General Physics: Electromagnetism, Correction 7

Exercise 1 :

Consider the circuit below. The internal resistance of the battery is r = 1 Ω, the resistance of the resistor is R = 10 Ω. What is the EMF of the battery, if the heat power delivered on the resistor is P = 40 W? How much of heat energy W_B will be delivered in the battery during 1 hour?

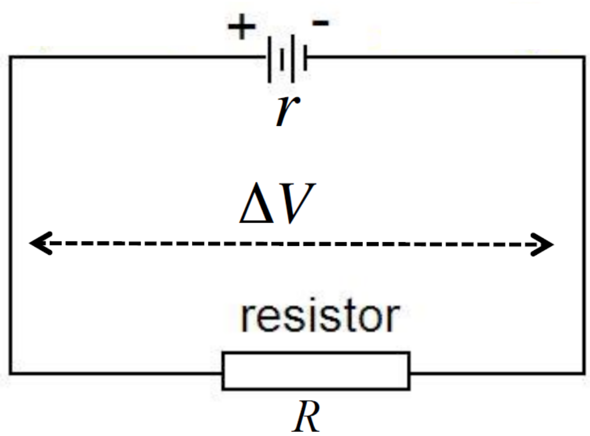


Figure 1: Simple circuit with a battery and a resistor.

Solution 1 :

The relation between the potential difference ΔV or simply V applied to the resistor and the EMF of the battery is given by:

$$\Delta V = EMF - I \cdot r \iff EMF = \Delta V + I \cdot r \tag{1}$$

where I is the current in the circuit.

For the electric power P delivered in the resistor one can write:

$$P = \frac{\Delta V^2}{R} = I^2 R \tag{2}$$

Therefore we find the following expressions for ΔV and I:

$$\Delta V = \sqrt{PR} \tag{3}$$

$$I = \sqrt{\frac{P}{R}} \tag{4}$$

Substituting ΔV and I in **(8)** we get:

$$EMF = \sqrt{PR} + \sqrt{\frac{P}{R}} \cdot r = \sqrt{40W \cdot 10\Omega} + \sqrt{\frac{40W}{10\Omega}} \cdot 1\Omega = 22V \tag{5}$$

The same current goes through the resistor and through the battery. Therefore, the heat delivered in the battery:

$$W_B = I^2 \cdot r \cdot t = \frac{P}{R} \cdot r \cdot t = \frac{40W}{10\Omega} \cdot 1\Omega \cdot 3600s = 14.4\,kJ \tag{6}$$

Exercise 2 :

Two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb in both cases? For which circuit will the bulbs be brighter?

Hint: The more power consumed, the brighter the bulb.

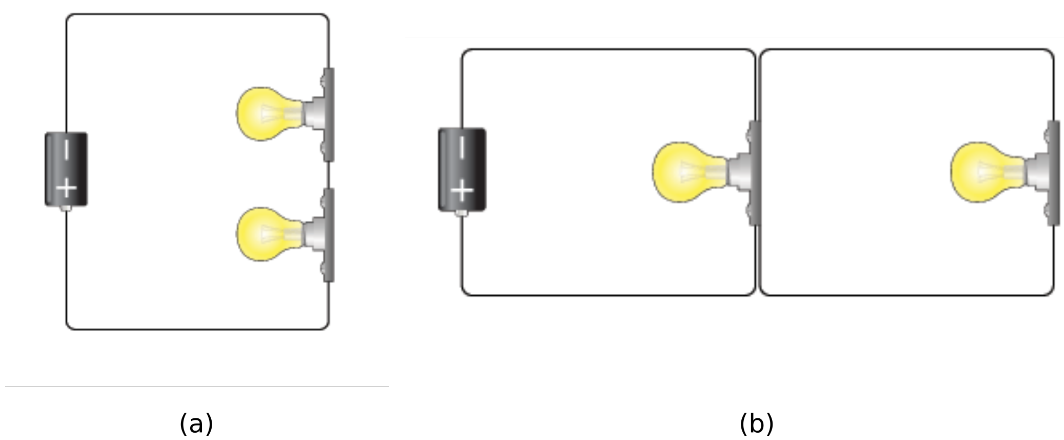


Figure 2: Series (a) and parallel (b) circuit of two light bulbs powered by a battery.

