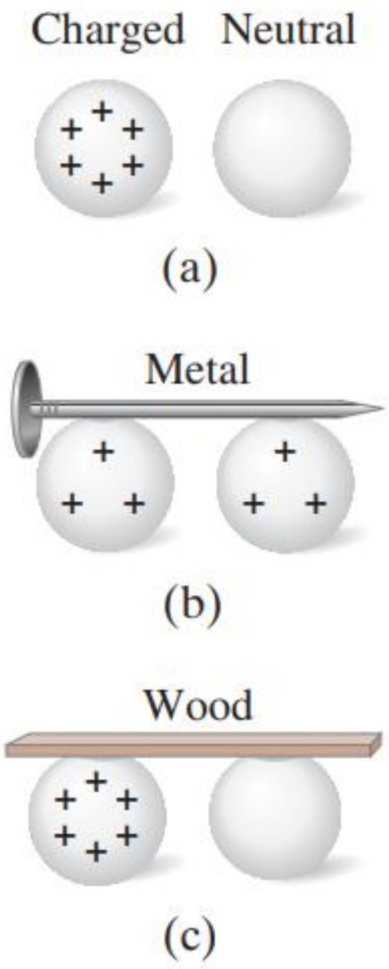
DEMO

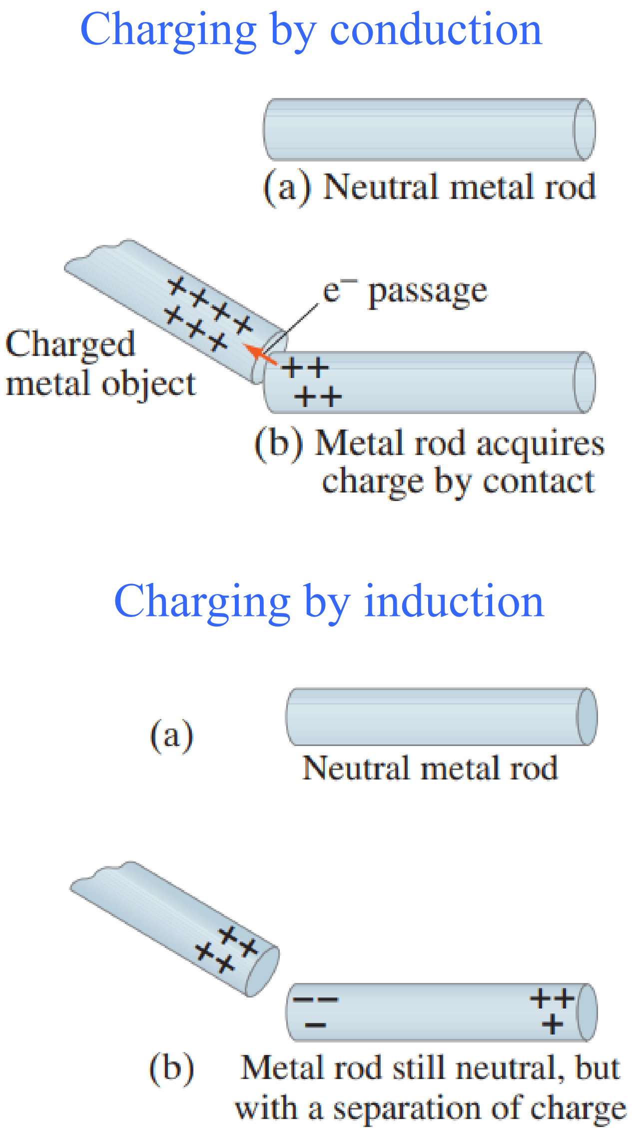
https://auditoires-physique.epfl.ch/experiment/425/electrostatique-canette-de-soda

EPFL

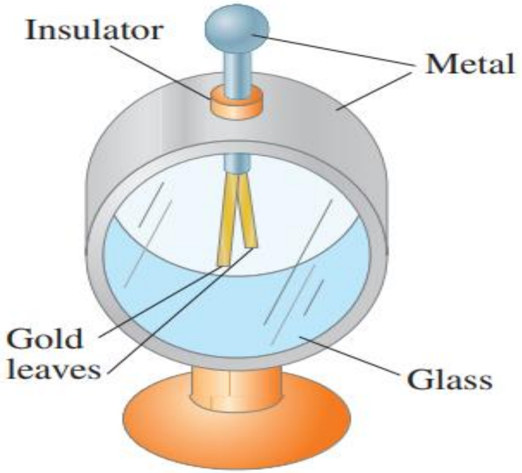
Some new empirical observations about conductors

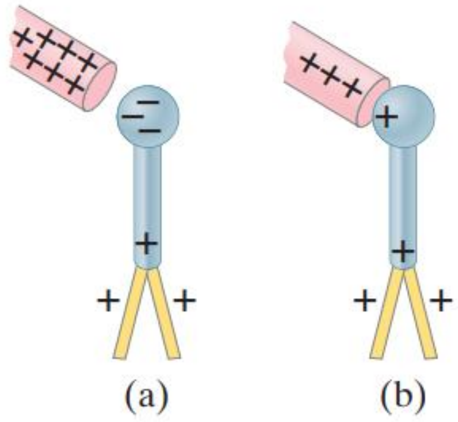
(a) A charged metal sphere and a neutral metal sphere.
(b) The two spheres connected by a conductor (a metal nail), which conducts charge from one sphere to the other.
(c) The original two spheres connected by an insulator (wood); almost no charge is conducted





An electroscope is a device that can be used for detecting charge.





Electroscope charged (a) by induction, (b) by conduction.

How to charge a conductor: Contact

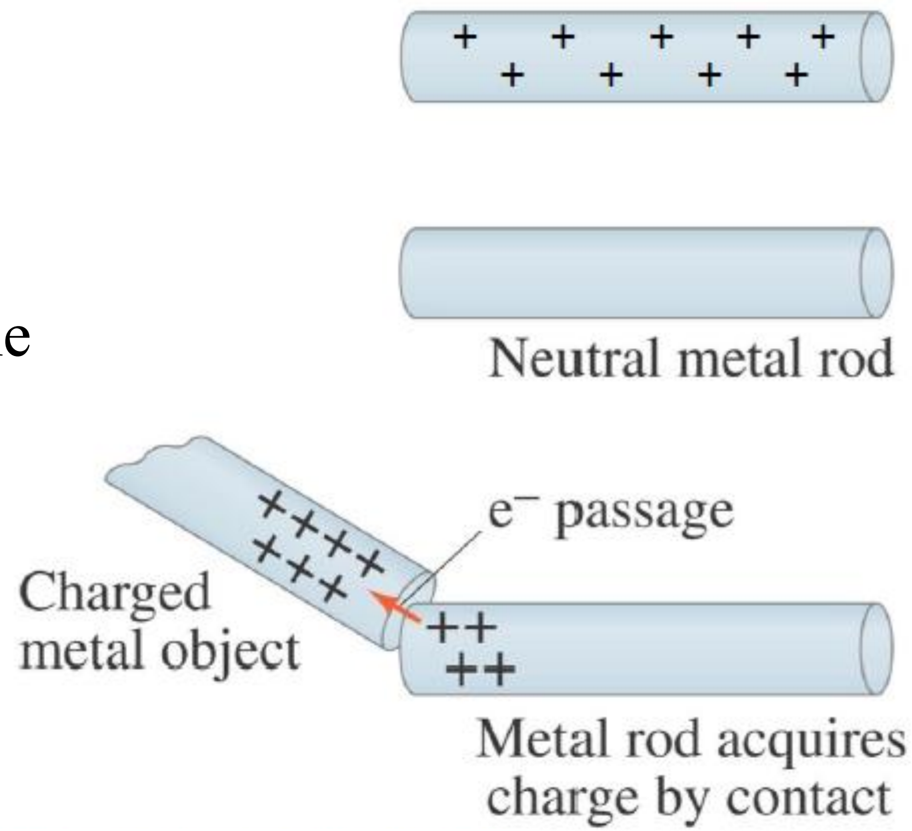
•Get charges on one metal rod: e.g. by separation (remove –e)

[for example, by Volta's Electrophorus, see later]

•Move the charged metal object in contact with the other initially neutral metal object.

