

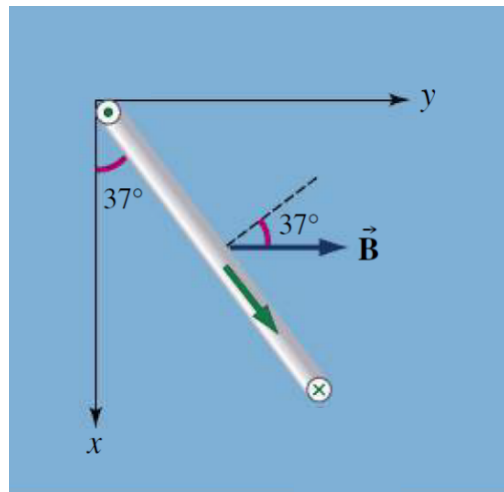
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## General Physics: Electromagnetism, Problem Set 9

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### Exercise 1 :

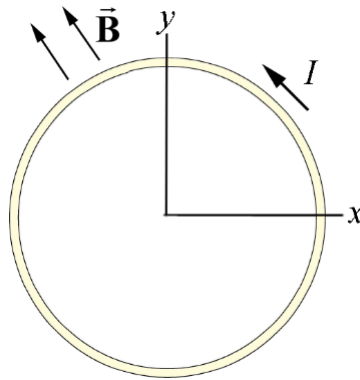
A rigid wire loop of square shape has sides of length 20 cm. It has five turns and is carrying a current of 2 A (indicated by green arrow). The normal to the loop makes an angle of  $37^\circ$  with a uniform magnetic field  $B = 0.5\hat{y}$  T.



- Find the magnetic dipole moment.
- Find the magnitude and direction of the torque acting on the loop.
- Find the work that an external agent must provide to rotate the frame from its position of minimum energy to the given position.

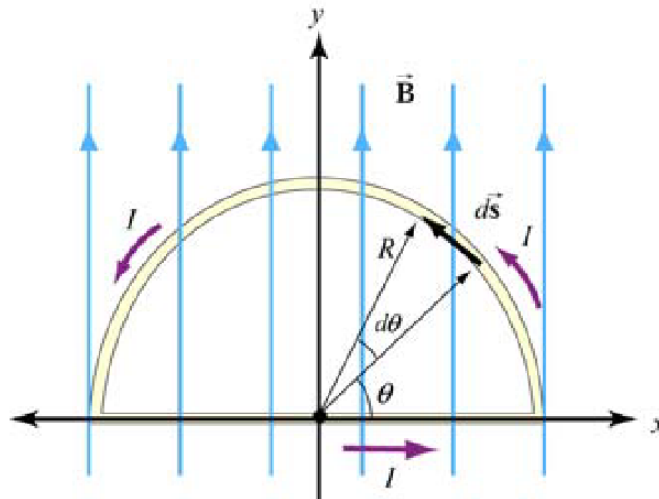
### Exercise 2 :

A wire ring lying in the  $xy$ -plane with its center at the origin carries a counterclockwise current  $I$ . There is an external uniform magnetic field  $B = B_x \hat{i} + B_y \hat{j}$  such that  $B_y > 0$  and  $B_x < 0$ . The magnetic moment vector  $\vec{\mu}$  is perpendicular to the plane of the loop and has magnitude  $\mu = IA$  and the direction is given by right-hand-rule with respect to the direction of the current. What is the direction and magnitude of the torque on the loop?



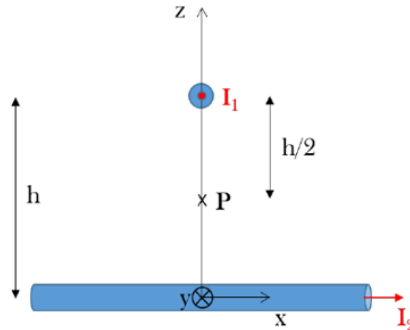
### Exercise 3 :

Consider a closed semi-circular loop lying in the  $xy$  plane carrying a current  $I$  in the counterclockwise direction, as shown in the figure below. A uniform magnetic field pointing in the  $+y$  direction is applied. Find the magnetic force acting on the straight segment and the semicircular arc.



