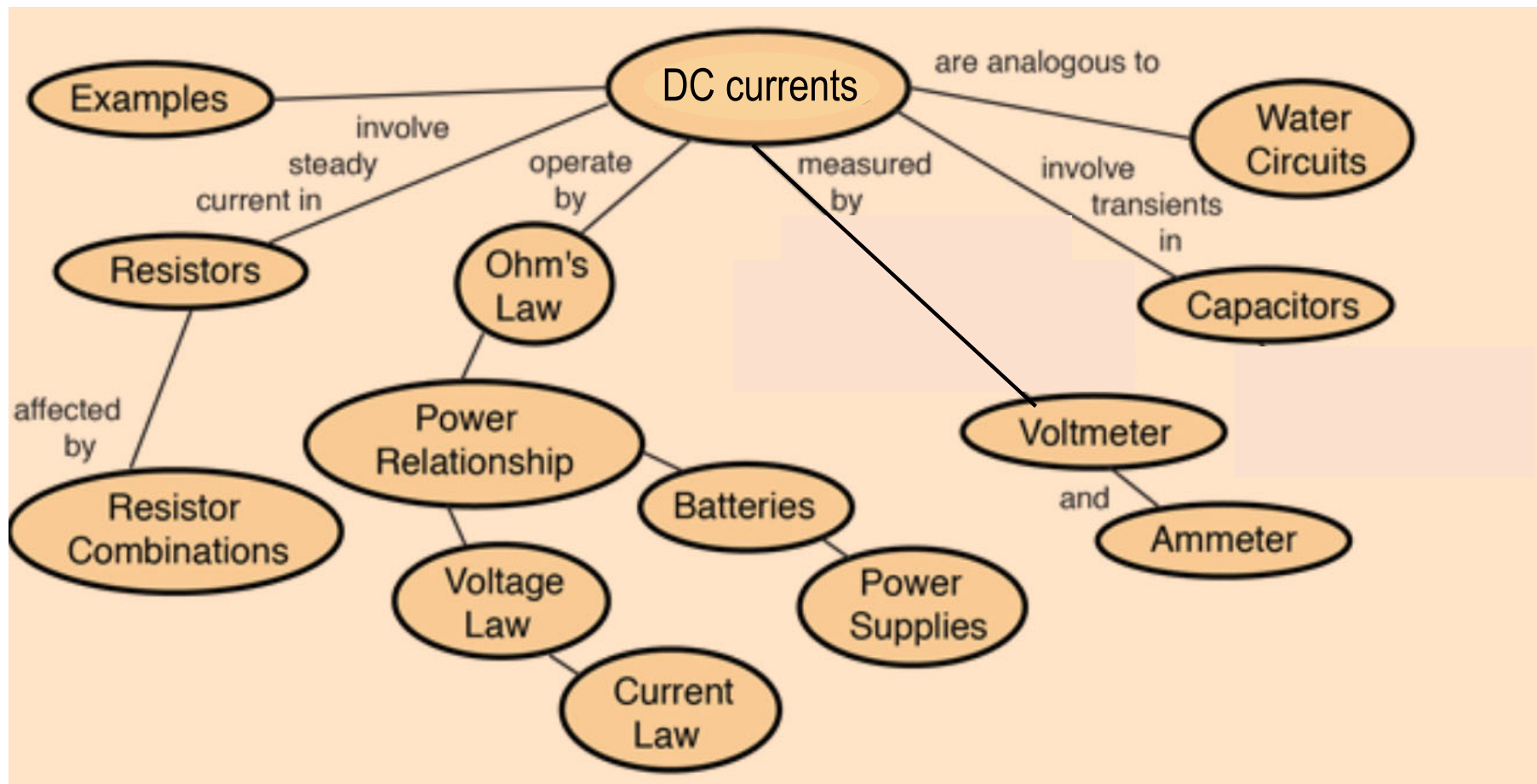


3. Stationary (DC) currents in conductors, resistors and capacitors

Mindmap, adapted from: <http://hyperphysics.phy-astr.gsu.edu/hbase/electric/dccircon.html>



3. Stationary (DC) currents in conductors, resistors and capacitors

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3. DC currents in conductors, resistors and capacitors