Solution 2 :

(a) In series, the total resistance is given by the sum of the resistances, so we get

$$R_{\text{eq}} = R_1 + R_2$$
$$V = IR_{\text{eq}}$$
$$V = I(R_1 + R_2)$$
$$I = V/(R_1 + R_2) = 24\ \text{V}/(100\ \Omega + 100\ \Omega) = 0.12\ \text{A}$$

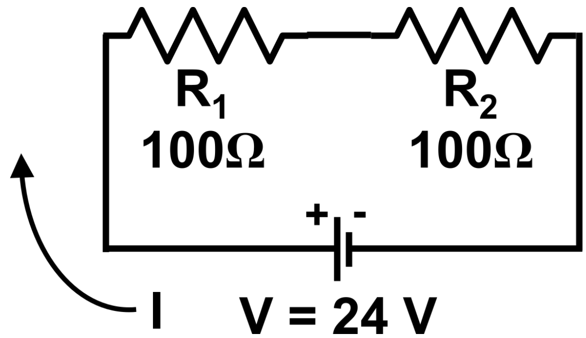


Figure 3: Schematic for the series case.

(b) In parallel, the total resistance is given by a parallel combination, so we get

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{100\ \Omega} + \frac{1}{100\ \Omega}$$
$$R_{\text{eq}} = 50\ \Omega$$
$$V = IR_{\text{eq}} \Rightarrow I = V/R_{\text{eq}}$$
$$I = \frac{24\ \text{V}}{50\ \Omega} = 0.48\ \text{A}$$
$$I_1 = I_2 = I/2 = 0.24\ \text{A}$$

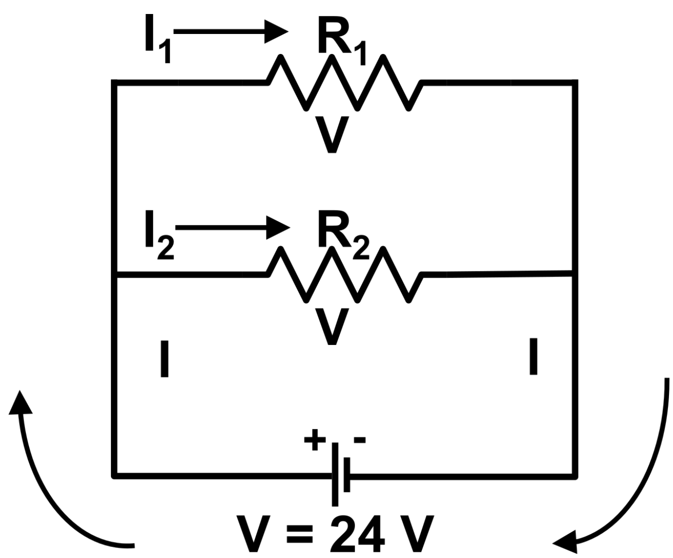


Figure 4: Schematic for the parallel case.

The power consumed by the light bulbs is given by $P = RI^2$. Therefore, $P = (0.12\ \text{A})^2 \times 100\ \Omega = 1.44$ W in the series case and $P = (0.24\ \text{A})^2 \times 100\ \Omega = 5.76$ W for the parallel case, so the light bulbs are brighter when connected in parallel to the battery.

Exercise 3 :

Determine the equivalent resistance of the "ladder" of equal resistors R shown in the figure below. In other words, what resistance would an *ohmmeter* read if connected between points A and B?