### Exercise 4 :

Two long wires of radius  $a$  are perpendicularly oriented as shown in figure below. The upper wire has a current  $I_1$  in the  $\hat{y}$  direction and the lower cable has a current  $I_2$  in the  $\hat{x}$  direction.



- Find the magnetic field along the  $z$  axis, between  $z = a$  and  $z = h$ . Use the fact that the magnitude of the magnetic field around a straight wire carrying current  $I$  is given by  $B = \frac{\mu_0 I}{2\pi r}$ , where  $r$  is the perpendicular distance from the wire.
- For  $I_1 = 100$  A and  $I_2 = 150$  A, with the distance  $h = 2.5$  cm, what is the magnitude of the magnetic field at point  $P$ ?
- Describe the direction of the compass needle placed in point  $P$ .  
**Hint:** The magnetic field of Earth is about  $5 \cdot 10^{-5}$  T.

### Exercise 5 :

Consider two infinitely long wires carrying currents are in the  $x$  direction.

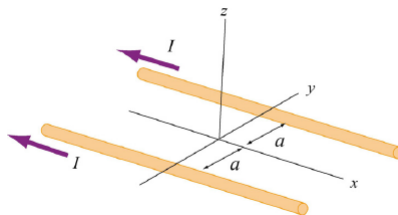


Figure 1: Non-uniform current density

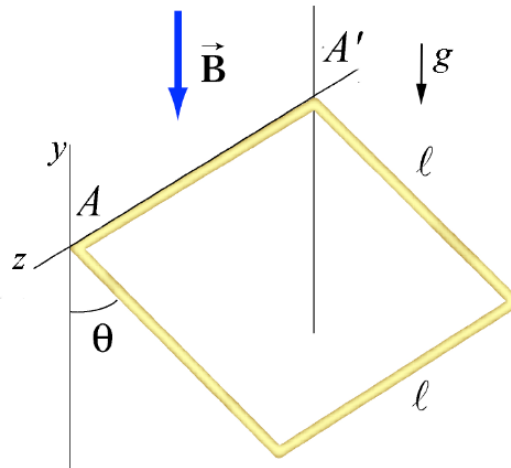
- Draw a schematic of the magnetic field pattern in the  $yz$ -plane.
- Find the distance  $d$  along the  $z$ -axis where the magnetic field is maximum. Use the fact that the magnitude of the magnetic field around a straight wire carrying current  $I$  is given by  $B = \frac{\mu_0 I}{2\pi r}$ , where  $r$  is the perpendicular distance from the wire.

### Exercise 6 :

Find the minimum diameter of the wires  $d$  that can transmit  $P = 225$  MW of electricity with only a 2.0% loss. Their length is  $l = 185$  km. Assume there are two wires to make a complete circuit (the length is thus doubled). The wires are to be made of aluminum ( $\rho = 2.6 \cdot 10^{-8} \Omega \cdot m$ ) and the voltage is  $V = 660$  kV.

### Exercise 7 :

A square loop of wire, of length  $l$  on each side, and mass  $m$ , pivots about an axis  $AA'$  that corresponds to a horizontal side of the square, as shown in the figure on the left below. The external magnetic field  $\vec{B}$  of magnitude  $B$  is directed vertically downward, and uniformly fills the region in the vicinity of the loop. A current  $I$  flows around the loop. The gravitational torque on the loop and the magnetic torque on the loop sum to zero when the loop makes an angle  $\theta$  with the  $z$ -axis. The magnitude of the gravitational field is  $g = 9.8 \text{ m s}^{-2}$ .



- In what direction does the current need to flow in order that the magnetic torque acts in an opposite direction from the gravitational torque?
- Calculate the magnitude of the magnetic torque on this loop of wire in terms of the quantities given.
- Suppose that the mass of the loop  $m = 0.4$  kg and the length of a side is  $l = 1.0$  m . Suppose that when current in the loop is  $I = 2.0$  A , the torques on the loop balance when  $\theta = 45^\circ$  . What is the magnitude of the magnetic field?