# General Physics: Electromagnetism, Problem Set 5

## Exercise 1 :

A wire having a uniform linear charge density $\lambda$ is bent into the shape shown in Fig.1. Find the electric potential and the electric field at point $O$.

- **Hint 1**: use the superposition principle and compute the total potential as the sum of three separate contributions in the three different regions. Remember that the electric potential at distance $r$ generated by a continuous charge distribution is $V(r) = k \int \frac{dq}{r}$.
- **Hint 2**: express the infinitesimal charge $dq$ in the curved region as a function of the infinitesimal angle $d\theta$;
- **Hint 3**: for the electric field computation, use the symmetry of the wire to cancel out some contributions.

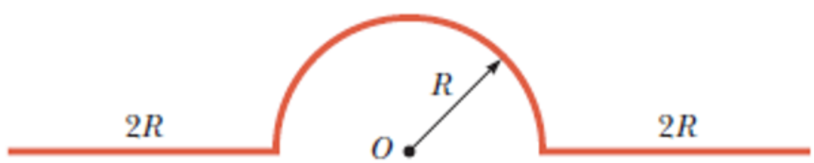


Figure 1: A bent wire with uniform charge density $\lambda$.

## Exercise 2 :

Time of flight mass spectrometer (TOF MS) determines mass-to-charge ratio of ions by measuring time of their flight in a field free region (see Fig.2). It consists of a metal plate **A**, two metal grids **B** and **C**, which are transparent for ions, and a Detector. The grids **B** and **C** are grounded (i.e potential V=0), the plate **A** can be put at the fixed positive potential $V_0 = +1000\ V$ by closing the switch $S$. The ion Detector, which is very close to **C**, is at some high negative potential. Positively singly charged ions are initially very close to the plate **A** (they do not interact with each other). At a well-defined time $t = 0$ the switch $S$ is **on** and $V_0$ is being applied to **A** and ions begin to move toward the detector.

- **Hint**: the potential energy of a charge $q$ in a potential $V_0$ is given by $E_{\text{potential}} = qV_0$.

(a) Show that ions of different masses ($q = +1$) will arrive to the detector at different time. Derive an expression for time-of-flight (the time between B and C).

(b) Estimate the time-resolution of the detector, required to distinguish masses of uranium isotopes: $^{238}U$ ($m_1 = 3.95 \cdot 10^{-25}$ kg) and $^{235}U$ ($m_2 = 3.90 \cdot 10^{-25}$ kg). The time-resolution represents the time variation needed to detect this mass variation.

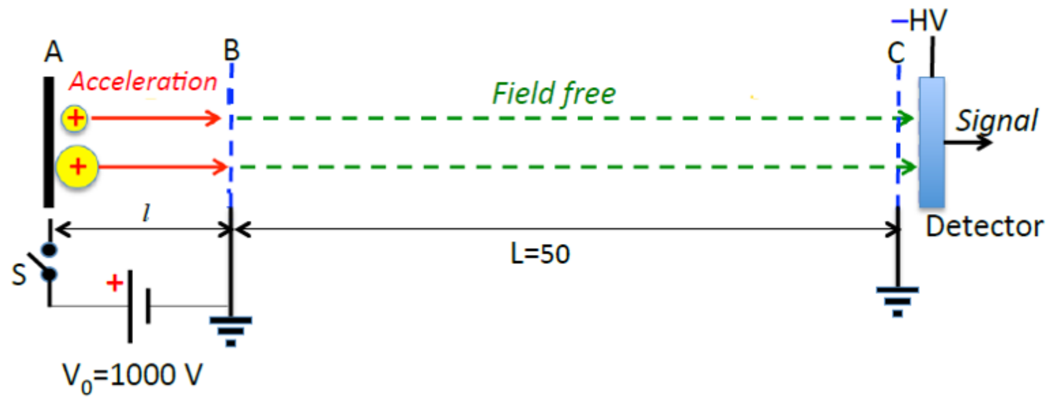


Figure 2: Time of flight-based mass spectrometer schematic.

## Exercise 3 :

Two particles, with charges 20.0 nC and −20.0 nC, are placed at the points with coordinates (0, 4.00 cm) and (0, -4.00 cm), as shown in Fig.3. A particle with charge 10.0 nC is located at the origin.

