General Physics: Electromagnetism, Correction 8

Exercise 1 :

Electronic devices (like computers for example) usually use RC circuits to protect against current failure (as shown in the Figure 1). If the power source stops working (which can be represented by opening the switch S), the capacitor will supply voltage in the circuit until it discharges. If the protective circuit has to maintain the supply voltage at at least 75% of the full voltage for 0.20 s. What is the resistance R needed to maintain this voltage? The capacity of the capacitor is 8.5 μF. Suppose that the electronic device attached to the circuit consumes negligible current.

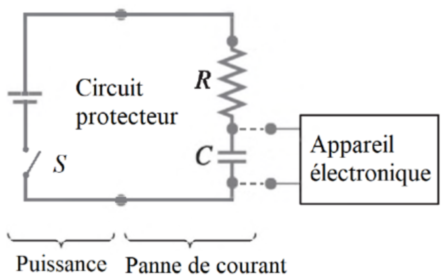


Figure 1: Scheme of an example of a protection circuit in electronic devices.

Solution 1 :

If everything works correctly, the capacitor is completely charged by the power supply. Hence, the

voltage in the capacitor is the same as the voltage of the power supply and there will not be any current going through the resistance. If there is an interruption, the voltage in the capacitor will decrease exponentially (i.e. the capacitor is discharging). We want the voltage at the terminals of the capacitor to be 75% of the full voltage after 0.20 s. The equation of the capacitor discharge is:

$$V = V_0 e^{-\frac{t}{RC}} \tag{1}$$

$$0.75 V_0 = V_0 e^{-\frac{0.20\ s}{RC}} \tag{2}$$

$$0.75 = e^{-\frac{0.20\ s}{RC}} \tag{3}$$

$$ln(0.75) = -\frac{0.20\ s}{RC} \tag{4}$$

$$R = -\frac{0.20\ s}{ln(0.75) \cdot C} \tag{5}$$

$$\tag{6}$$

$$R = -\frac{0.20\ s}{ln(0.75) \cdot 8.5 \cdot 10^{-6}\ F} = \boxed{81790\,\Omega} \tag{7}$$

Note that the electronic device has to be directly connected to the capacitor.

Exercise 2 :

Consider a circuit below that contains several resistors $R = 5\ \Omega$, two batteries with $emf = E$ each, switch S and capacitor $C = 10^{-6}$F. At time t = 0 the capacitor has no charge and the switch closes.

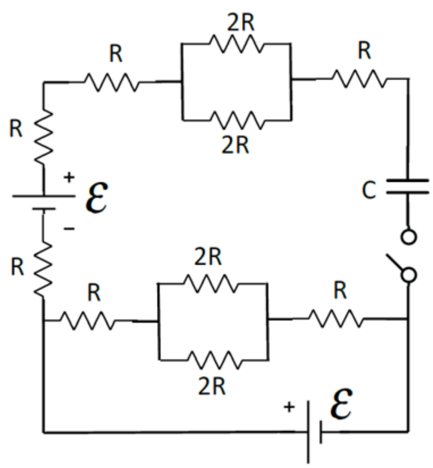


Figure 2: Circuit with two batteries, a switch and multiple resistors.

(a) Draw a simplified equivalent circuit of this initial complex circuit.

(b) Determine the time-constant for charging the capacitor (use the solution of the differential equation for a RC circuit powered by a battery from the lecture).

Solution 2 :

(a) The simplified circuit reads:

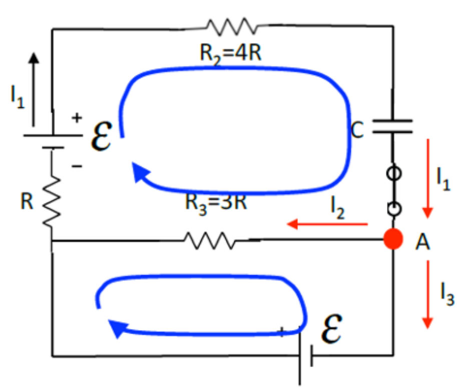


Figure 3: Simplified circuit

where R_2 was calculated the following way:

$$R_2 = R + R + \left(\frac{1}{2R} + \frac{1}{2R}\right) + R = 4R \tag{8}$$

and R_3 was calculated the following way:

$$R_3 = R + \left(\frac{1}{2R} + \frac{1}{2R}\right) + R = 3R \tag{9}$$

(b) First we apply Kirchhoff's rules and then we solve the differential equations that link charge and current on C.