•Some of the electrons from the initially charged metal object will be transfer to the second one.

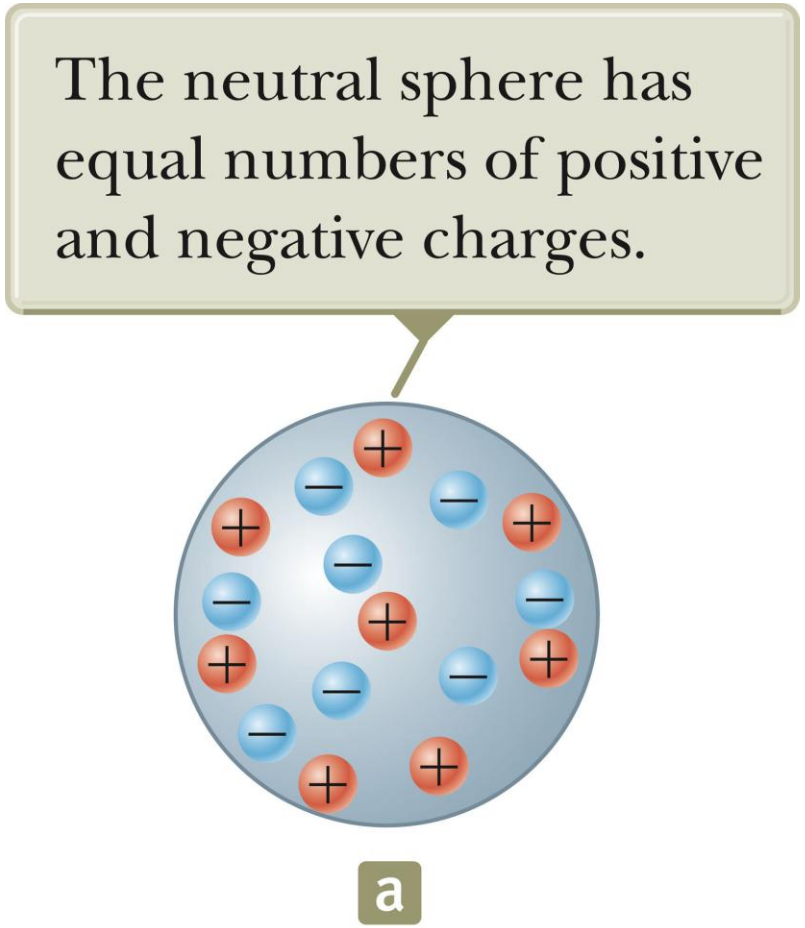
•The process is almost instantaneous and will terminate till a new equilibrium will be reached.



How to charge a conductor/insulator: Induction, 1

- <u>It works for both metals and insulators.</u>
- Charging by induction requires no contact with the object inducing the charge.
- That is in contrast to charging an object by rubbing (that is, by conduction), which does require contact between the two objects.

- Assume we start with a neutral metallic sphere.
  - The sphere has the same number of positive and negative charges.



How to charge a conductor/insulator: Induction, 1

•Conduction electrons in a conductor move freely

•A charged object nearby exerts an electric force on these electrons

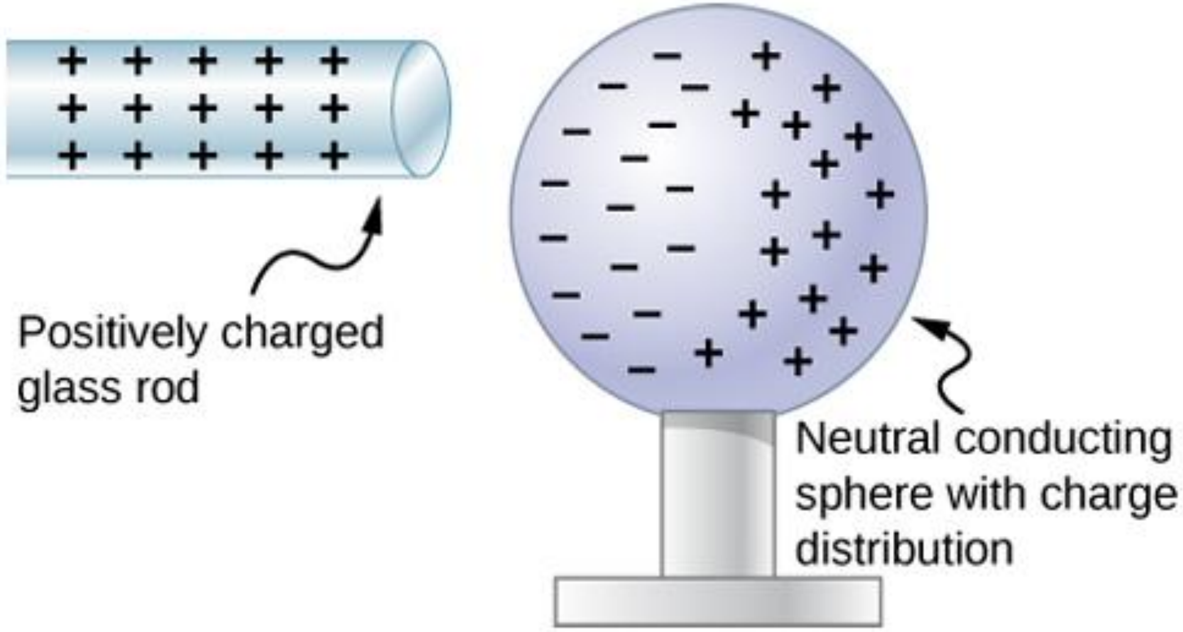
- *Example:* positively charged rod attracts electrons in the metallic sphere → they shift toward the rod

•Conductor remains overall neutral, but:

- Near side: excess negative charge
- Far side: excess positive charge

•This redistribution = **polarization**

•Works similarly for negative charges (opposite direction)



Induced polarization. A positively charged glass rod is brought near the left side of the conducting sphere, attracting negative charge and leaving the other side of the sphere positively charged. Although the sphere is overall still electrically neutral, it now has a charge distribution, so it can exert an electric force on other nearby charges. Furthermore, the distribution is such that it will be attracted to the glass rod.

How to charge a conductor/insulator: Induction, 1

Induced Dipoles & Attraction

•Charge on nearby object → shifts electrons in conductor

•Creates an **electric dipole** ("two ends")

•Insulating objects can still be attracted to charged ones.

