General Physics: Electromagnetism, Problem Set 3

Exercise 1 :

A disk of radius R has a uniform surface charge density σ.

1. Calculate the electric field at a point P that lies at a distance x from the disk's center along the central axis perpendicular to the plane of the disk (see next figure). Hint: Use the result obtained in the last exercise session about the electric field generated by a uniformly charged ring.

2. What happens if we let the radius of the disk grow so that the disk becomes an infinite plane of charge?

3. Show that the electric field at distances x that are large compared with R approaches that of a particle with charge $Q = \sigma\pi R^2$.

   Hint: Use the fact that $x/(x^2 + R^2)^{1/2} = (1 + R^2/x^2)^{-1/2}$ and the approximation for the binomial expansion $(1 + \delta)^n \approx 1 + n\delta$ when $\delta \ll 1$.

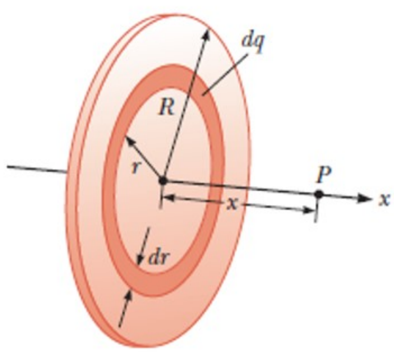

Figure 1: Disk of radius R with uniform surface density σ. The elementary charge dq is given by a circular crown of width dr located at distance r from the center. The field point is at distance x on the central axis of the ring.

Exercise 2 :

(a) A uniform electric field is given by

$$\vec{E} = E\cos\theta\,\hat{x} + E\sin\theta\,\hat{y},$$

throughout space. A rectangular surface is defined by the four points $(0,0,0)$, $(0,0,H)$, $(L,0,0)$, and $(L,0,H)$. What is the flux of the electric field through the surface?

(b) An electric field points in the z direction everywhere in space. The magnitude of the electric field depends linearly on the x position:

$$\vec{E} = (a - bx)\,\hat{z},$$

where a and b are constants. What is the flux of the electric field through a square of side L located in the positive xy-plane, with one corner at the origin?

(c) A negative electric charge $-Q$ is located at the origin. Calculate the flux of the electric field through a spherical surface of radius R centered at the origin.

Exercise 3 :

An insulating sphere of radius R contains a total charge Q, which is uniformly distributed throughout its volume. Determine an expression for the electric field as a function of distance r from the center of the sphere.

Exercise 4 :

A sphere of radius r_0 carries a volume charge density ρ_E (see Figure 2). A spherical cavity of radius $r_0/2$ is then scooped out and left empty, as shown.

(a) What is the magnitude and direction of the electric field at point A?

(b) What is the direction and magnitude of the electric field at point B?

Points A and C are at the centers of the respective spheres.
Hint: in order to take a symmetric Gaussian surface consider two spheres: a big one centered in A, with radius r_0 and with volume charge density ρ_E and a second smaller one centered in C, with radius $r_0/2$ and with volume charge density $-\rho_E$.

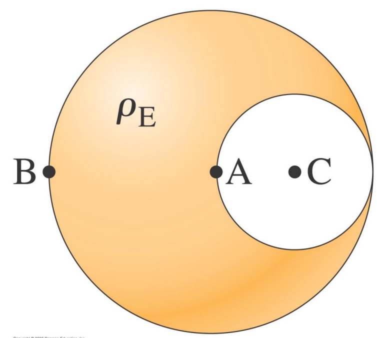

Figure 2: A charged sphere of radius r_0 containing a cavity of radius $r_0/2$.

Exercise 5 :

A solid insulating sphere of radius a carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge $-2Q$. Using Gauss's law, find the electric field in the regions labeled ①, ②, ③, and ④ in Figure 3 and the charge distribution on the shell when the entire system is in electrostatic equilibrium.

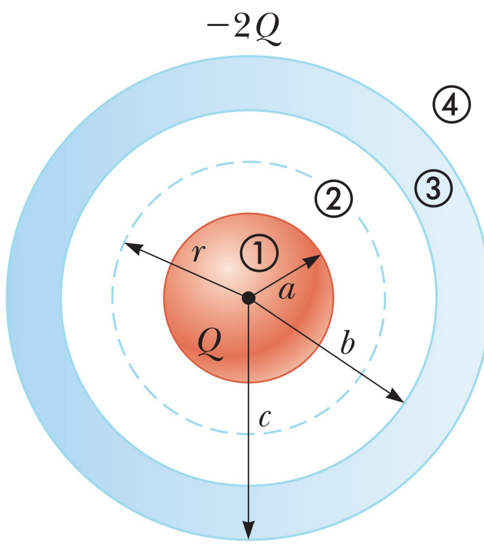

Figure 3: An insulating sphere of radius a and carrying a charge Q surrounded by a conducting spherical shell carrying a charge $-2Q$.