1) Consider the two loops as shown in Figure 3. The choice is arbitrary, but this coincides with positive potentials of both batteries which is more convenient.

2) Define the directions of the currents in the upper loop. Here it is taken in the direction of the loop. At the junction point A the current I_1 splits into I_2 and I_3. The direction of the current I_3 is therefore defined. It is equally possible to consider the lower loop or the second junction point.

3) Kirchhoff's rules are applied for the junction A and the two loops. Remember, the **current** is **positive**, if it is in the **same direction** as the **loop**.

$$I_1 = I_2 + I_3 \tag{10}$$

$$E = I_1 R_2 + Q/C + I_2 R_3 + I_1 R \ \ \text{(upper loop)} \tag{11}$$

$$E = -I_2 R_3 \ \ \text{(lower loop)} \tag{12}$$

The equation for charging the capacitor is:

$$\frac{dQ}{dt} = I_1 \tag{13}$$

4) Solving the system of equations:

Plug in (13) and (12) into (11):

$$E = \frac{dQ}{dt} \cdot (R_2 + R) + \frac{Q}{C} - E \longrightarrow \frac{dQ}{dt} \cdot 5RC + Q = 2EC \tag{14}$$

$$\frac{dQ}{dt} \cdot 5RC = 2EC - Q \tag{15}$$

After solving the differential equation (15) which is the same type of equation as for a trivial RC circuit:

$$\frac{dQ}{dt} \cdot rc = \varepsilon c - Q \longrightarrow Q = \varepsilon \cdot c \cdot (1 - e^{-\frac{t}{rc}}) \tag{16}$$

Here $r = 5R$ and $\varepsilon = 2E$. Therefore:

$$Q = 2EC \cdot (1 - e^{-\frac{t}{5RC}}) \tag{17}$$

Finally, the time constant τ for charging the capacitor is:

$$\tau = 5RC = 5 \cdot 5\Omega \cdot 10^{-6}F = 25 \cdot 10^{-6}\ s = 25\ \mu s \tag{18}$$

Exercise 3 :

A conductive plate of width w has current flowing in the $-\hat{x}$ direction. The plate is placed in an homogeneous magnetic field $\overrightarrow{B} = -B\hat{z}$, as shown in the figure below. Suppose the current is carried by electrons that move with a velocity $\overrightarrow{v} = v\hat{x}$.

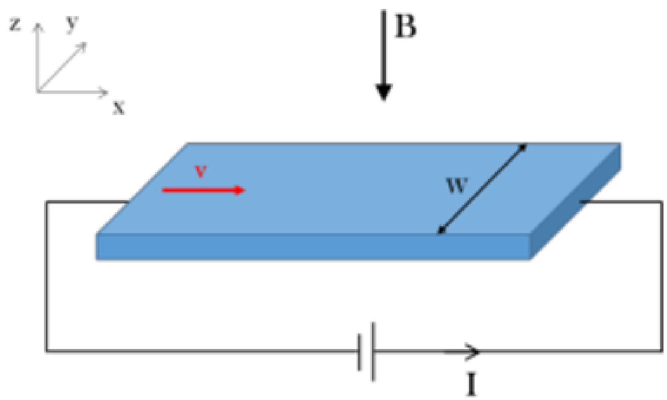


Figure 4: Circuit with a conductive plate in a magnetic field.

a) In which direction will the electrons be deviated ?

b) The deviation of electrons polarises the plate, therefore creating a potential difference between the two sides of the plate and hence, an electric field. The equilibrium is reached when the force generated by the electric field on each electron is equal and of opposite direction to the force on each electron generated by the magnetic field. The electric field at this equilibrium is called *Hall field* E_H. For $B = 0.1$ T and $v = 1.3 \cdot 10^6\ ms^{-1}$ find the value and direction of E_H.

c) Is it possible, by measuring the potential difference between the two sides of the plate, to determine whether the charge carriers in the conductor are electrons or positive ions?

d) Suppose now that the magnetic field is unknown. Would it be possible to determine the magnetic field by adding a component to the circuit? What are the limitations of this measurement system?