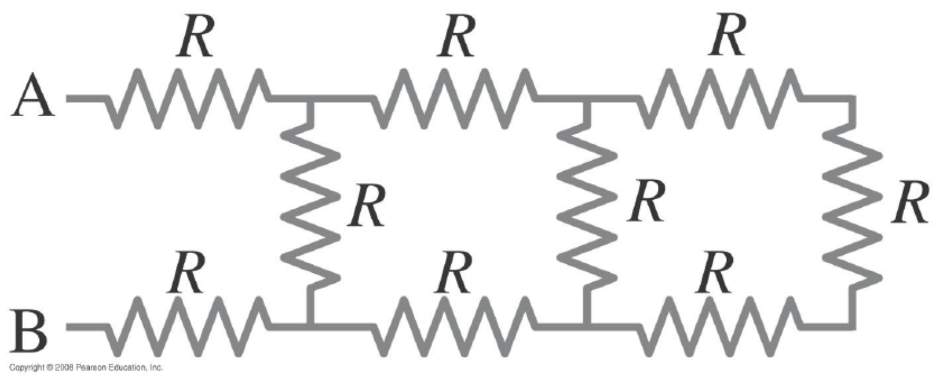
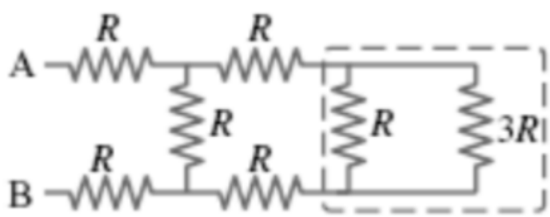


Figure 5: "Ladder" of resistors.

Solution 3 :

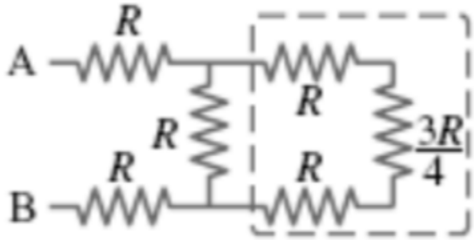
The three resistors on the far right are in series, so their equivalent resistance is $3R$. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the figure below.



The equivalent resistance of the dashed box is found as follows:

$$R_{\text{eq}} = \left(\frac{1}{R} + \frac{1}{3R}\right)^{-1} = \frac{3}{4}R \tag{7}$$

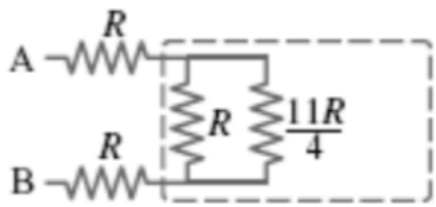
This equivalent resistance of $\frac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in the figure below.



The equivalent resistance of that dashed box is:

$$R_{\text{eq2}} = 2R + \frac{3}{4}R = \frac{11}{4}R \tag{8}$$

R_{eq2} is in parallel with the next resistor to the left, as shown in the figure below.



The equivalent resistance of that dashed box is found as follows:

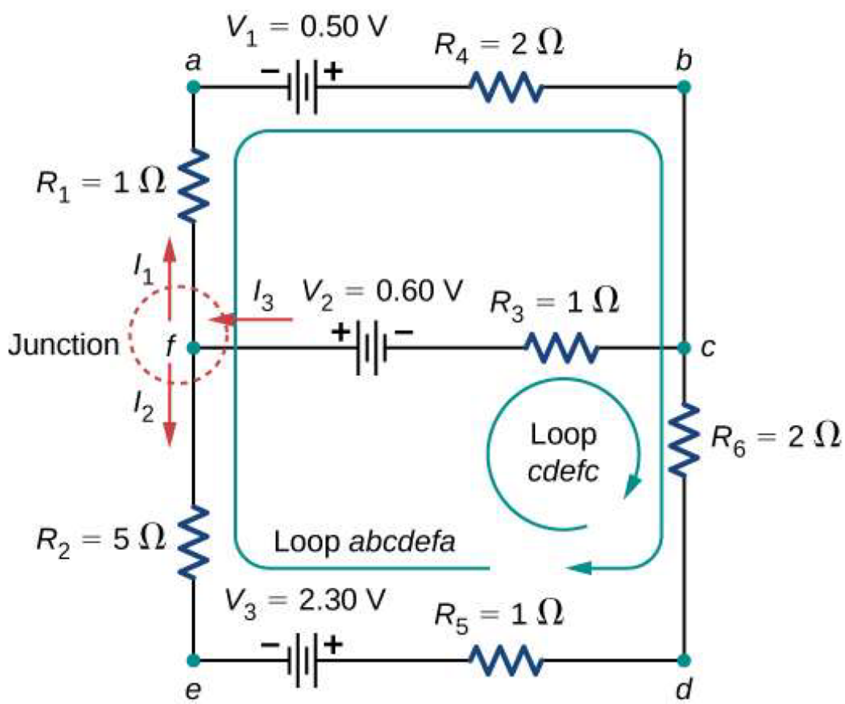
$$R_{\text{eq3}} = \left(\frac{1}{R} + \frac{4}{11R}\right)^{-1} = \frac{11}{15}R \tag{9}$$