•Mechanism: **polarization of atoms/molecules** → dipole interaction

Polarization in Insulators

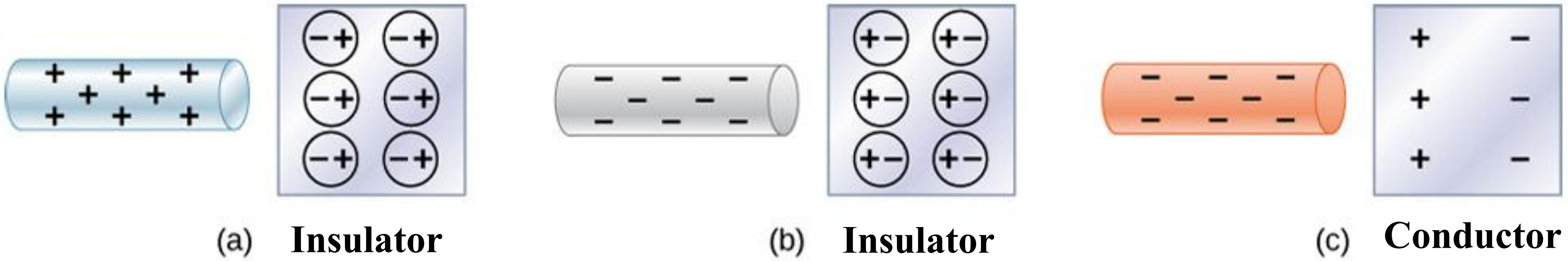
•Charged rod near insulator → atoms/molecules slightly distorted

- Opposite charge shifts closer/Like charge shifts farther

•Attraction stronger than repulsion → **net attraction**

•**Polar molecules** (e.g. water):

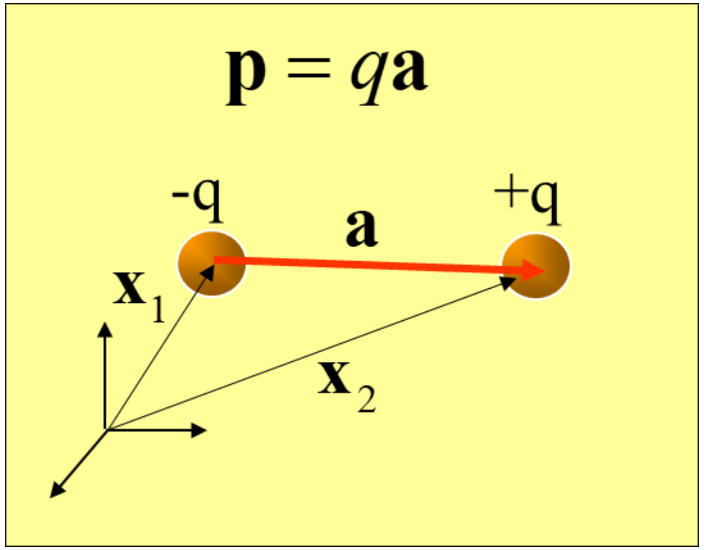
- Already have inherent charge separation
- Even more affected by external charges



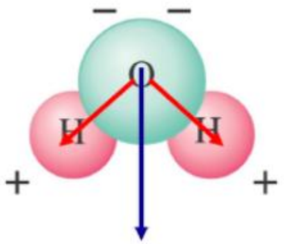
Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

DEMO https://auditoires-physique.epfl.ch/experiment/426/dipoles-electriques

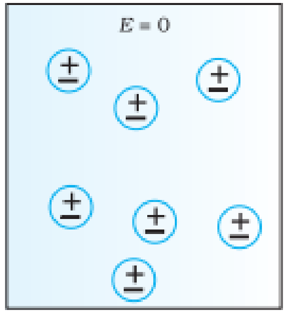
Electric Dipoles (dielectric materials)



"**Dielectric**" materials:
composed of microscopic **electric dipoles** (induced or permanent)