Solution 3 :

a) The motion of the electrons is governed by the Lorentz force, which reads:

$$\vec{F}_B = q\vec{v} \times \vec{B}, \tag{19}$$

where $q = -e$, $\vec{B} = -B\hat{z}$, $\vec{v} = v\hat{x}$. Hence, we have

$$\vec{F}_B = evB\hat{x} \times \hat{z} = -evB\hat{y}. \tag{20}$$

The electron will thus be subjected to a force in the direction $-\hat{y}$. This gives an acceleration and a deviation of the path in the direction of $-\hat{y}$.

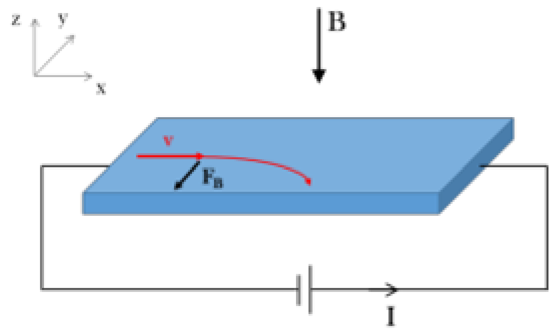


Figure 5: Schematics of the Hall effect. A conductive plate is placed into a magnetic field. This causes a deviation of the electrons in the plate due to the action of the Lorentz force.

b) At equilibrium we have $\vec{F}_B + \vec{F}_E = 0$ that means

$$-evB\hat{y} - eE_H\hat{y} = 0, \tag{21}$$

that is $|E_H| = vB = 1.3 \cdot 10^5\ Vm^{-1}$ in the direction of $-\hat{y}$.

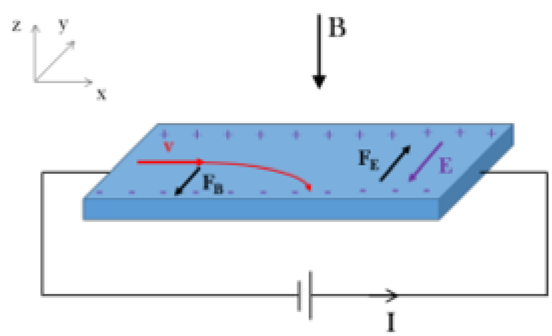


Figure 6: Schematics of the Hall effect. The motion of the electrons polarizes the plates creating an electric field, the Hall field.

c) The magnetic force depends on the speed of the charge carrier as well as the charge. If the current is created by positively charged ions instead of electrons, as in some semiconductors, the velocity would be in the same direction as the current.

Since the ions are positively charged, and the velocity has the opposite sign, the Lorentz force remains in the same direction. However, there would be an accumulation of positive charge on the opposite side with respect to the situation studied in the previous point. This leads to an electric field pointing in the opposite direction than before.

Measuring the potential difference would therefore reveal the charge of the particles, due to the reversed electric field.

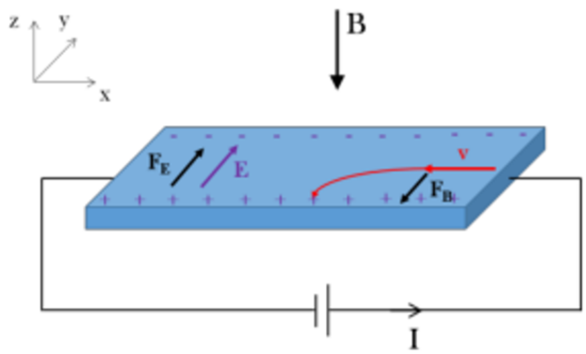


Figure 7: Schematics of the Hall effect. The electric field changes sign if the charge of the carriers is positive instead of negative.