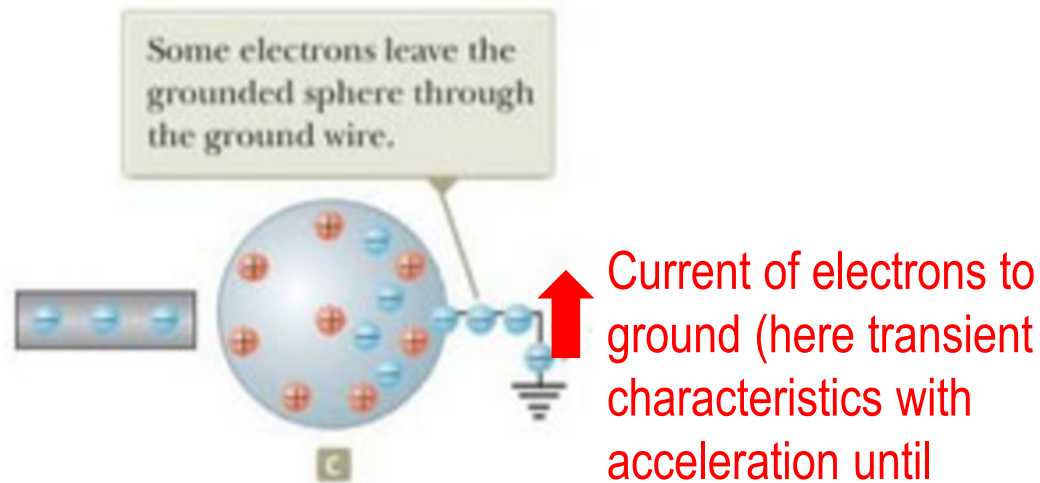
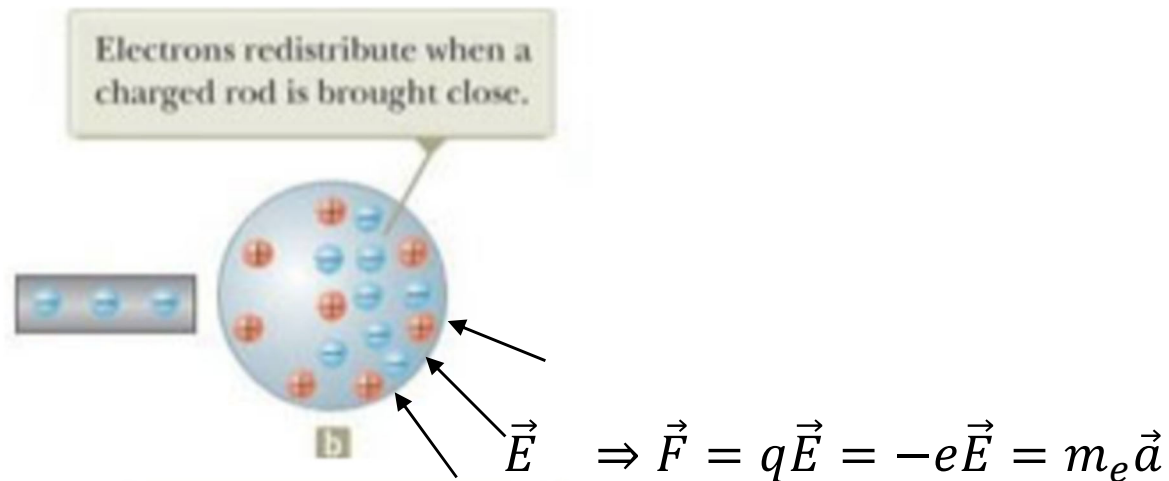
At the end of chapter 3 you will apply

- the definition of currents originating from moving charges,
- the continuity equation of charge currents,
- the difference between perfect conductors and metals,
- Ohm's law,
- the Drude model with relaxation/scattering times and mobility,
- the electromotive force,
- and Kirchhoff's laws