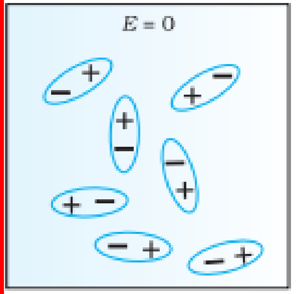


Non-Polar molecules

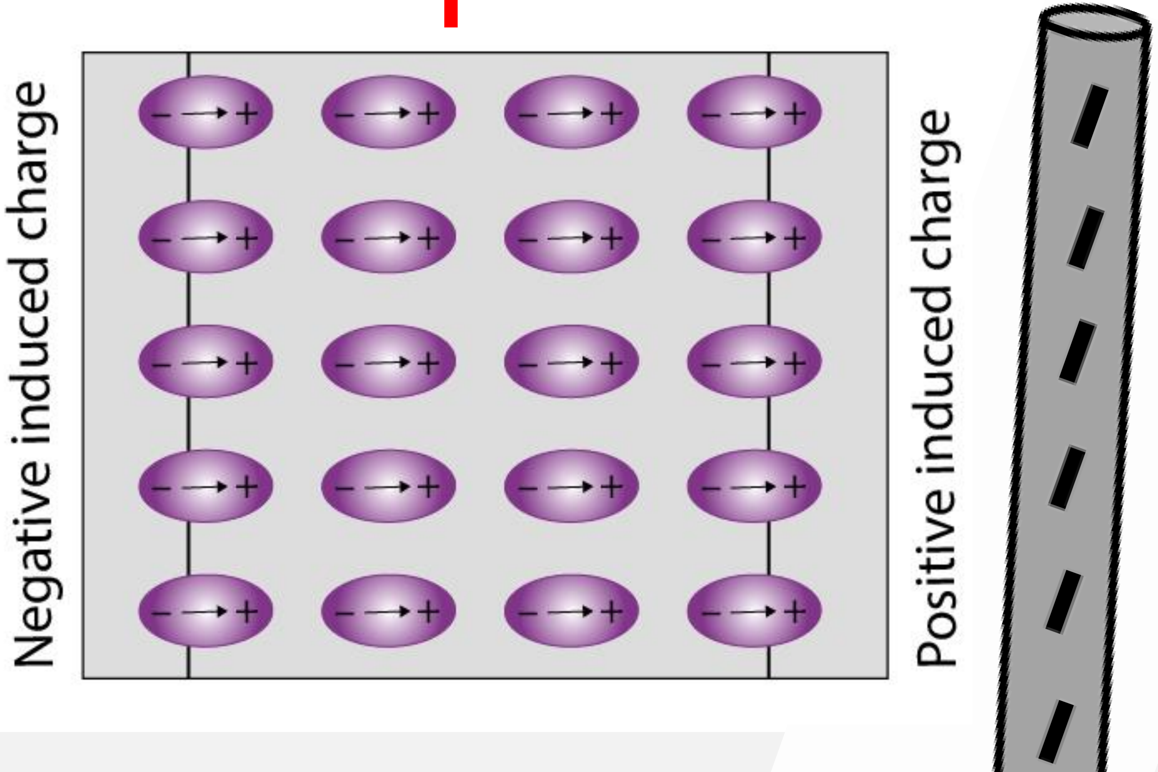


(a) Non-polar molecules

Polar molecules



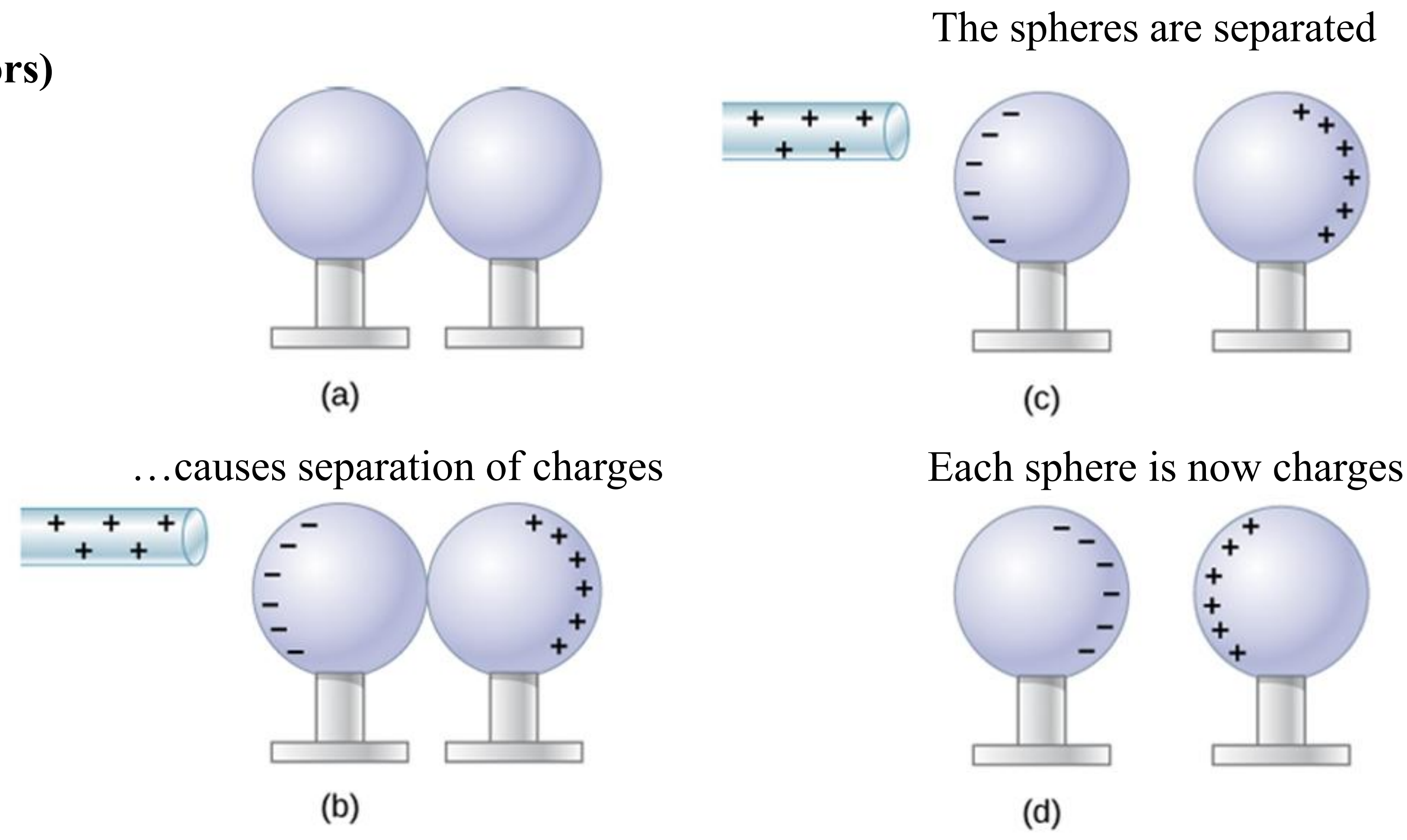
(b) Polar molecules



How to charge a conductor: Induction, 2

Charging by Induction (Conductors)

- Two neutral metal spheres in contact, insulated from ground
- Charged rod brought near → electrons shift
    - Sphere near rod: negative charge
    - Far sphere: positive charge
- **Result:** separation of charges without direct transfer



Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charges. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

How to charge a conductor: Induction, 3

Charging by Induction with Grounding

•Neutral metal sphere polarized by nearby charged rod

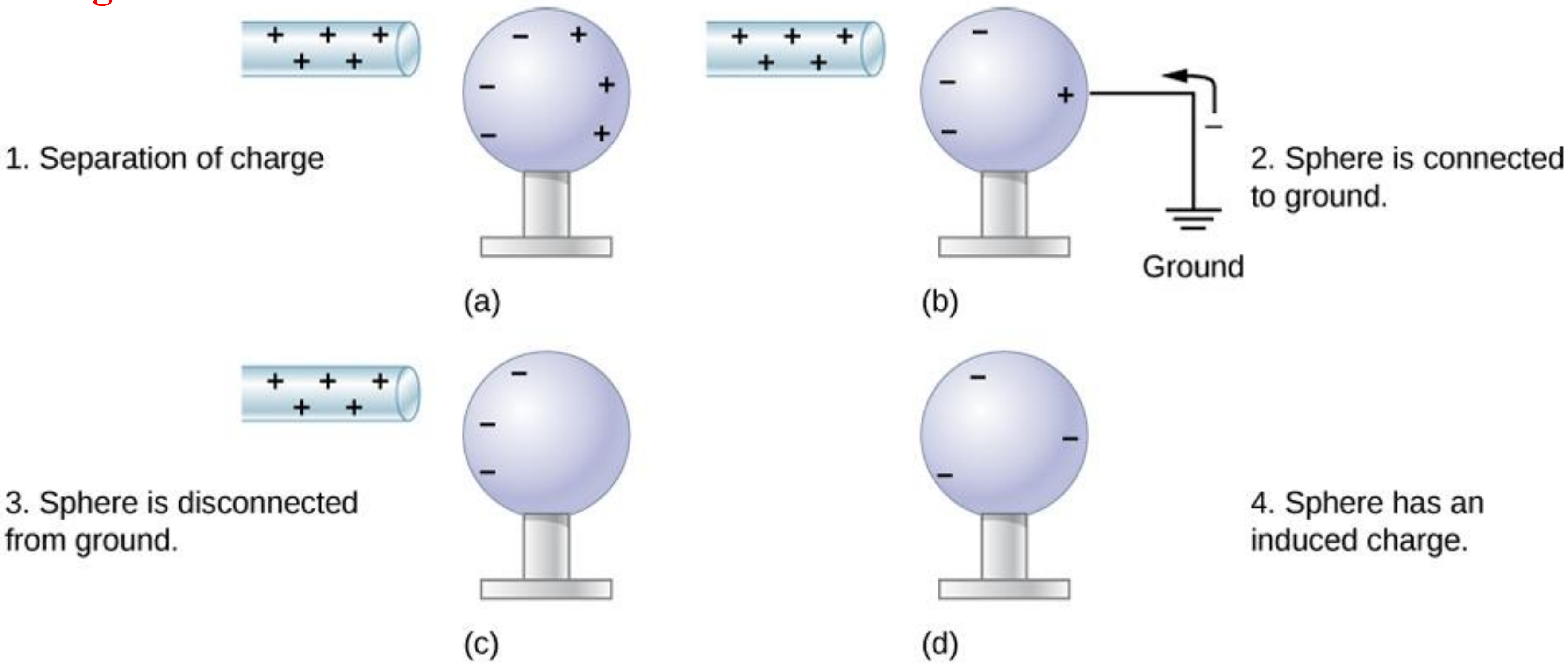
•Sphere connected to **ground** → electrons flow in/out