d) A possible strategy is to add a voltmeter on the sides of the plate (see Figure 8)

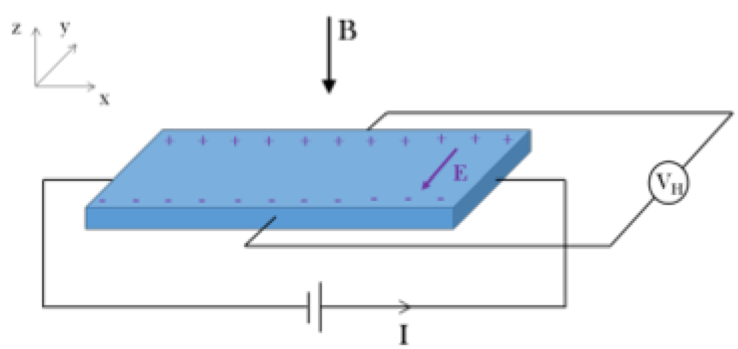


Figure 8: Schematics of the Hall effect. Measuring the voltage in the plate can reveal the magnetic field perpendicular to the velocity of the carriers and to the Hall field.

With this we can measure V_H finding the Hall field $E_H = V_H/w$. Following the answer found in part **b)**, we can calculate the amplitude of the magnetic field in the $\hat{z}$ direction as:

$$B_z = \frac{-E_H}{v} = \frac{-V_H}{wv}. \tag{22}$$

The issue is that this will only measure the component of the field along $\vec{z}$, because the equation gives us the component of the field perpendicular to the speed of the electrons and perpendicular to the electric field. Hence, if the magnetic field is tilted with respect to the $\vec{z}$ axis, we would only measure part of the field, and not the total field.

Exercise 4 :

1. A rectangular copper strip $d = 1.5$ cm wide and $t = 0.10$ cm thick carries a current of 5.0 A. Find the Hall voltage for a magnetic field of $B = 1.2$ T applied in a direction perpendicular to the strip. For finding the charge carrier density, assume that one electron per atom is available for conduction and use the molar mass $M = 0.0635$ kg/mol and density $\rho = 8920$ kg/m^3 of copper.

   Hint: To find the drift velocity v_d of the charge carriers with charge q, you can consider a uniform charge carrier density $n = N/V$. The current through the strip is then given by $I = qnAv_\mathrm{d}$, where $A = d \cdot t$ is the cross-sectional area of the strip.

2. Now, consider a strip of the same dimensions, but made of silicon with a typical charge carrier density of $n = 1.0 \cdot 10^{20}$ electrons/m^3. Find the Hall voltage for a magnetic field of $B = 1.2$ T. Which material is better suited for measuring magnetic fields?

Solution 4 :

1. The Hall voltage is given by
$$\Delta V_\mathrm{H} = v_\mathrm{d} B d \tag{23}$$
   Using $I = qnAv_\mathrm{d}$ from the hint, we can find $v_\mathrm{d} = I/qnA$. Because we assume that one electron per atom is contributing to the current, we can find the charge carrier density n by using
$$n = \frac{N}{V} = \frac{N_\mathrm{A} m}{MV} = \frac{N_\mathrm{A}\rho}{M} = 8.46 \times 10^{28} \text{ electrons}/m^3, \tag{24}$$
   where N is the total number of atoms in the strip, $N_\mathrm{A} = 6.02 \cdot 10^{23}$ mol^{-1} is the Avogadro constant and m is the total mass of the strip. Combining these equations together and using the values given in the problem ($q = e = 1.60 \cdot 10^{-19}$ C), we get
$$\Delta V_\mathrm{H} = v_\mathrm{d} B d = \frac{IBd}{qnA} = \frac{IBdM}{qAN_\mathrm{A}\rho} = \frac{IBM}{qtN_\mathrm{A}\rho} = 0.44\ \mu\text{V}. \tag{25}$$

2. For the second part of the problem, we keep all the values the same, but use the charge carrier density of silicon $n = 1.0 \cdot 10^{20}$ electrons/m^3 instead, so we get
$$\Delta V_\mathrm{H} = v_\mathrm{d} B d = \frac{IBd}{qnA} = \frac{IB}{qnt} = 375 \text{ V}. \tag{26}$$
   The resulting Hall voltage for a strip of silicon is much larger, so we can apply a small current, like $I = 0.1$ mA and still have a sizable Hall voltage $\Delta V_\mathrm{H} = 7.5$ mV, which is easily measurable. Therefore, a device made of a semiconducting material is much more suitable for precise measurements of magnetic fields.