R_{eq3} is in series with the remaining two resistors, the ones connected directly to A and B. The final resistance is given as:

$$R_{\text{tot}} = 2R + R_{\text{eq3}} = 2R + \frac{11}{15}R = \frac{41}{15}R \tag{10}$$

Exercise 4 :

Calculate the current in each resistor inside the circuit shown below.



Solution 4 :

This circuit is sufficiently complex that the currents cannot be found using Ohm 's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules.

One equation comes from Kirchhoff's junction rule applied to the junction f.

$$I_1 + I_2 - I_3 = 0 \tag{11}$$

Another equation comes for Kirchoff's loop rule applied to the loop $abcdefa$ following the direction given in the schematic.

$$I_1(R_1 + R_4) - I_2(R_2 + R_5 + R_6) - V_1 + V_3 = 0 \tag{12}$$

The final equation can be taken from Kirchhoff's loop rule applied to the $cdefc$ loop.

$$I_2(R_2 + R_5 + R_6) + I_3R_3 - V_2 - V_3 = 0 \tag{13}$$

Substituting the numerical values, we have :

$$I_1 + I_2 - I_3 = 0 \tag{14}$$

$$3I_1 - 8I_2 = -1.8 \tag{15}$$

$$8I_2 + I_3 = 2.90 \tag{16}$$

With these equations, we can solve by substituting $I_3 = I_1 + I_2$ in the second loop equation:

$$9I_2 + I_1 = 2.9 \tag{17}$$

From the first loop equation, we have:

$$I_1 - \frac{8}{3}I_2 = -0.6 \tag{18}$$

Subtracting these two equations, we get:

$$9I_2 + \frac{8}{3}I_2 = 2.9 + 0.6 \rightarrow I_2 = 0.3A \tag{19}$$

Then,

$$9I_2 + I_1 = 2.9 \rightarrow I_1 = 2.9 - 9 \cdot 0.3 = 0.2A \tag{20}$$

and,

$$I_3 = I_2 + I_1 \rightarrow I_3 = 0.5A \tag{21}$$

The current in each resistor is as follows:

$$\boxed{R_1\text{: 0.2A} \quad R_2\text{: 0.3A} \quad R_3\text{: 0.5A} \quad R_4\text{: 0.2A} \quad R_5\text{: 0.3A} \quad R_6\text{: 0.3A}}$$

A method to check the calculations is to compute the power dissipated by the resistors and the power supplied by the voltage sources:

$$\begin{aligned}
P_{R_1} &= I_1^2 R_1 = 0.04\,\text{W}, \\
P_{R_2} &= I_2^2 R_2 = 0.45\,\text{W}, \\
P_{R_3} &= I_3^2 R_3 = 0.25\,\text{W}, \\
P_{R_4} &= I_1^2 R_4 = 0.08\,\text{W}, \\
P_{R_5} &= I_2^2 R_5 = 0.09\,\text{W}, \\
P_{R_6} &= I_2^2 R_6 = 0.18\,\text{W}, \\
P_{\text{dissipated}} &= 1.09\,\text{W}.
\end{aligned}$$

$$P_{\text{source}} = I_1 V_1 + I_2 V_3 + I_3 V_2 = 0.10\,\text{W} + 0.69\,\text{W} + 0.30\,\text{W} = 1.09\,\text{W}.$$

The power supplied equals the power dissipated by the resistors.

Exercise 5 :

Calculate the current in each resistor inside the circuit shown below.