to explain and analyze electric circuits based on resistors and capacitors

3.1 Currents

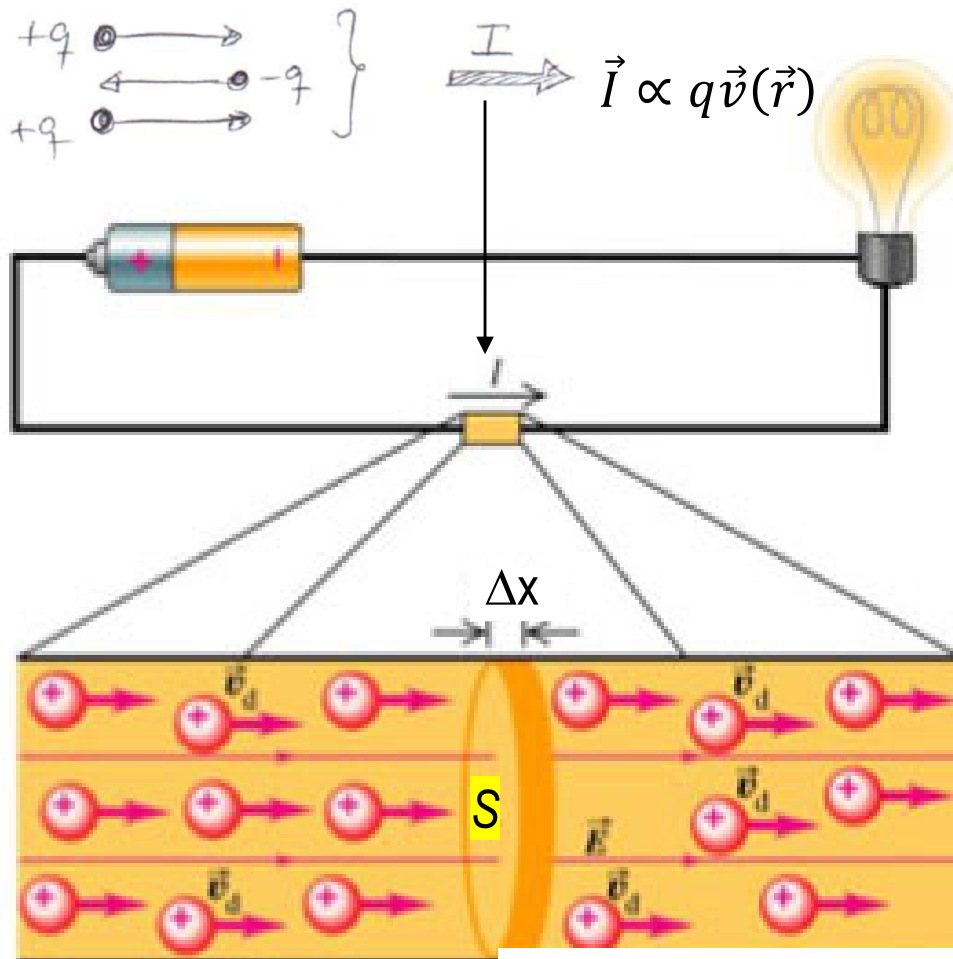
Charging of a conductor by induction is based on charge flow (current) to ground:



We look first in the **description of a steady current** and will then explain it by the **Drude model**.

3.1.1. Definition of a DC current: movement of positive charge

- The sign of the current is that of positive charges



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Here: $v = v_d = v_{\text{drift}}$

- Note that the same current can be realized by ev or $(2e)v/2$, i.e., charge carriers with $q = 1e$ at velocity v or e.g. $q = 2e$ at $v/2$.

- Current I :** "Flux of charges ΔQ per unit time Δt through cross-sectional plane S "

- Carrier density: charges per unit volume n
- Their velocity $v = \frac{\Delta x}{\Delta t}$

$$\langle I \rangle = \frac{\Delta Q}{\Delta t} = \frac{(nS\Delta x)q}{\Delta t} = nSqv$$

$\langle I \rangle$: average over the discrete charge carriers of different polarity