Exercise 5 :

Mass spectrometers, which separate ions based on mass, are often used by chemists to determine the composition of a sample. The aim of the exercise is to explore the working principle of mass spectrometers.

1. The first section of the mass spectrometer is the **accelerator**. A particle of mass m and charge q is released from rest near one plate of a charged parallel-plate capacitor. The particle accelerates towards the other plate of the capacitor. The potential difference between the plates is ΔV.

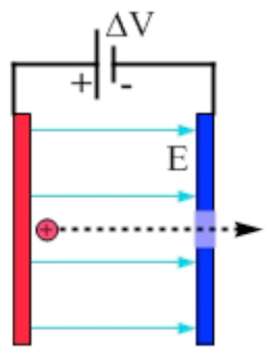


Figure 9: Charged particles are released from rest near the left-hand plate of a parallel plate capacitor, accelerate across the gap, and emerge via a hole cut in the right-hand plate.

a) Apply energy conservation to obtain a relation between, the potential difference across the capacitor, and the speed of the particle when it emerges from a small hole in the second plate.

Solution 5 :

To find the velocity of the particle when it emerges, we use conservation of energy

$$K_i + U_i + W_{nc} = K_f + U_f \tag{27}$$

The particle starts with no initial kinetic energy $K_i = 0$. We define the particle electric potential energy to be zero at the negative plate $U_f = 0$ and $U_i = q\Delta V$. There is no work done by non-conservative forces $W_{nc} = 0$. Consequently,

$$q\Delta V = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2q\Delta V}{m}}. \tag{28}$$

After leaving the accelerator, the particle passes through the **velocity selector**. The velocity selector is a second parallel-plate capacitor in which the plates are parallel to the particle's velocity. In addition to the electric field E inside the capacitor there is also a magnetic field B, directed perpendicular to both the electric field and the velocity v of the particle. The combined effect of the two fields is such that a particle with just the right speed experiences no net force and passes undeflected through the velocity selector.

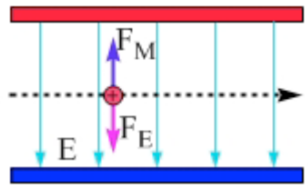


Figure 10: A charged particle with just the right velocity passes undeflected through the velocity selector because the magnetic force balances the electric force.

a) Determine the speed of the undeflected particle.

The electric force on the positive charge is also directed down, because $\vec{F}_E = q\vec{E}$. For the particle to experience no net force, the magnetic force must exactly balance the electric force. The magnitude of the magnetic force is given by $F_B = qvB\sin(\theta)$. Because the velocity and magnetic field are at $\theta = \pi/2$, $\sin(\theta) = 1$. Setting the magnitude of the two forces equal $F_B = F_E$ gives $qvB = qE$. Therefore,

$$v = \frac{E}{B} \tag{29}$$

b) Is the magnetic field in figure above directed into or out of the page?

By the right-hand rule, to obtain a magnetic force directed up on a positive charge with a velocity to the right, the magnetic field is into the page.

c) What happens to particles traveling faster than the undeflected particles? What happens to particles traveling slower than the undeflected particles?

The magnetic force depends on speed, while the electric force does not. For particles going faster than the selected speed, the magnetic force exceeds the electric force. The net upward force deflects the fast particles up out of the beam (see Figure 19.14). For relatively slow particles, the magnetic force is less than the electric force. The net downward force deflects the slow particles down out of the beam.