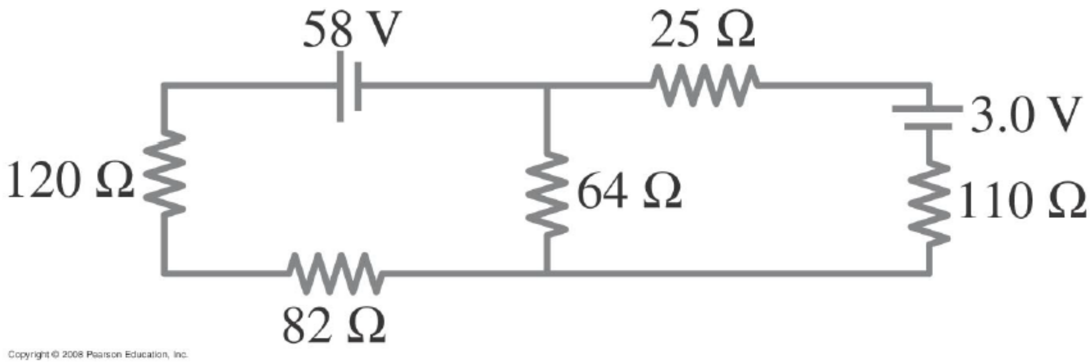
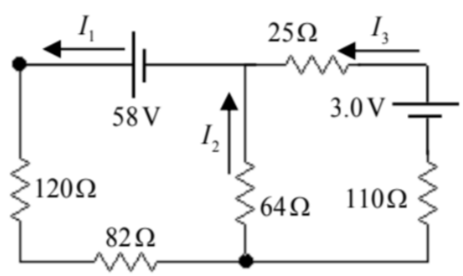


Figure 6: Circuit with two batteries and resistors.

Solution 5 :

There are three currents involved, and so there must be three independent equations to determine those three currents.



One equation comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.

$$I_1 = I_2 + I_3 \tag{22}$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$58V - I_1 \cdot 120\Omega - I_1 \cdot 82\Omega - I_2 \cdot 64\Omega = 0 \longrightarrow 202I_1 + 64I_2 = 58 \tag{23}$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0V - I_3 \cdot 25\Omega + I_2 \cdot 64\Omega - I_3 \cdot 110\Omega = 0 \longrightarrow -64I_2 + 135I_3 = 3 \tag{24}$$

Substitute equation **(25)** into the left loop equation **(26)**, so that there are two equations with two unknowns.

$$202I_1 + 64I_2 = 58 \;\rightarrow\; 202(I_2 + I_3) + 64I_2 = 58 \;\rightarrow\; 266I_2 + 202I_3 = 58 \tag{25}$$

Solve the right loop equation **(27)** for I_2 and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$-64I_2 + 135I_3 = 3 \;\rightarrow\; I_2 = \frac{135I_3 - 3}{64} \tag{26}$$

$$266I_2 + 202I_3 = 58 \;\rightarrow\; 266\frac{135I_3 - 3}{64} + 202I_3 = 58 \;\rightarrow\; \boxed{I_3 = 0.09235A} \tag{27}$$

$$I_2 = \frac{135I_3 - 3}{64} = \frac{135 \cdot 0.09235A - 3}{64} = \boxed{0.1479A} \tag{28}$$

$$I_1 = I_2 + I_3 = 0.1479A + 0.09235A = \boxed{0.24025A} \tag{29}$$

The current in each resistor is as follows:

120Ω: 0.24A	82Ω: 0.24A	64Ω: 0.15A	25Ω: 0.092A	110Ω: 0.092A

Exercise 6 :

A good battery of a car is used to start a second car with a low battery. The good battery has an EMF of 12.5 V and an internal resistance of 0.020 Ω. Let's suppose that the low battery has an EMF of 10.1 V and an internal resistance of 0.10 Ω. The cables have a resistance $R_j = 0.0026$ Ω each and can be connected as shown in the figure below. Let's suppose that all the rest of the car can be represented as a $R_s = 0.15$ Ω.

1. Find the current I_3 flowing into the starter motor if only the low battery is connected;
2. Find the current flowing into the starter motor if now also the good battery is connected.

