PHYS-201(e)
General Physics: Electromagnetism

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Mock Exam

19.12.2025

Name: ____________________ Sciper: ____________

- **P**ut your name and Sciper number on every paper sheet (like the sticker in the first page).
- **Please give at least a brief explanation of the calculations you are undertaking. Any answer requires a justification, even a brief one.**
- The exam takes place from 9:15 to 12:15 a.m.
- Please put your valid ID on the table.
- There are six problems. Each problem provides different points as indicated. Check that you have received all problems.
- One page of handwritten notes is allowed as well as a calculator without equation solver. The items will be checked in the course of the written exam.
- If you want to leave before the end of the written exam, please raise your hand and wait until an assistant arrives at your place.
- Please respect your colleagues and do not leave between 11:45 and 12:15 am. Use remaining time to check your solutions.
- Please remain seated while the papers are collected at the end of the exam.
- The copy must be written in blue or black pen or fountain pen, using indelible ink; pencil, in particular, is not allowed, except for graphs.
- Number each page and indicate the total number of double sheets on the first page at the end of the exam.
- For a graph to be considered correct, it must have a legend, the axes must be named and, if it makes sense, scaled.
- Useful constants: $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$, $\mu_0 = 4\pi \times 10^{-7}\mathrm{H\,m^{-1}}$, $c = 3 \times 10^{8}\mathrm{m\,s^{-1}}$.

Problem 1:

(Tot 8 pts)

A very long insulating cylindrical object consists of an inner cylinder of radius a, which carries a uniform volumetric charge density ρ, and a concentric outer cylindrical shell (very thin), of radius b, which has an equal but opposite total charge, uniformly distributed on the surface.

1. Calculate the electric field making use of Gauss's law and draw the used Gaussian surface for:
   (a) $0 \leq r \leq a$,
   (b) $a < r \leq b$,
   (c) $r > b$,

   being r the radial distance from the center of the inner cylinder.
2. Plot the electric field as a function of r.
3. Calculate and plot the electric potential everywhere, taking $V = 0$ on the outer cylinder.
4. Calculate the electrostatic energy per unit length of the object. Remember the expression for the electrostatic energy per unit volume:

   $$u_E = \frac{1}{2}\,\varepsilon_0\,|\vec{E}|^2,$$

   where $\vec{E}$ is the electric field.
5. Explain qualitatively what would happen to the electric field and potential if the outer cylinder was conductive and neutral ($\rho_{\text{outer}} = 0$).

Problem 2:

(Tot 6 pts)

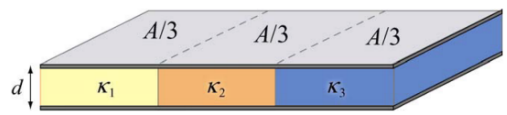
A parallel plate capacitor consists of two rectangular plates, each with an area A, separated by a distance d. We assume $A \gg d$, such that the electric field between the plates is considered uniform and the electric field outside the capacitor is zero.

1. Using Gauss's law, derive the expression for the capacitance of a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

The parallel plate capacitor is now filled with three dielectric blocks, each made of a distinct dielectric material. The arrangement of the dielectric materials between the plates follows the geometries shown in the three sub-questions below. Each material is characterized by its respective dielectric constant: κ_1, κ_2, and κ_3.

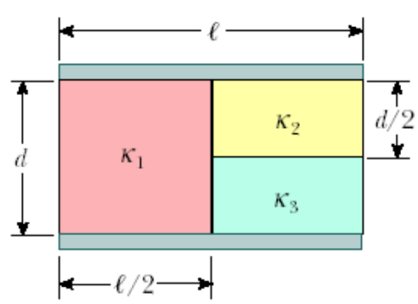
For each of the configurations below, find the expression of the capacitance of the system in terms of A, d, κ_1, κ_2, and κ_3. You can still assume that $A \gg d$ such that the non-uniform electric field near the edges of the plates can be neglected.

2. Each of the three dielectric blocks is placed adjacent to the others, with each occupying 1/3 of the volume between the plates, as in the figure below.



3. Each of the three dielectric blocks is placed on top of the others, with each occupying 1/3 of the volume between the plates, as in the figure below.

4. The block with dielectric constant κ_1 occupies 1/2 of the volume between the plates, while the blocks with dielectric constants κ_2 and κ_3 each occupy 1/4 of the volume, as in the figure below.



Problem 3:

(Tot 9 pts)

Consider the circuit shown in the left of the figure.

1. Simplify the circuit by computing the equivalent values of resistance R_{eq}, inductance L_{eq} and capacitance C_{eq} and draw it for both configurations of the switch.