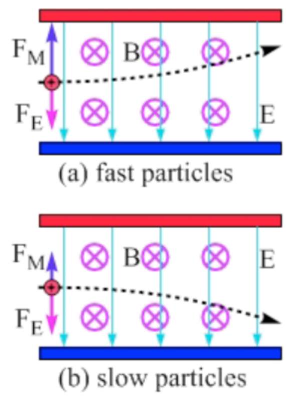


Figure 11: A velocity selector deflects fast particles in one direction (up, in this case) and (b) slow particles in the opposite direction (down, here).

Exercise 6 :

Particles that pass undeflected through the velocity selector of the mass spectrometer in the exercice above are sent into the **mass separator**, which consists of a uniform magnetic field that is perpendicular to the velocity of the particles. Suppose that we want to separate the fissile Uranium isotope U_{235} from the heavier non-fissile U_{238}, such that a mixed beam of U_{235} and U_{238} ions is sent into the mass spectrometer. The ions exit the velocity selector into a homogeneous magnetic field region, with $B = 0.5$ T perpendicular to the beam, as shown in the figure below. The charge of both U isotope ions is $q = 3.2 \cdot 10^{-19}$ C, and the acceleration voltage is 100 kV ($m_{U_{235}} = 3.903 \cdot 10^{-25}\ kg$, $m_{U_{238}} = 3.953 \cdot 10^{-25}\ kg$).

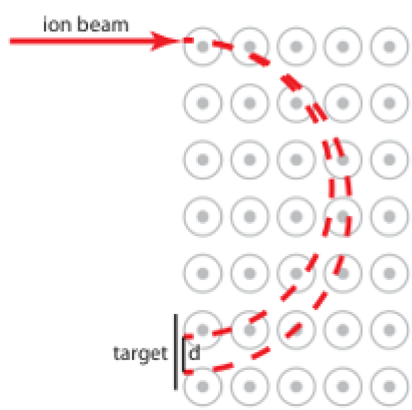


Figure 12: Ion beam in the magnetic field.

a) Evaluate the different speeds of the two ion species $v_{U_{235}}$ and $v_{U_{238}}$ when they enter the magnetic field region.

b) Evaluate the spatial separation d between the two isotopes when they exit the mass spectrometer after the semicircular path in the magnetic field region.

c) What acceleration voltage would be needed in order to get separation of $d = 2\ cm$?

Solution 6 :

a) In accordance with kinetic energy theorem for a charged particle in electric field:

$$qV = \frac{1}{2}mv^2 \longrightarrow v = \sqrt{\frac{2qV}{m}} \tag{30}$$

Hence, we find the following velocities for the two isotopes:

$$v_{U_{235}} = \sqrt{\frac{2qV}{m_{U_{235}}}} = \sqrt{\frac{2 \cdot 3.2 \cdot 10^{-19} \cdot 100 \cdot 10^3}{3.903 \cdot 10^{-25}}} \approx 4.049 \cdot 10^5\ m/s \tag{31}$$

$$v_{U_{238}} = \sqrt{\frac{2qV}{m_{U_{235}}}} = \sqrt{\frac{2 \cdot 3.2 \cdot 10^{-19} \cdot 100 \cdot 10^3}{3.953 \cdot 10^{-25}}} \approx 4.024 \cdot 10^5\ m/s \tag{32}$$

b) The Lorentz force $q\vec{v} \times \vec{B}$ provides the centrifugal force. The magnetic field and the velocity are always perpendicular, so that $|q\vec{v} \times \vec{B}| = qvB$. Therefore,

$$qvB = \frac{mv^2}{r} \tag{33}$$

and we can isolate the radius of the particle trajectories:

$$r = \frac{mv}{qB} = \sqrt{\frac{2mV}{q}}\frac{1}{B} \tag{34}$$

So the spatial separation d between the two isotopes after completing their semicircular trajectories is:

$$d = 2(r_{238} - r_{235}) = \frac{2}{B}\left(\sqrt{\frac{2m_{U_{238}}V}{q}} - \sqrt{\frac{2m_{U_{235}}V}{q}}\right) \approx 1.26\ cm \tag{35}$$

c)

$$d = \frac{2}{B}\left(\sqrt{\frac{2m_{U_{238}}V}{q}} - \sqrt{\frac{2m_{U_{235}}V}{q}}\right) \longrightarrow V = \frac{q}{2}\left(\frac{Bd}{2(\sqrt{m_{U_{238}}} - \sqrt{m_{U_{235}}})}\right)^2 \approx 251\ kV \tag{36}$$