- Ground acts as infinite charge reservoir

•Ground removed **before** rod is taken away

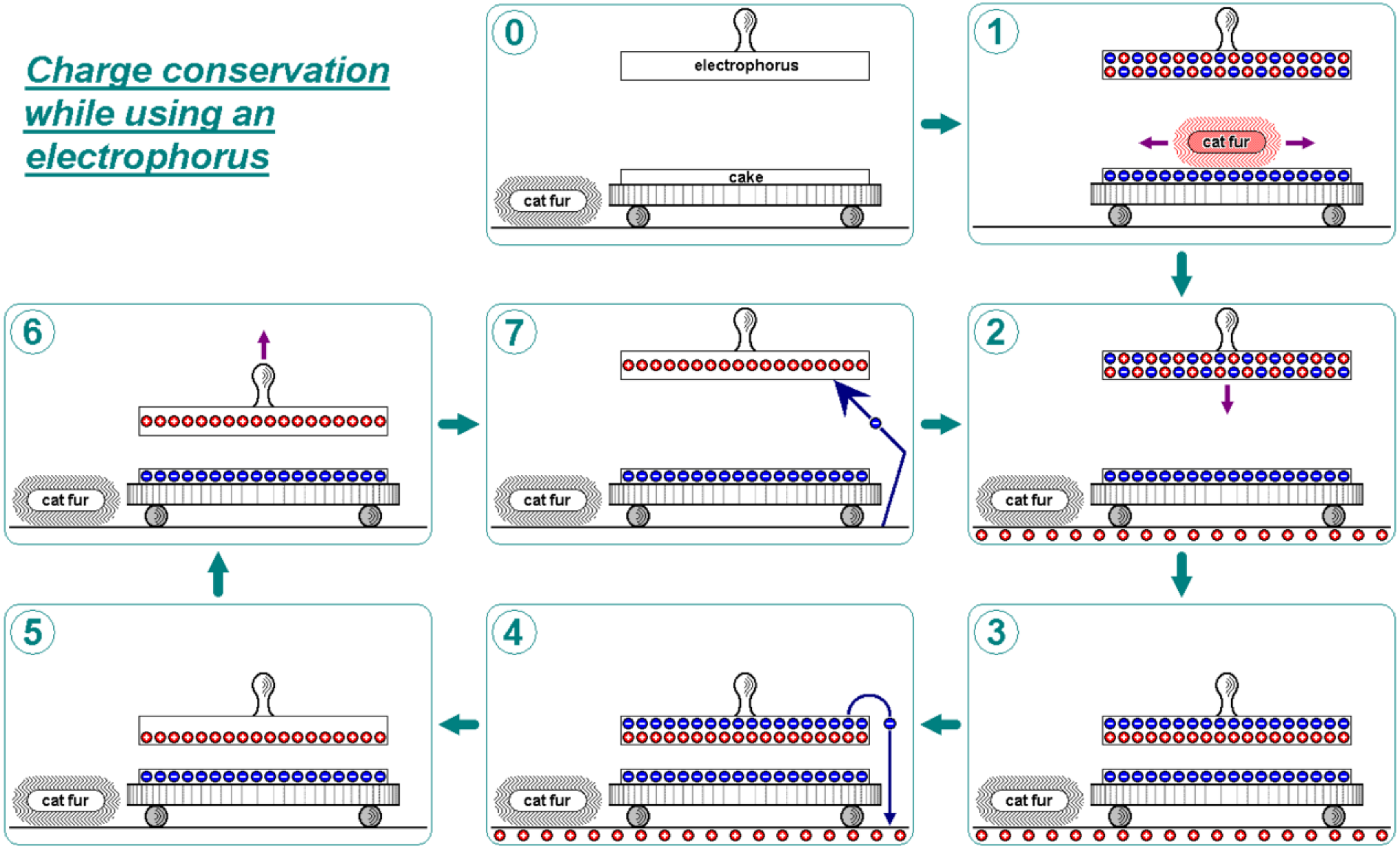
•Sphere left with **net charge opposite** to the rod

•Rod keeps its own charge (no direct transfer)



Charging by induction using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from Earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.

Volta's Electrophorus

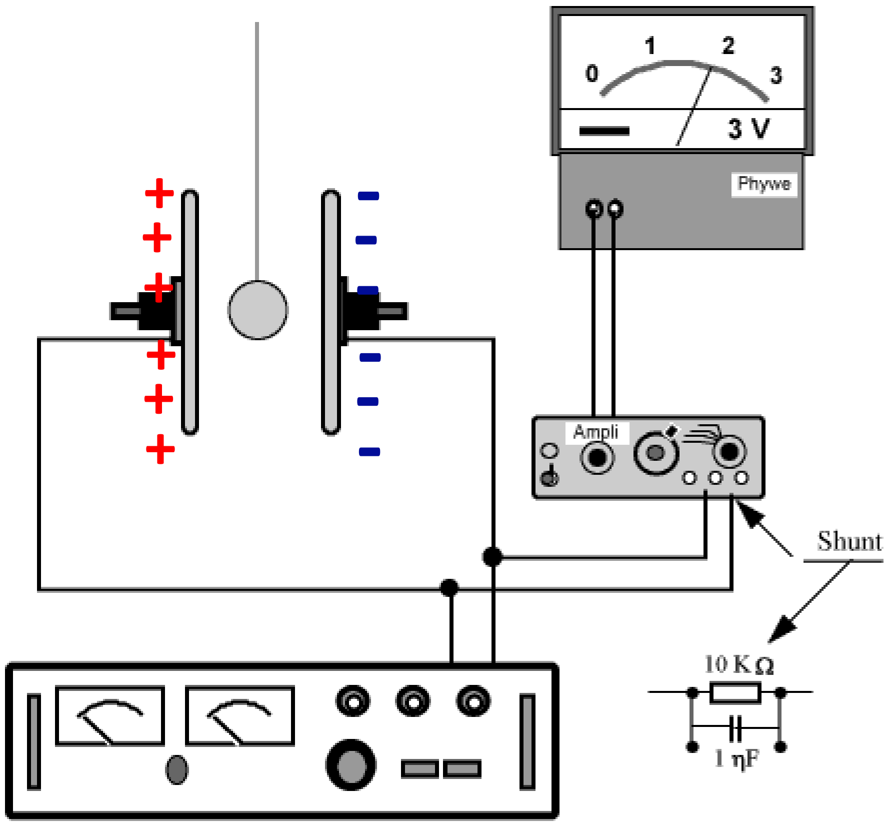


Author: Qniemiec (Wikipedia)

DEMO https://auditoires-physique.epfl.ch/experiment/304/electrophore-de-volta

Electrostatic Pendulum

Pendule électrostatique



Alimentation Ht = 10 à 15 KV
Ampli = 60 dB / pleine échelle = 3 ηA
Multimètre = 3 V -

A conductive sphere suspended from a wire oscillates between the plates of a capacitor.

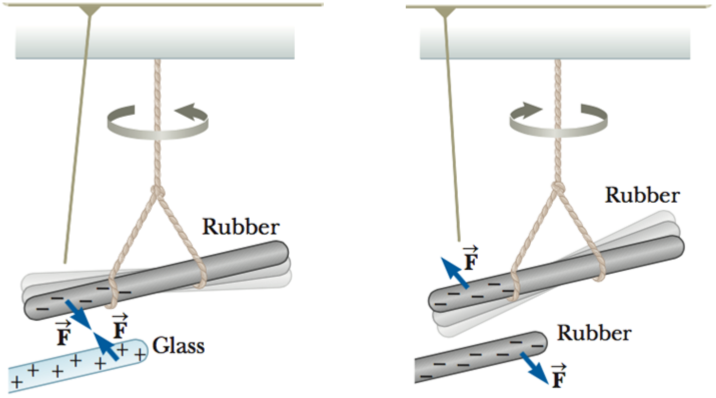
As the capacitor is charged with the HV supply, the ball oscillates from plate to plate, changing charge with each contact with a plate.
It is observed that the oscillation frequency increases with the applied voltage.

Secondly, a dielectric plate is inserted between the ball and one of the charged plates. The observed result is that the pendulum remains "stuck" to the dielectric.

DEMO https://auditoires-physique.epfl.ch/experiment/420/pendule-electrostatique

Electric *Charge* Some new empirical observations

The concept of charge came from observations of interactions



After rubbing of the rods they attract or repeal – experience a FORCE

The FORCE increases, if more intense rubbing of one or of both rods.

$$F \propto Q_1 \cdot Q_2$$

F – force; **Q_1** and **Q_2** are CHARGES of the rods

- Charges exist in two different qualities:

  POSITIVE (+) and NEGATIVE (-)

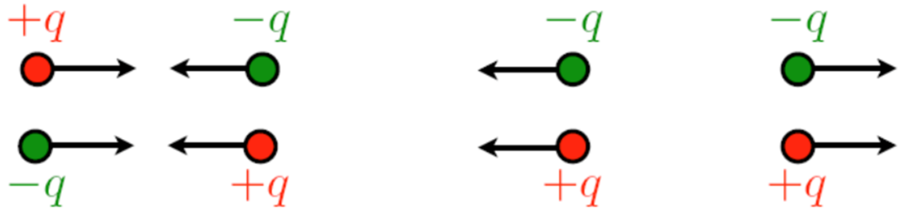
- Charge changes by increments: the smallest are **±e**

  Charge is QUANTIZED

https://auditoires-physique.epfl.ch/experiment/431/electrostatique-peau-de-chat-loi-de-coulomb **DEMO**

The Coulomb interaction between charges

- We observe a charge because it produces a force
- Repulsion/attraction of charges depends on their sign.
- The force between two charges Q_1 and Q_2 is repulsive if the two charges have the same sign.



