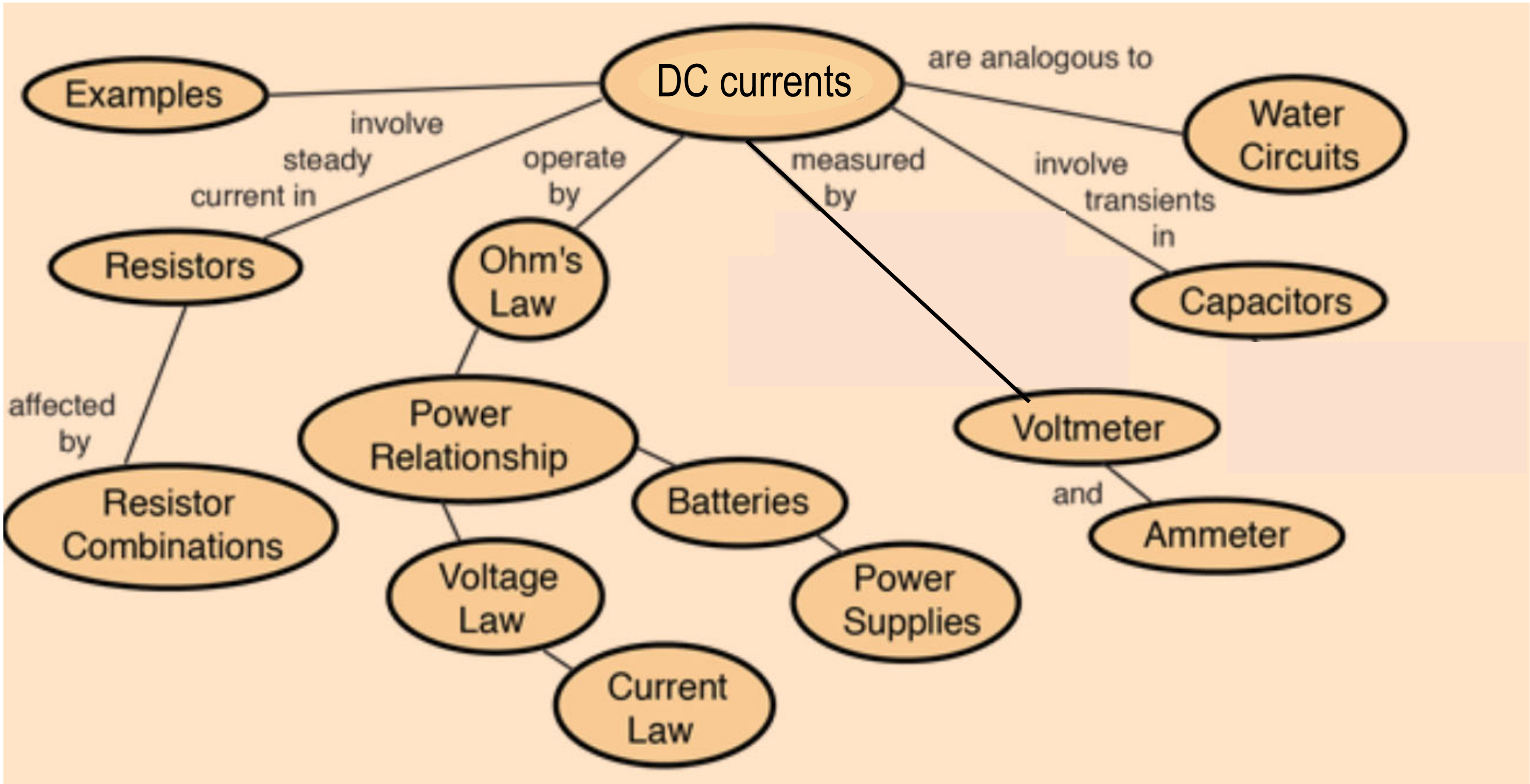
3. Stationary (DC) currents in conductors, resistors and capacitors

Mindmap, adapted from: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/dccircon.html



3. Stationary (DC) currents in conductors, resistors and capacitors

Contents:

3. DC currents in conductors, resistors and capacitors

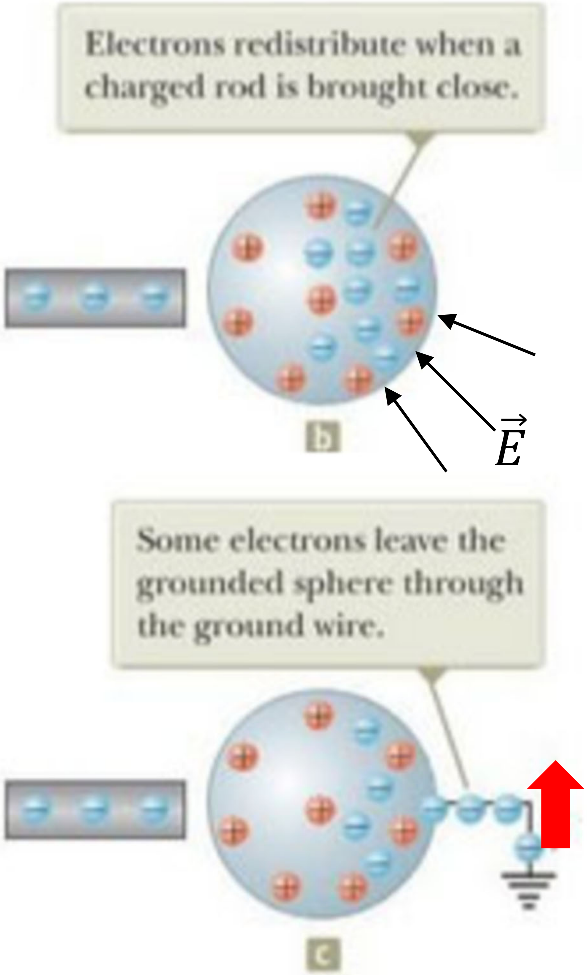
At the end of chapter 3 you will apply

- the definition of currents originating from moving charges,
- the continuity equation of charge currents,
- the difference between perfect conductors and metals,
- Ohm's law,
- the Drude model with relaxation/scattering times and mobility,
- the electromotive force,
- and Kirchhoff's laws

to explain and analyze electric circuits based on resistors and capacitors

3.1 Currents

Charging of a conductor by induction is based on charge flow (current) to ground:



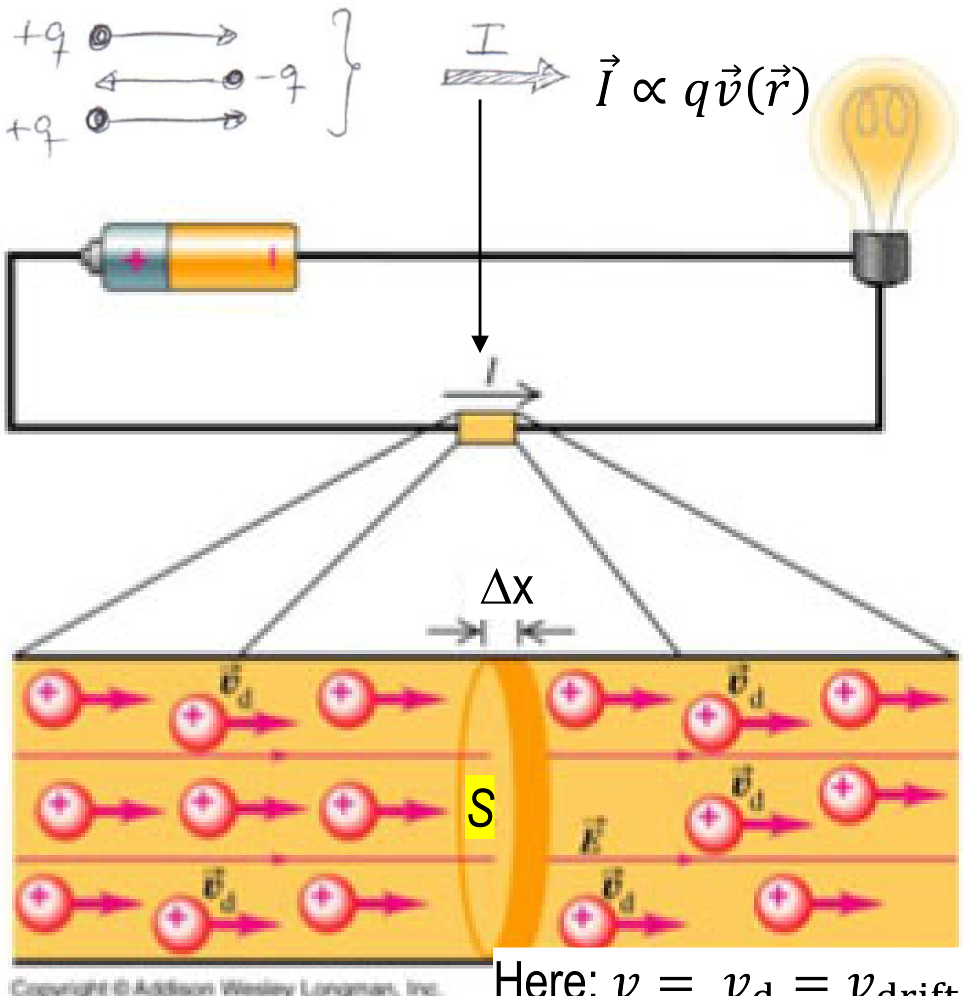
$\vec{E} \quad \Rightarrow \vec{F} = q\vec{E} = -e\vec{E} = m_e\vec{a}$