Exercise 7 :

A circuit hangs vertically on the hook of a balance as shown in Figure 13. The balance is calibrated to zero (i.e. we subtract the weight of the circuit). The circuit is partially suspended in a magnetic field perpendicular to the page. The upper part of the circuit is out of the field. The circuit consists of a 1 V battery and a resistor of $0.5\,\Omega$. The width of the circuit is $a = 0.1\,m$ and a length $h = 0.07\,m$ is suspended in the magnetic field. If the balance measures a 'weight' of 3.5 g, what is the magnitude of the magnetic field?

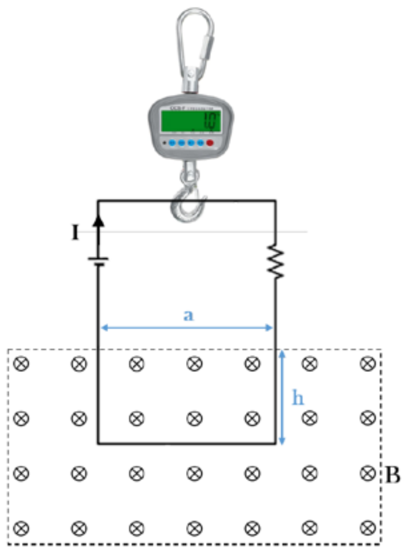


Figure 13: Circuit partially suspended in a magnetic field

Solution 7 :

At equilibrium, the gravitational force and magnetic force will be equal. The magnetic force is defined as:

$$\vec{F}_B = I \oint d\vec{l} \times \vec{B} \tag{37}$$

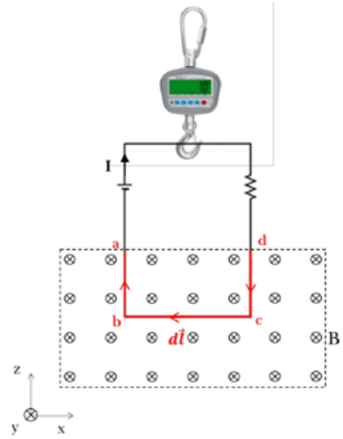
where $d\vec{l}$ is the path through which the current I passes. In our case, we need to consider three different parts of the circuit:

$$\vec{F}_B = I \left(\oint_{ab} d\vec{l} \times \vec{B} + \oint_{bc} d\vec{l} \times \vec{B} + \oint_{cd} d\vec{l} \times \vec{B} \right) \tag{38}$$

From the figure:

$$\vec{F}_B = I \left| \vec{B} \right| \left(h(\hat{z} \times \hat{y}) - a(\hat{x} \times \hat{y}) - h(\hat{z} \times \hat{y}) \right) \tag{39}$$

$$\vec{F}_B = -I \left| \vec{B} \right| a(\hat{x} \times \hat{y}) \longrightarrow \vec{F}_B = -I \left| \vec{B} \right| a\hat{z}. \tag{40}$$



Now we can find the expression for $\left|\vec{B}\right|$ by using the fact that $\vec{F}_B = \vec{F}_g$ at equilibrium:

$$-I\left|\vec{B}\right|a\hat{z} = -mg\hat{z} \Longrightarrow \left|\vec{B}\right| = \frac{mg}{Ia} \tag{41}$$

The current I is found by the Ohm law:

$$I = \frac{V}{R} = \frac{1\ V}{0.5\ \Omega} = 2\ A \tag{42}$$

Finally, we find that the magnitude of the magnetic field is:

$$\left|\vec{B}\right| = \frac{mg}{Ia} = \frac{3.5 \cdot 10^{-3}\ kg \cdot 9.81\ m/s^2}{2\ A \cdot 0.1\ m} = 0.17\ T \tag{43}$$