From now on make use of this simplified equivalent circuit (see the right figure). The switch of the circuit is maintained in position (a) for 2 seconds.

2. Derive the electrical current in the coil L_{eq} at time $t = 2$ s **by explicitly solving the first order differential equation**.

The switch is then put in position (b) instantaneously (without interrupting the electrical current in the coil L_{eq}). At time $t = 2$ s, the equivalent capacitor C_{eq} carries no charge. In this position (b) of the switch:

3. Determine the charge $Q(t)$ carried by the capacitor C_{eq} as a function of the time t, by explicitly solving the second order differential equation.
4. Determine the current $I(t)$ flowing in the equivalent circuit as a function of the time.
5. Derive the frequency of oscillation and plot $Q(t)$ and $I(t)$ of the two previous points.
6. Imagine now that the obtained equivalent circuit, with the switch kept in position (b), is used to generate an electromagnetic wave propagating in vacuum. Compute the module of the wavevector of the generated wave.

Numerical values: $V = 0.2$ V, $R_1 = 0.01\ \Omega$, $R_2 = R_3 = 0.04\ \Omega$, $L_1 = 30$ mH, $L_2 = 24$ mH, $C_1 = C_2 = 1.0\ \mu$F, $C_3 = 1.2\ \mu$F.

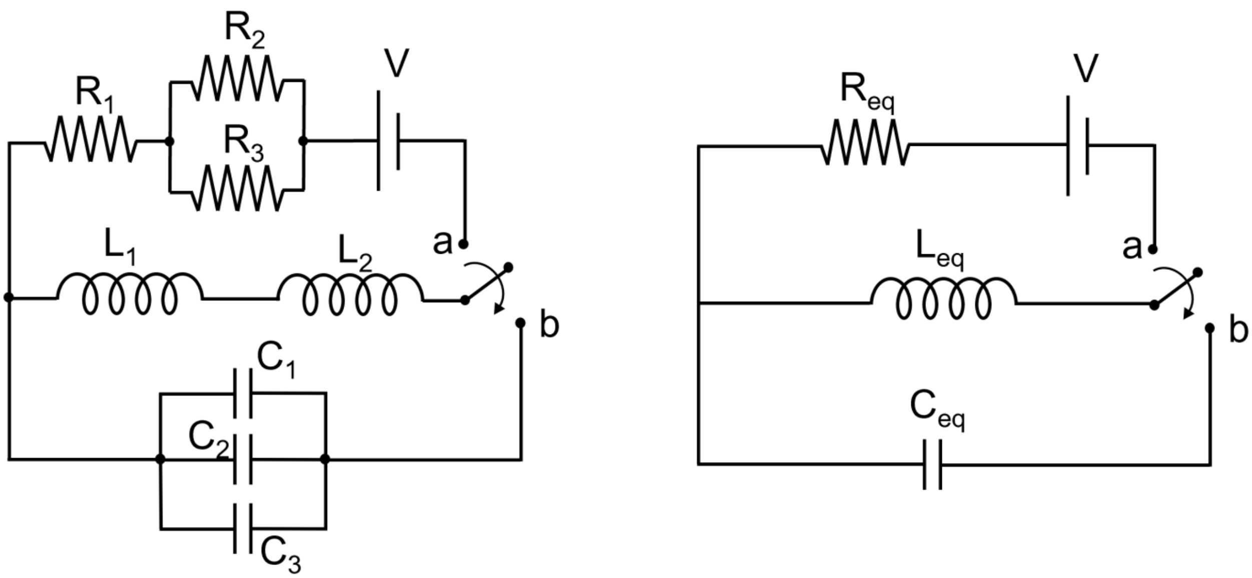


Figure 1: RLC network (on the left) and its simplified equivalent version (on the right).

Problem 4:

(Tot 9 pts)

Consider the rectangular circuit of sides a and b shown in Figure 2, where a bar of cross section S is placed on two rails connected by a wire. The bar is fixed with two screws so it cannot move.