1. What is the total energy of the system of the three charges? Consider first a system in vacuum and what is the energy cost to introduce one charge, then a second and finally the third.
2. A fourth particle, with a mass of $2.00 \times 10^{-13}$ kg and a charge of 40.0 nC, is released from the rest at the point (3.00 cm, 0). Find its speed after it has moved freely to a very large distance.

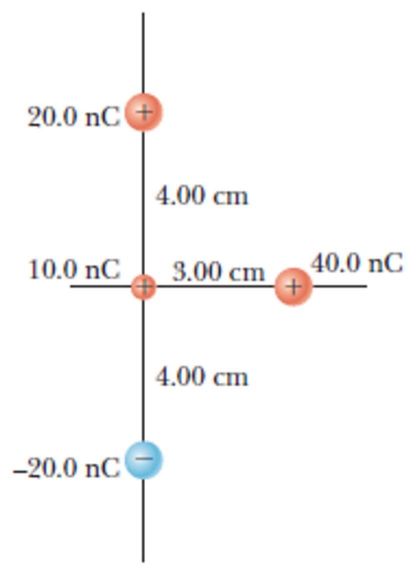


Figure 3: Schematic of the charges and their positions.

## Exercise 4 :

A nonconducting sphere of radius $r_0$ carries a total charge $Q$ distributed uniformly throughout its volume. Determine the electric potential as a function of the distance $r$ from the center of the sphere for: (a) $r > r_0$ and (b) $r < r_0$ . Take $V = 0$ at $r = \infty$; (c) Plot $V$ versus $r$ and $E$ versus $r$.

- **Hint**: calculate the electric field first, and then the potential from it.

## Exercise 5 :

Refer to Fig.4 and consider the three following cases.

(a) An insulating spherical shell, centered at the origin $O$ of the Cartesian axes, is uniformly charged (with internal radius $a$, external radius $b$ and total charge $+Q$). Calculate at any point in space the electric potential $V(\overrightarrow{r})$ and the electric field $\overrightarrow{E}(\overrightarrow{r})$. Graphically represent the functions $V(\overrightarrow{r})$ and the radial component of $\overrightarrow{E}(\overrightarrow{r})$.

(b) A conductive spherical shell, centered at the origin $O$ of the Cartesian axes, is electrically neutral and floating (internal radius $a$ and external radius $b$). A $+Q$ charge is placed in the center of the shell (in $O$). Calculate at any point in space the electric potential $V(\overrightarrow{r})$ and the electric field $\overrightarrow{E}(\overrightarrow{r})$. Graphically represent functions $V(\overrightarrow{r})$ and the radial component of $\overrightarrow{E}(\overrightarrow{r})$.

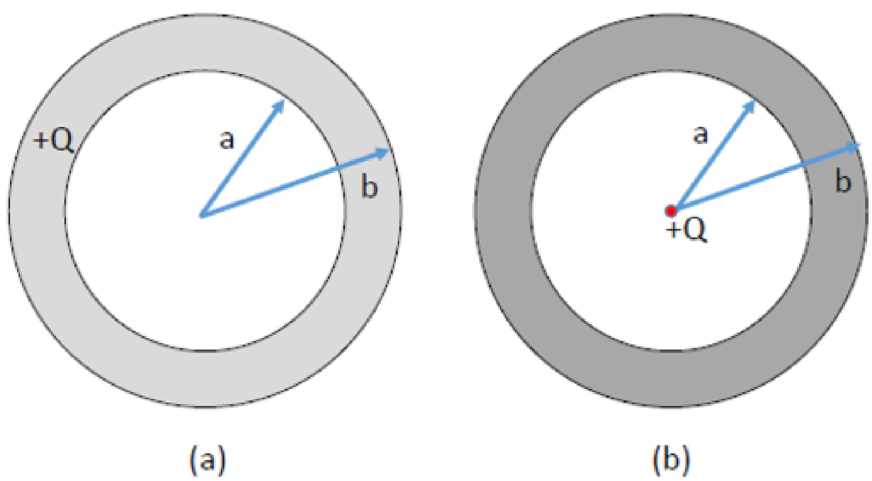


Figure 4: a) Insulating spherical shell uniformly charged with a total charge +Q. b) Neutral conductive spherical shell with a +Q charge in the centre. The shell is floating, i.e. electrically disconnected from the environment.

- **Hint 1**: use Gauss's law to compute the electric field first and then the potential by integrating the electric field.