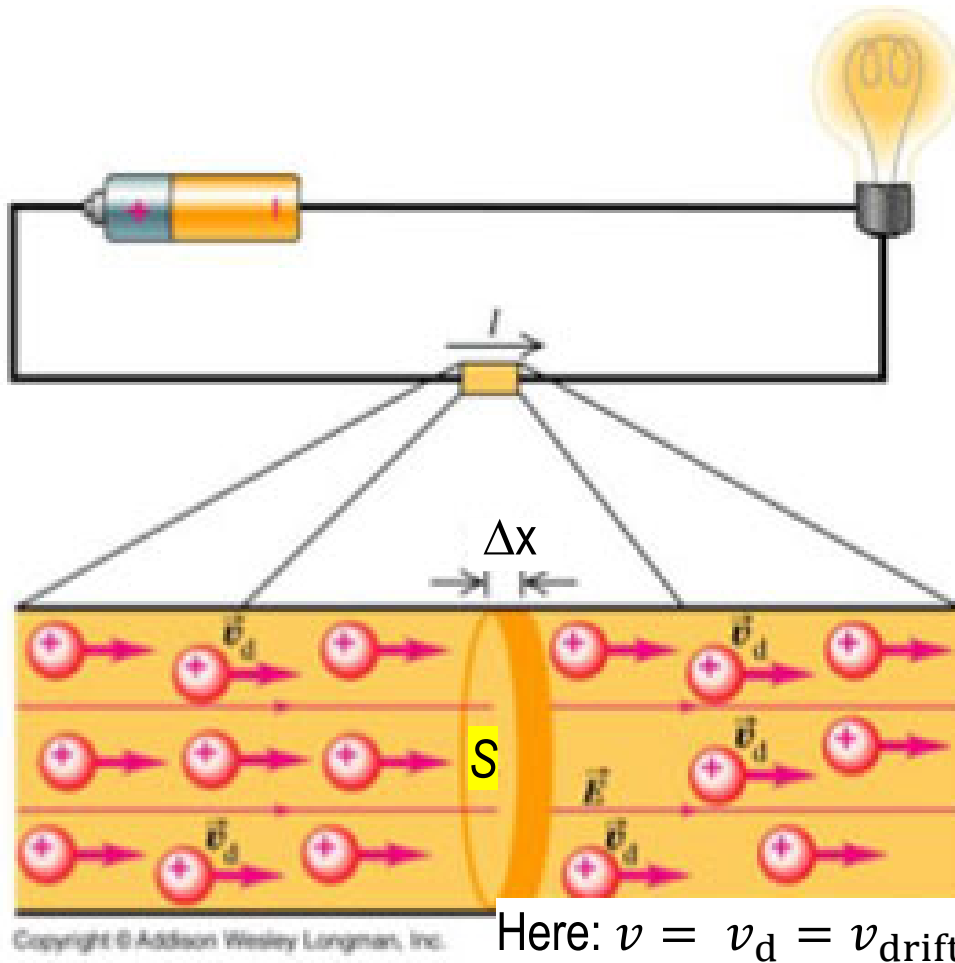
General definition:

$$I(t) = \frac{dQ}{dt} \quad [I] = \text{A} = \text{C/s (Ampere)}$$

- If $I = \text{const.}$: DC current

3.1.2 Current density (“charge flux through surface”)

- Current defined via integration over current density:



normal vector
on surface element

$$I = \iint_{\text{surface } S} nq\vec{v}d\vec{a} = \iint_{\text{surface } S} \vec{j}(\vec{r})d\vec{a}$$



(equivalent to “flux of charges” through surface S)

Current density defined as:

Density of moving charges
(pos. or neg. value)

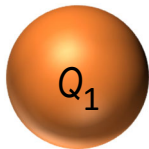
$$\vec{j}(\vec{r}) = nq\vec{v}(\vec{r}) = \rho_e\vec{v}(\vec{r}) \quad [j] = \text{A/m}^2$$

3.1.3 Stationary current and charge conservation

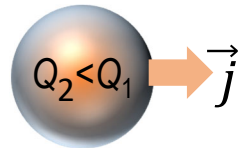
- Stationary (or steady or DC) current: $\frac{dI}{dt} = 0$

- Charge Conservation: $Q = \iiint_{\text{volume}} \rho_e(\vec{r}, t) dx dy dz \Big| \cdot \frac{\partial}{\partial t}$ (consider the charges in the specific volume V sketched)
 $\Rightarrow \frac{dQ}{dt} = \iiint_{\text{volume}} \frac{\partial \rho_e}{\partial t} dx dy dz$

Imagine first, accumulated positive charges in a volume V :



When their number decreases ($dQ/dt < 0$), a current I (or density j) goes out:



The outgoing current reads:

$$I(t) = \frac{dQ}{dt} = - \oiint_{\text{surface of volume}} \vec{j} \cdot d\vec{a}$$

(here: "flux of charges" through **closed** surface)

Divergence theorem $\rightarrow = - \iiint_{\text{volume}} \vec{\nabla} \cdot \vec{j} dx dy dz$

- The comparison provides the **Continuity Equation**: $\frac{\partial \rho_e}{\partial t} = -\vec{\nabla} \cdot \vec{j}$ ρ_e is evaluated at a given point (x,y,z,t)
- Stationary case**: $\vec{\nabla} \cdot \vec{j} = 0$ or $\oiint \vec{j} \cdot d\vec{a} = 0 \Rightarrow$ **The charge density ρ_e does not vary in time;** ("the charges that enter a volume, leave it").

(Do not confuse the "stationary case of current flow" with "stationary charges ($j = 0$)" considered in "electrostatics")