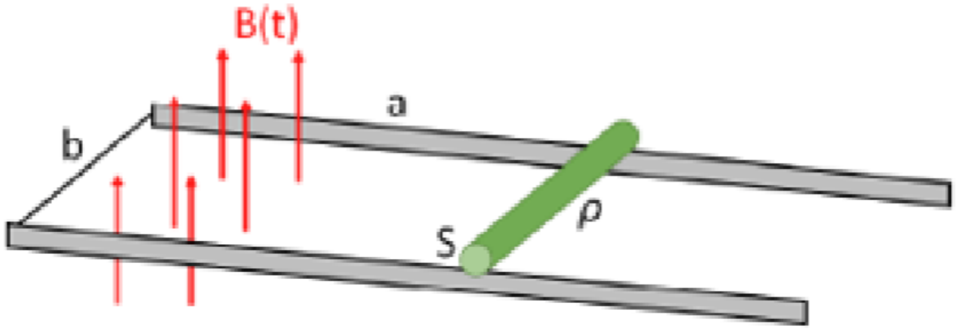


Figure 2: Circuit constituted by a rail closed on one side by a wire b and on the other by a rod.

An external magnetic field B is applied perpendicular to the circuit area with a time dependence shown in Figure 3.

Figure 3: Time dependence of the external magnetic field B.

1. Plot the potential induced on the circuit as a function of time.

In the circuit only the bar has a considerable resistance, due to a resistivity ρ.

2. Evaluate the current that flows through the bar starting from $t = 5\,\mathrm{s}$

Assume that at $t = 5\,\mathrm{s}$ the screws loosen, so that the bar becomes free to slide along the rails. The bar does not rotate and maintains electrical contact.

3. Evaluate the magnitude of the time dependent force you should apply on the bar in order to keep it in its place
   Hint: for simplicity you can set t to zero at $t = 5\ s$.

Problem 5:

(Tot 7 pts)

A rectangular coil of 60 turns, dimensions 0.10 m by 0.20 m and total resistance $10\,\Omega$ rotates with angular speed 30 rad/s about the y axis in a region where a 1.0-T magnetic field is directed along the x axis. The rotation is initiated so that the plane of the coil is perpendicular to the direction of $\vec{B}$ at t=0.

1. Calculate the maximum induced emf in the coil.

2. Calculate the maximum rate of change of magnetic flux through the coil.

3. Calculate the induced emf at t=0.050 s.

4. Calculate the torque exerted on the loop by the magnetic field at the instant when the emf is maximum.

Problem 6:

(Tot 6 pts)

An inductor consists of two very thin conducting cylindrical **shells**, one of radius a and one of radius b as depicted in the figure below, both of length h. The inner shell carries a current I in the positive z direction (out of the page), and the outer shell carries a current I in the negative z direction (into the page), as depicted in the figure. In both shells, the current is distributed uniformly around the circumference. We neglect the thickness of the shells and assume that $h \gg a, b$ such that the non-uniform magnetic field near the edges of the cylinder can be neglected.

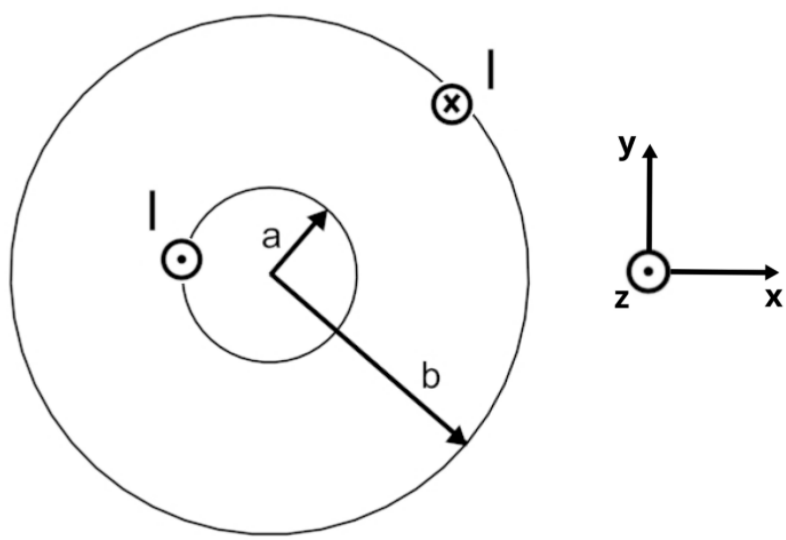


Figure 4: Time dependence of the external magnetic field B.

1. Use Ampere's law to find the magnetic field as a function of the radius r (distance from the center). Indicate the direction of the magnetic field as well.
2. Calculate the magnetic energy density u_B as a function of the radius r.
3. Calculate the inductance of this long inductor recalling that the magnetic energy is $U_B = \frac{1}{2}LI^2$ and using your results for the magnetic energy density in (2).
4. Calculate the inductance of this long inductor recalling that the magnetic flux is $\Phi_B = LI$and using your results for the magnetic field in (1). Clearly indicate the surface over which you evaluate the magnetic flux.