Point charge:

- is a hypothetical charge located at a single point in space;
- is a mathematic concept to simplify discussion;
- in practice, any charge is a point charge if its spatial distribution size is much smaller than the distance at which the charge is considered.

Coulomb’s Law

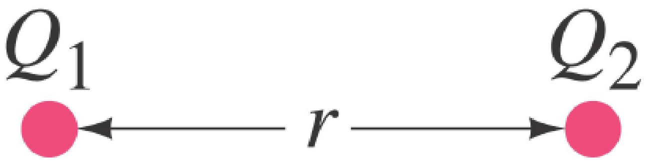
- **Coulomb’s law describes the interaction between charges**. It is the central law for electrostatics and plays there a similar role as Newton’s gravitational law does for classical mechanics.
- In order to derive a general law, we have to consider the following points: the direction of the force, the dependence on the magnitude and sign of the charges, and the dependence on the distance.

$F \propto Q_1 \cdot Q_2$ F – force; Q_1 and Q_2 are CHARGES

How does the force depend on distance ?

Experiment shows that the electric force between two charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F \propto \frac{1}{r^2}$$



- power of 2 is tested to very high precision: 1×10^{-16}!
- resembles Newton’s inverse square law of gravitation.

$$F = k\frac{Q_1 Q_2}{r^2}$$

Charles-Augustin de Coulomb

1736-1806

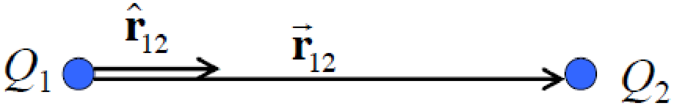
Magnitude of the force between two charges.

In vector form, the force on Q_2 due to Q_1 is

$$\vec{\mathbf{F}}_{21} = k\frac{Q_1 Q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Symmetrical, abs of $F_{12}=F_{21}$

where $\hat{\mathbf{r}}_{12}$ is the unit vector from Q_1 to Q_2

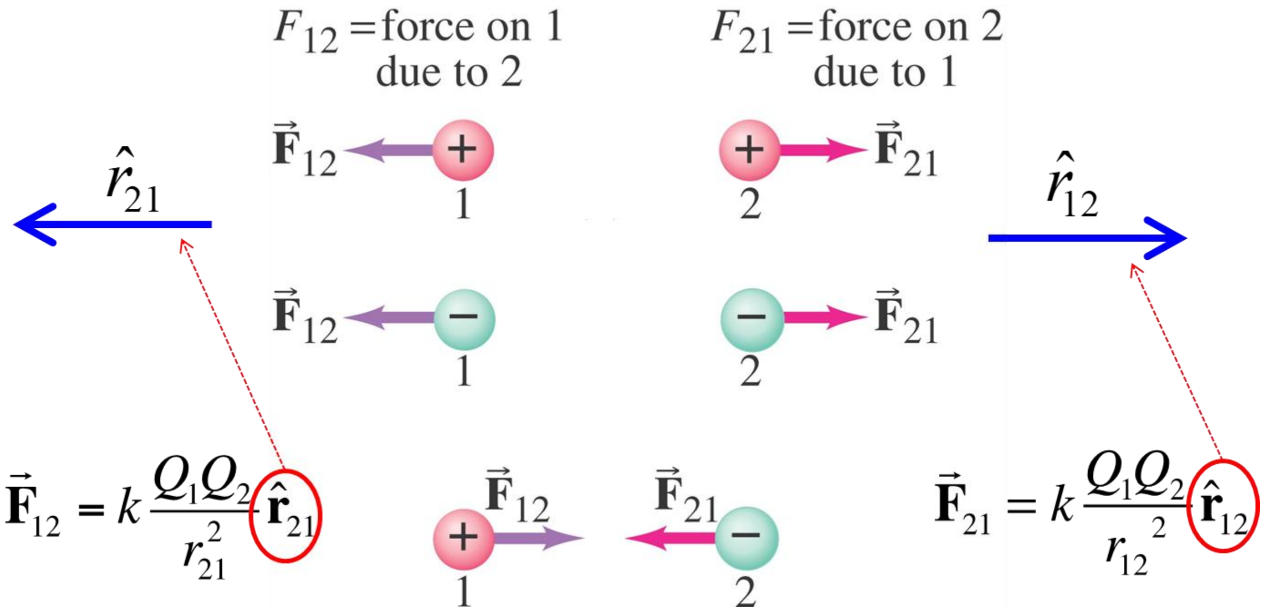


$$\hat{\mathbf{r}}_{21} = \vec{\mathbf{r}}_{12} / \left|\vec{\mathbf{r}}_{12}\right|$$

Coulomb's Law

Here below, we report a basic representation of the Coulomb force between two charges of varying polarity

The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



k **is a fundamental constant value of which depends on our definition of unit charge**

in SI units, k is expressed as

$$k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

$$\epsilon_0 \approx 8.8 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

is the permittivity of free space

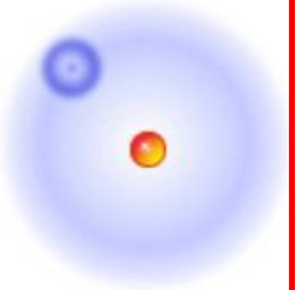
Example:

The electrostatic force holding the electron in the orbit (e.g. H):

$r = 10^{-10}$ m; $k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$

$$|\mathbf{F}_{21}| = k\frac{Q_1Q_2}{{r_{12}}^2}$$

$$\sim 10^{10}\frac{1.6^2 \times 10^{-38}}{10^{-20}}N \sim 10^{-8}N$$



Electric *vs* Gravitational force

Gravitational force between two **masses**

$$\mathbf{F} = G_G \frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

Attractive
Only one type of mass (positive)

$$G_G \cong 7\times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$$

Exemple: Forces between an electron and a proton

$m_p \cong 1.7\times 10^{-27}$ kg $q_p \cong 1.6\times 10^{-19}$ C

$m_e \cong 9\times 10^{-31}$ kg $q_e \cong -1.6\times 10^{-19}$ C

$$\frac{\mathbf{F}_{electrostatic}}{\mathbf{F}_{gravitation}} = \frac{G_C}{G_G}\frac{q_e q_p}{m_e m_p} \cong 10^{39} \text{ (at all distances)}$$

(the electrostatic force is dominant)

Electrostatic force between two **charges**

$$\mathbf{F} = G_C \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Attractive or répulsive
Two types of charges (positive or negative)

$$G_C \cong 9\times 10^{9}\ \mathrm{Nm^2C^{-2}}$$

Exemple: Forces between the earth and the sun

$m_t \cong 6\times 10^{24}$ kg $q_t \cong -10$ C (?)