We look first in the **description of a steady current** and will then explain it by the **Drude model**.

Current of electrons to ground (here transient characteristics with acceleration until *E*-field vanishes). The red upward arrow indicates the direction of the technical current *I* defined via positive charges.

3.1.1. Definition of a DC current: movement of positive charge

- The **sign of the current** is that of **positive charges**



Here: $v = v_\mathrm{d} = v_\mathrm{drift}$

- Note that the same current can be realized by *ev* or (2*e*)*v*/2, i.e., charge carriers with *q* = 1*e* at velocity *v* or e.g. *q* = 2*e* at *v*/2.

- **Current *I*:** "Flux of charges Δ*Q* per unit time Δ*t* through cross-sectional plane *S*"

- Carrier density: charges per unit volume n
- Their velocity $v = \frac{\Delta x}{\Delta t}$

$$\langle I \rangle = \frac{\Delta Q}{\Delta t} = \frac{(nS\Delta x)q}{\Delta t} = nSqv$$

$\langle I \rangle$: average over the discrete charge carriers of different polarity

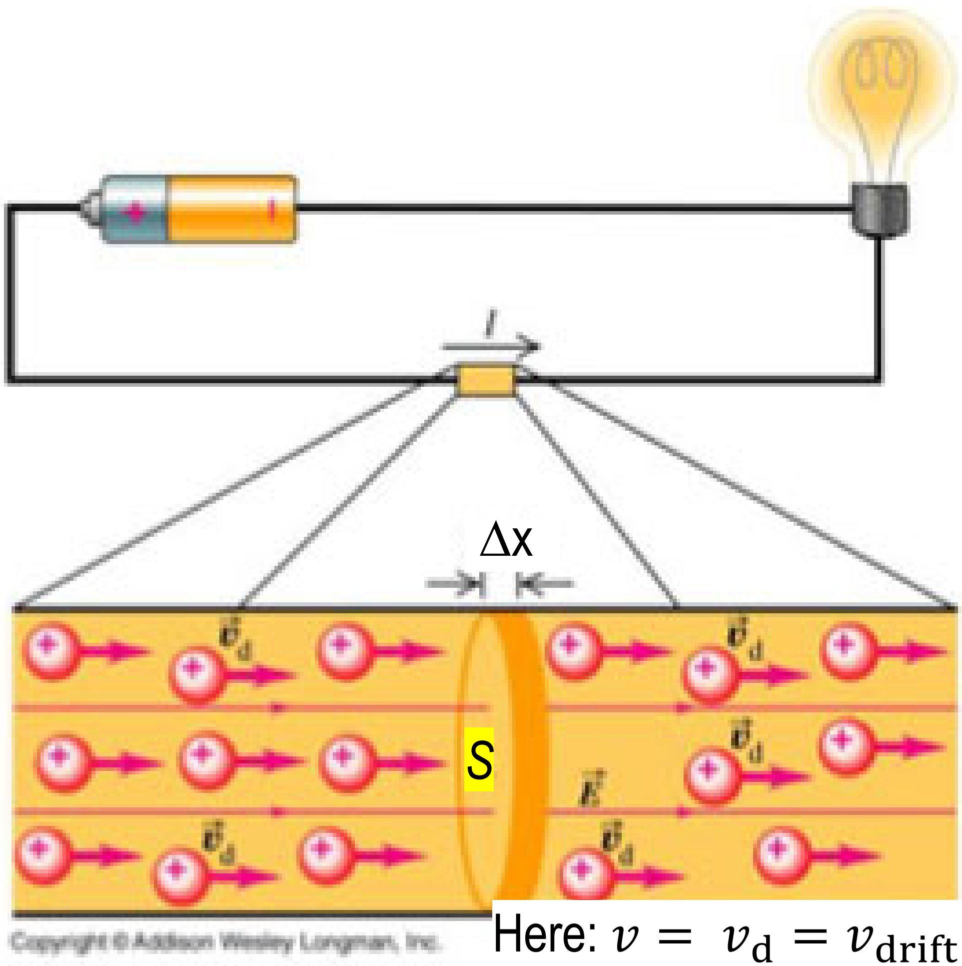
General definition:

$$I(t) = \frac{dQ}{dt}$$

[*I*] = A = C/s (Ampere)

- If *I* = const. : DC current

3.1.2 Current density ("charge flux through surface")



Here: $v = v_d = v_{drift}$

- Current defined via integration over current density:

normal vector
on surface element

$$I = \iint_{\text{surface } S} nq\vec{v}d\vec{a} = \iint_{\text{surface } S} \vec{j}(\vec{r})d\vec{a}$$

(equivalent to "flux of charges" through surface *S*)

Current density defined as:

Density of moving charges
(pos. or neg. value)

$$\vec{j}(\vec{r}) = nq\vec{v}(\vec{r}) = \rho_e\vec{v}(\vec{r})$$

$[j] = A/m^2$

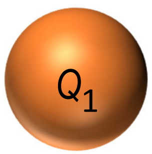
3.1.3 Stationary current and charge conservation

- **Stationary (or steady or DC) current:** $\frac{dI}{dt} = 0$

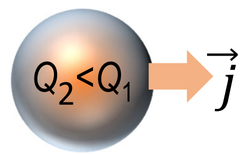
- **Charge Conservation**: $Q = \iiint_{\text{volume}} \rho_e(\vec{r}, t)\,dxdydz \;\Big|\cdot \frac{\partial}{\partial t}$ (consider the charges in the specific volume *V* sketched)

$$\Rightarrow \frac{dQ}{dt} = \iiint_{\text{volume}} \frac{\partial \rho_e}{\partial t}\,dxdydz$$

Imagine first, accumulated positive charges in a volume *V*:



When their number decreases (dQ/dt<0), a current *I* (or density *j*) goes out:



The outgoing current reads:

$$I(t) = \frac{dQ}{dt} = -\oiint_{\substack{\text{surface}\\\text{of volume}}} \vec{j}\cdot d\vec{a}$$

(here: "flux of charges" through **closed** surface)

Divergence theorem → $$= -\iiint_{\text{volume}} \vec{\nabla}\cdot\vec{j}\,dxdydz$$

- The <u>comparison</u> provides the **Continuity Equation:** $\frac{\partial \rho_e}{\partial t} = -\vec{\nabla}\cdot\vec{j}$ — ρ_e is evaluated at a given point (*x*,*y*,*z*,*t*)

- **Stationary case:** $\vec{\nabla}\cdot\vec{j} = 0 \text{ or } \oiint \vec{j}\cdot d\vec{a} = 0$ => **The charge density ρ_e does not vary in time;**

("the charges that enter a volume, leave it").

(Do not confuse the "stationary case of current flow" with "stationary charges (*j* = 0)" considered in "electrostatics")