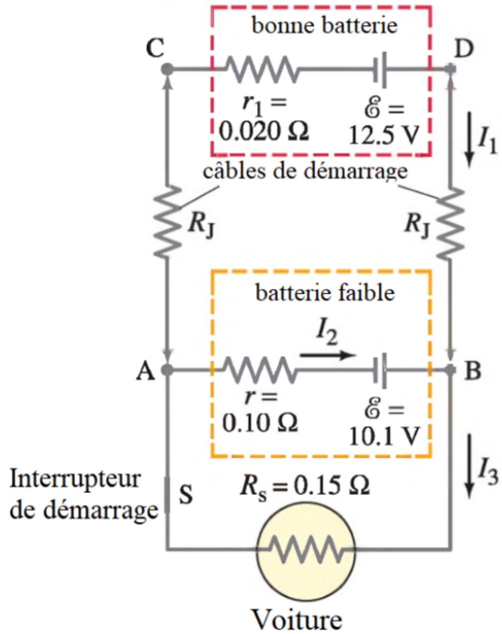


Figure 7: Schematic of the equivalent circuit.

Figure 8: Two batteries, a good one and a low one, connected as shown in the circuit on top.

Solution 6 :

1. The circuit containing just the low battery and not the jumper cables is simple: one EMF of $10.1V$ connected to two resistances in series, $0.10\Omega + 0.15\Omega = 0.25\Omega$. The current is then:

$$I = \frac{V}{R} = \frac{10.1V}{0.25\Omega} = 40A \tag{30}$$

2. Apply Kirchhoff rule for the complete outer loop:

$$12.5V - I_1(2R_J + r_1) - I_3R_s = 0 \quad (31)$$
$$12.5V - I_1 \cdot 0.025\Omega - I_3 \cdot 0.15\Omega = 0 \quad (32)$$

The rule for the bottom loop with the low battery and the starter is:

$$10.1V - I_3 \cdot 0.15\Omega - I_2 \cdot 0.10\Omega = 0 \quad (33)$$

In point B we have: $I_1 + I_2 = I_3$. Using this relation, we can eliminate I_1 from equation **(35)** and hence we get the following system of equations:

$$12.5V - (I_3 - I_2) \cdot 0.025\Omega - I_3 \cdot 0.15\Omega = 0 \quad (34)$$
$$10.1V - I_3 \cdot 0.15\Omega - I_2 \cdot 0.10\Omega = 0 \quad (35)$$

By combining this two equations we find that $I_3 = 71A$. This current is better than what was found in **1.** The values for other currents are $I_2 = -5A$ and $I_1 = 76A$. Note that I_2 is in the opposite sense than the one indicated in the figure. A circuit like this efficiently charges the low battery.

Exercise 7 :

Consider a hollow cylinder of length L and inner radius a and outer radius b, as shown in the figure below. The material has resistivity ρ.

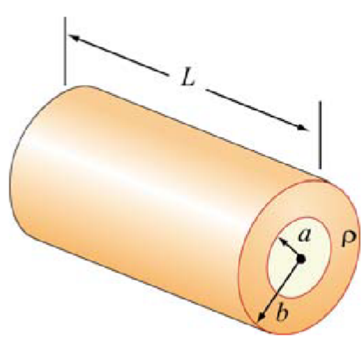


Figure 9: A hollow cylinder

1. Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?

2. If instead the potential difference is applied between the inner and outer surfaces so that current flows radially outward, what is the resistance measured?

Solution 7 :

1. When a potential difference is applied between the ends of the cylinder, current flows parallel to the axis. In this case, the cross-sectional area is $A = \pi(b^2 - a^2)$, and the resistance is given by
$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi(b^2 - a^2)} \tag{36}$$

2. Consider a differential element which is made up of a thin cylinder of inner radius r and outer radius $r + dr$ and length L. Its contribution to the resistance of the system is given by
$$dR = \frac{\rho dl}{A} = \frac{\rho dr}{2\pi r L} \tag{37}$$
where $A = 2\pi r L$ is the area normal to the direction of current flow. The total resistance of the system becomes
$$R = \int_a^b \frac{\rho dr}{2\pi r L} = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right) \tag{38}$$