$m_s \cong 2\times 10^{30}$ kg $q_s \cong +100$ C (?)

$$\frac{\mathbf{F}_{electrostatic}}{\mathbf{F}_{gravitation}} = \frac{G_C}{G_G}\frac{q_t q_s}{m_t m_s} \cong 10^{-32} \text{ (at all distances)}$$

The planets and stars
are almost electrically "neutral“
(the gravitational force is dominant)

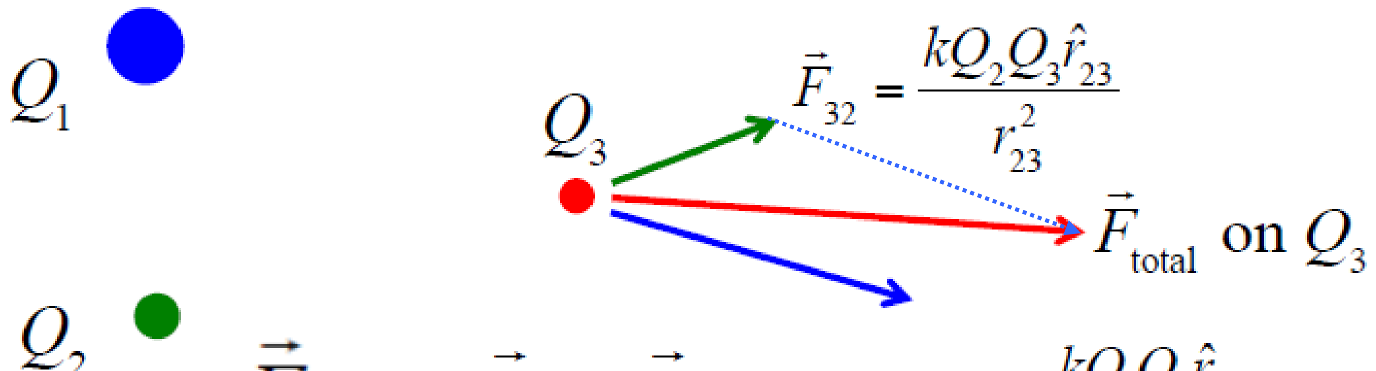
Superposition Principle

2 charges

Coulomb's Law: $F = k\dfrac{Q_1 Q_2}{r^2}$, $k = 8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}$

>2 charges (??) Superposition

The total electric force on a charge Q_3 from two charges Q_1, Q_2 is the vector sum of the forces from each remaining charge.



$$\vec{F}_{32} = \frac{kQ_2Q_3\hat{r}_{23}}{r_{23}^2}$$

$$\vec{F}_{total} = \vec{F}_{31} + \vec{F}_{32}$$

$$\vec{F}_{31} = \frac{kQ_1Q_3\hat{r}_{13}}{r_{13}^2}$$

It sounds trivial but **superposition** is not a logical necessity or consequence of Coulomb's law but **an empirically established feature** of classical electrodynamics that simplifies our life enormously

Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.00\ \mu C$, $q_2 = -2.00\ \mu C$, and $a = 0.100$ m. Find the resultant force exerted on q_3.

the magnitude of $\vec{\mathbf{F}}_{23}$:

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \frac{(2.00 \times 10^{-6}\ \text{C})(5.00 \times 10^{-6}\ \text{C})}{(0.100\ \text{m})^2} = 8.99\ \text{N}$$

Find the magnitude of the force $\vec{\mathbf{F}}_{13}$:

$$= (8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \frac{(5.00 \times 10^{-6}\ \text{C})(5.00 \times 10^{-6}\ \text{C})}{2(0.100\ \text{m})^2} = 11.2\ \text{N}$$

Find the x and y components of the force $\vec{\mathbf{F}}_{13}$:

$$F_{13x} = (11.2\ \text{N}) \cos 45.0° = 7.94\ \text{N}$$

$$F_{13y} = (11.2\ \text{N}) \sin 45.0° = 7.94\ \text{N}$$

Find the components of the resultant force acting on q_3:

$$F_{3x} = F_{13x} + F_{23x} = 7.94\ \text{N} + (-8.99\ \text{N}) = -1.04\ \text{N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.94\ \text{N} + 0 = 7.94\ \text{N}$$

Express the resultant force acting on q_3 in unit-vector form:

$$\vec{\mathbf{F}}_3 = (-1.04\hat{\mathbf{i}} + 7.94\hat{\mathbf{j}})\ \text{N}$$

Finalize The net force on q_3 is upward and toward the left in Figure 23.7. If q_3 moves in response to the net force, the distances between q_3 and the other charges change, so the net force changes. Therefore, if q_3 is free to move, it can be modeled as a particle under a net force as long as it is recognized that the force exerted on q_3 is *not* constant.

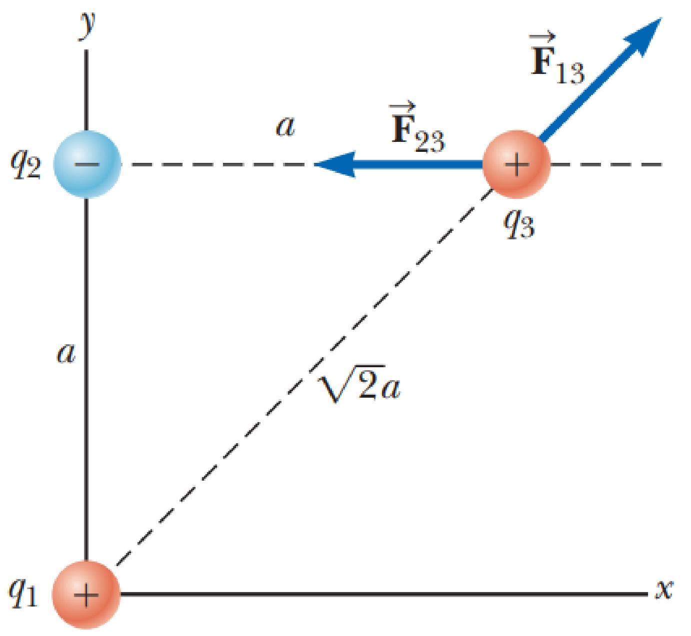


Figure 23.7 (Example 23.2) The force exerted by q_1 on q_3 is $\vec{\mathbf{F}}_{13}$. The force exerted by q_2 on q_3 is $\vec{\mathbf{F}}_{23}$. The resultant force $\vec{\mathbf{F}}_3$ exerted on q_3 is the vector sum $\vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$.

Where Is the Net Force Zero?

Three point charges lie along the x axis as shown in Figure 23.8. The positive charge $q_1 = 15.0\ \mu C$ is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu C$ is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3?

Analyze Write an expression for the net force on charge q_3 when it is in equilibrium:

$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{23} + \vec{\mathbf{F}}_{13} = -k_e \frac{|q_2||q_3|}{x^2}\hat{\mathbf{i}} + k_e \frac{|q_1||q_3|}{(2.00 - x)^2}\hat{\mathbf{i}} = 0$$

Move the second term to the right side of the equation and set the coefficients of the unit vector $\hat{\mathbf{i}}$ equal:

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

Eliminate k_e and $|q_3|$ and rearrange the equation:

$$(2.00 - x)^2|q_2| = x^2|q_1|$$

Take the square root of both sides of the equation:

$$(2.00 - x)\sqrt{|q_2|} = \pm x\sqrt{|q_1|}$$

Solve for x:

$$x = \frac{2.00\sqrt{|q_2|}}{\sqrt{|q_2|} \pm \sqrt{|q_1|}}$$

Substitute numerical values, choosing the plus sign:

$$x = \frac{2.00\sqrt{6.00 \times 10^{-6}\ \text{C}}}{\sqrt{6.00 \times 10^{-6}\ \text{C}} + \sqrt{15.0 \times 10^{-6}\ \text{C}}} = 0.775\ \text{m}$$

Finalize Notice that the movable charge is indeed closer to q_2 as we predicted in the Conceptualize step. The second solution to the equation (if we choose the negative sign) is $x = -3.44$ m. That is another location where the *magnitudes* of the forces on q_3 are equal, but both forces are in the same direction, so they do not cancel.

