General Physics: Electromagnetism, Problem Set 2

Exercise 1 :

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is $a_0 = 5.29 \times 10^{-11}$ m. The proton has charge $e = 1.60 \times 10^{-19}$ C and the electron has charge $-e$ and mass $m_\mathrm{e} = 9.11 \times 10^{-31}$ kg. $k_e = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.

1. Find the magnitude of the electric force exerted on each particle.
2. If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

Exercise 2 :

Charges q_1 and q_2 are located on the x axis at distances a and b respectively from the origin, as shown in the next figure.

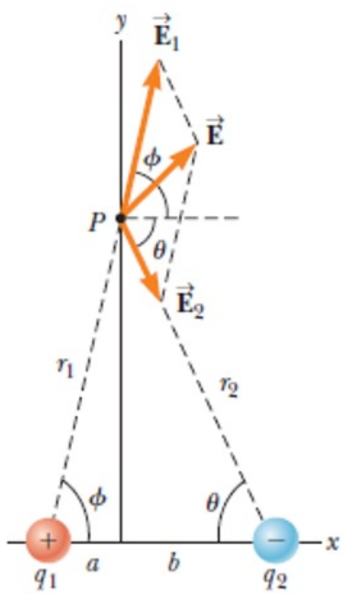

Figure 1: Positions of the charges q_1 and q_2 and of a field point P. The total electric field $\boldsymbol{E}$ at P equals the vector sum $\boldsymbol{E}_1 + \boldsymbol{E}_2$, where $\boldsymbol{E}_1$ is the field due to the positive charge q_1 and $\boldsymbol{E}_2$ is the field due to the negative charge q_2.

1. Find the components of the net electric field at the point P, which is at position $(0, y)$.

2. Evaluate the electric field at point P in the special case that $|q_1| = |q_2|$ and $a = b$.

3. Still for the special case of problem 2., find the electric field due to the electric dipole when point P is at distance $y \gg a$ from the origin.

Exercise 3 :

Two positive point charges are fixed on the x-axis:

$$Q_1 = +Q \quad \text{at position } x = -a, \qquad Q_2 = +2Q \quad \text{at position } x = +a.$$

The distance between them is therefore $2a$. A third "probe" charge q_3 of mass m is free to move. We want to find the position where this probe charge can stay at rest, and then study what happens if it is displaced slightly from this point.

1. Find the point x_0 (with $-a < x_0 < a$) along the x-axis where the total electric field due to Q_1 and Q_2 is zero.

2. Suppose the probe charge is at equilibrium x_0, and we displace it slightly along the x-axis to a new position $x = x_0 + \delta x$ with $|\delta x| \ll a$. Write the approximate expression for the electric field near x_0 and determine whether the resulting force on q_3 tends to restore it back to x_0 (stable equilibrium) or push it away (unstable equilibrium). Hint: Use a Taylor expansion $E_x(x_0+\delta x) \approx E_x(x_0)+\delta x \left.\frac{dE_x}{dx}\right|_{x_0}$. Since $E_x(x_0) = 0$ (equilibrium position), only the derivative matters.

3. Now place the probe at (x_0, y) with a small displacement y perpendicular to the axis ($|y| \ll a$). Compute the y-component of the electric field due to both Q_1 and Q_2, and decide whether the probe is stable or unstable in this direction. Hint: For a source charge Q_i at x_i, the y component of the field is $E_y^{(i)} = kQ_i \frac{y}{\left((x_0-x_i)^2+y^2\right)^{3/2}}$. For small y, use the approximation $(r^2 + y^2)^{3/2} \approx |r|^3$.

4. Summarize your results. For which sign of q_3 is the equilibrium stable along x? For which sign is it stable along y?

Exercise 4 :

A proton of mass $m_\text{p} = 1.67 \times 10^{-27}$ kg moves at 4.50×10^5 m/s in horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find:

1. the time interval required for the proton to travel 5.00 cm horizontally,

2. its vertical displacement during the time interval in which it travels 5.00 cm horizontally,

3. the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

Exercise 5 :

A ring of radius a carries a uniformly distributed positive total charge Q.

1. Calculate the electric field due to the ring at a point P lying at a distance x from its center along the central axis perpendicular to the plane of the ring (see next figure).
2. Suppose a negative charge is placed at the center of the ring and displaced slightly by a distance $x \ll a$ along the axis. When the charge is released, what type of motion does it exhibit?

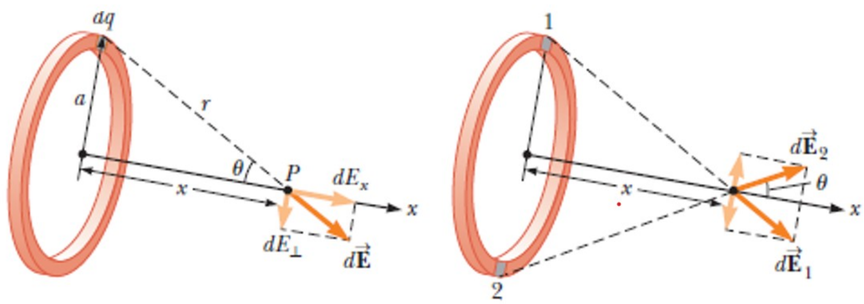

Figure 2: Ring with uniform positive charge Q and field point P at a distance x on the central axis of the ring. The infinitesimal electric field $d\boldsymbol{E}$ produced by an elementary charge dq on the ring in the point P can be divided in a component along the central axis dE_x and in a component lying on the ring's plane $dE_{\perp}$. The total electric field in the point P is given by the sum of the infinitesimal electric fields produced by all the elementary charges on the ring.