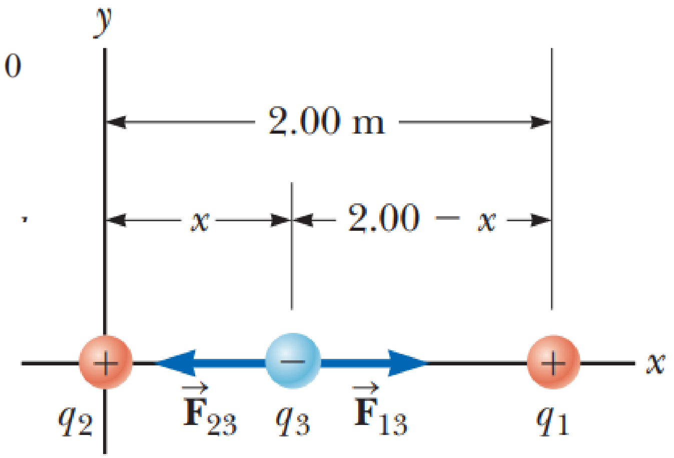
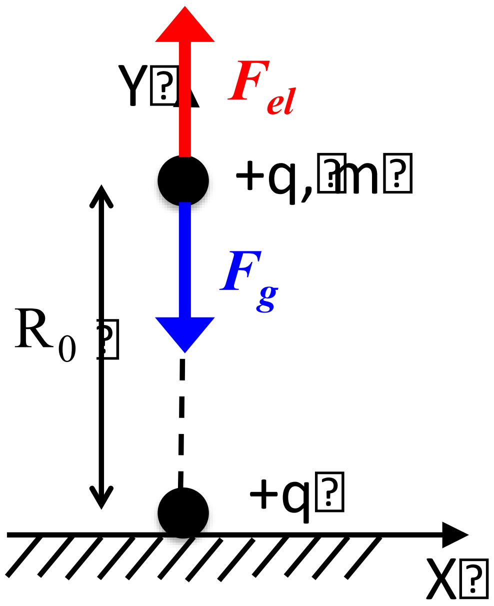


Figure 23.8 (Example 23.3) Three point charges are placed along the x axis. If the resultant force acting on q_3 is zero, the force $\vec{\mathbf{F}}_{13}$ exerted by q_1 on q_3 must be equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{23}$ exerted by q_2 on q_3.

Suppose q3 is constrained to move only along the x axis. From its initial position, it is pulled a small distance along the x axis. When released, does it return to equilibrium, or is it pulled farther from equilibrium? That is, is the equilibrium stable or unstable? And what about a displacement of q3 along the y-axis?

Solving problems

Two small identical charges +q are placed strictly on a vertical line (see figure) and rest at the equilibrium. The lower charge is fixed to the surface of the earth ground. The upper charge has the mass m and can freely move.



1. Evaluate the distance R_0 at equilibrium

Forces: $F_g = mg; \quad F_{el} = k\frac{q^2}{R^2}$

At equilibrium:

$$F_g = F_{el} \quad \Rightarrow \quad mg = k\frac{q^2}{R_0^2} \quad \Rightarrow \quad R_0 = \sqrt{\frac{kq^2}{mg}}$$

Average exam score: 4.9/5

Solving problems

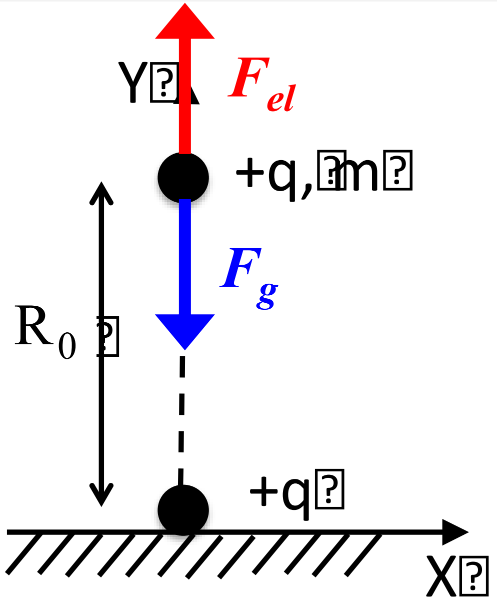
2. Where the upper charge will be after long time, if it is slightly pushed out of equilibrium horizontally (along X)?

a) Fall on the ground.

b) Pushed to the sky

c) Rotate around the Earth

d) Return to the equilibrium position



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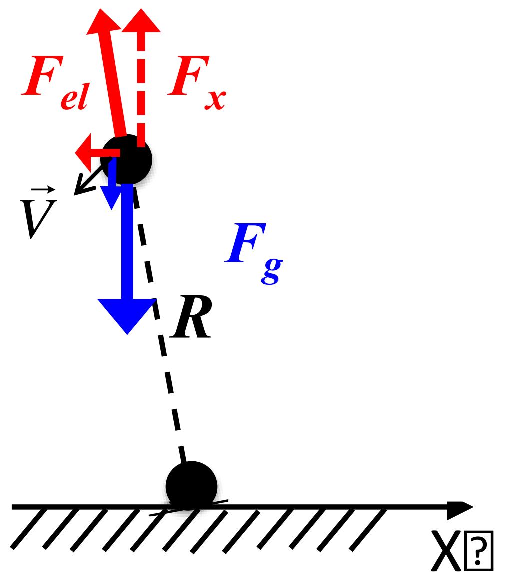

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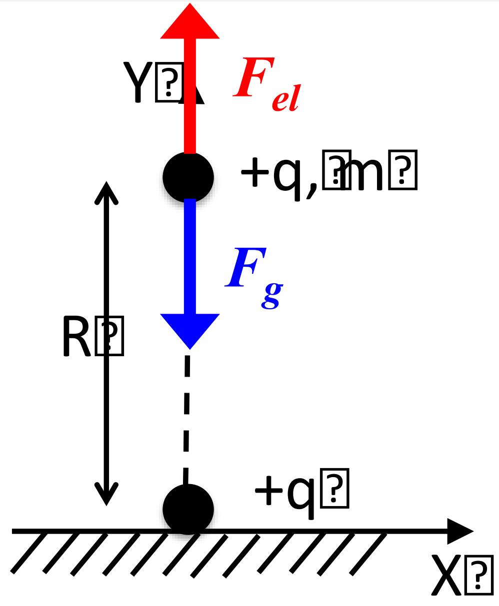
For horizontal shift: R > R_0 =>

$F_g > F_{el}$ => F_Y and F_X => **a)**

Average exam score: 3.75/5

Solving problems

3. The upper charge is slightly pushed strictly along the Y-axis. Write the equation of the motion y”=f(y) for the upper charge.



$$F_\Sigma = ma = \frac{kq^2}{R^2} - mg; \quad R = R_0 + y;$$

$$a = \frac{kq^2}{mR^2} - g = \frac{kq^2}{m}\frac{1}{(R_0+y)^2} - g$$

$$\frac{1}{(R_0+y)^2} = \frac{1}{R_0{}^2(1+y/R_0)^2} =$$

Taylor expantion for small x:

$$(1+x)^\alpha \simeq 1+\alpha x$$

$$\alpha = -2 \Rightarrow 1-2x$$

$$a = \left(\frac{kq^2}{mR_0{}^2} - g\right) - \frac{2kq^2}{mR_0{}^3}y = -\frac{2kq^2}{mR_0{}^3}y$$

Remember you have already found for the equilibrium condition:

$$R_0 = \sqrt{\frac{kq^2}{mg}}$$

$$a \equiv \frac{d^2y}{dt^2} = -\omega^2 y; \quad \omega = \sqrt{\frac{2kq^2}{mR_0{}^3}}.$$

Average exam score: 1.25/5