Question 1: "I had a question about the conventions to use in the exam for denoting a potential difference.
If I define delta(V) as the difference between potential A and potential B,
is the sign (+/-) of delta V something important governed by a convention,
or is it not important, and is it perfectly possible to obtain the potential difference with a
different sign than in the answer key?"

Illustration: Problem sheet 4, Ex. 4

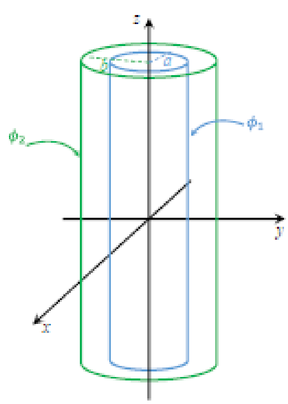
b) Due to the azimuthal symmetry of the problem, the potential does not depend on φ so: $\frac{\sigma\varphi}{\partial\varphi} = 0$. Since the cylinders are infinitely long and each of them represents an equipotential surface, ϕ does not depend on z and: $\frac{\partial\phi}{\partial z} = 0$. Therefore, using $\vec{E} = -\vec{\nabla}\phi$, the potential difference can be found as

$$\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \vec{\nabla}\phi \cdot d\vec{r} \tag{5}$$

Here, the direction of the line element $d\vec{r}$ can be chosen parallel or antiparallel with $\vec{E}$ but has to be consistent on both sides of the equation. Proceeding with the integration gives:

$$\int_a^b \frac{\sigma a}{\epsilon_0 r}\hat{r}\cdot d\vec{r} = -(\phi(b) - \phi(a))$$
$$(\phi(b) - \phi(a)) = -\frac{\sigma a}{\epsilon_0}\ln\frac{b}{a} \tag{6}$$
$$(\phi_2 - \phi_1) = -\frac{\sigma a}{\epsilon_0}\ln\frac{b}{a}$$

Are boundaries defined arbitrarily or do they follow a convention?



Inner cylinder,
positively charged,
E-field radially
outward, high potential

a is taken here
here the upper
boundary; in (5)
this was not the case

$$\vec{\nabla}^2\phi(r) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \underbrace{\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}}_{=0} + \underbrace{\frac{\partial^2\phi}{\partial z^2}}_{=0}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)$$

$$0 = \frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)$$

$$A = r\frac{\partial\phi}{\partial r} \quad (*)$$

$$\phi(r) = A\ln(r) + B$$

$$\Delta V = \phi(a) - \phi(b)$$
$$= A\ln(a) + B - A\ln(b) - B$$
$$\Delta V = A\ln\left(\frac{a}{b}\right)$$

Unit vector missing

$$\frac{\sigma\cdot a}{r\varepsilon_0} = -\left(\frac{\partial\phi(r)}{\partial r}\vec{e}_r\right)$$

$$(*) \Rightarrow A = r\frac{\partial\phi}{\partial r}$$

$$\frac{A}{r} = \frac{-\sigma\cdot a}{r\varepsilon_0} \Rightarrow A = \frac{-\sigma a}{\varepsilon_0}$$

$$\Rightarrow \Delta V = -\frac{\sigma a}{\varepsilon_0}\ln\left(\frac{a}{b}\right)$$

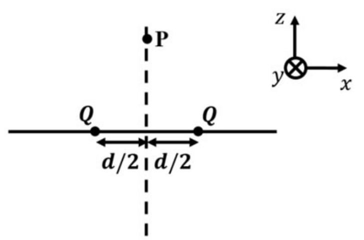
Problem 3

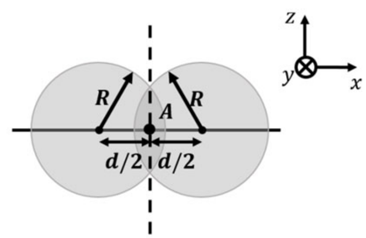
a) Consider two positive point charges Q separated by a distance d as shown in the sketch below on the left. Find the electric field $\vec{E}$ (magnitude and direction) at point P situated at a distance z above the midpoint between the charges, as a function of Q, z and d. The midpoint is indicated by the broken line. (3 P)

b) Now consider that there are two three-dimensional solid spheres of radius R. Positive electrical charges are uniformly distributed in the volume of each sphere. The total amount of charges in each sphere is Q. These two spheres are brought closer and made to penetrate each other. The cross-sectional view is shown in the sketch below on the right. This means that in the overlapping region, the charge density is twice that of the rest. The centres of the spheres are separated by a distance $d = \frac{8}{5}R$.

(I) Find the expression of the electric field $\vec{E}$ at a point P situated at a distance z above the midpoint between the two spheres in terms of Q, z, and R. Consider the two cases for which the point P is either inside or outside the two spheres. (4 P)

(II) Calculate the electrical potential at the mid-point between the centres of the two spheres (point A in the sketch on the right) in terms of Q and R. (2 P)





Question 2:

"Why are we considering the potential at infinity on the exercise b) II of this problem?"

Question 3:

I don't understand how to apply this formula.
I don't understand how to integrate my E because I don't really understand what is the dl.
By example in the mock exam, I don't understand why we integrate according to the x direction and not another one like y,z or e_r.

$$\phi(\vec{r}) = \phi(\infty) - \int_{\infty}^{\vec{r}} \vec{E} d\vec{l}$$

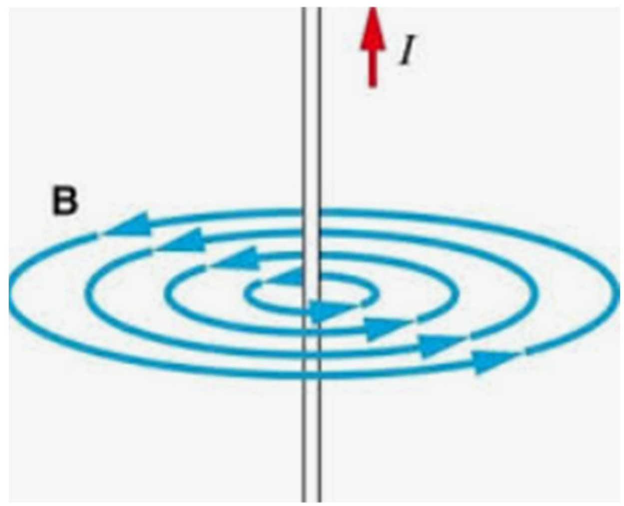
We use the superposition principle of the scalar potentials of two charge distributions and $\phi(\infty)=0$:

$$\phi(\infty) - \frac{1}{2}\phi(A) = -\int_{R}^{\infty} \vec{E} \cdot dx\,\hat{x}$$

previous results →

$$= -\int_{\frac{d}{2}}^{R} \frac{Qx}{4\pi\varepsilon_0 R^3} dx - \int_{R}^{\infty} \frac{Q}{4\pi\varepsilon_0 x^2} dx$$

Question 4a:
Could you re-explain Ampère-Laplace law/Biot-Savart law :
For example, how can we find u(t) and u(l) in problem 3 of the [test blanc No.1], and solve the law to obtain the B-field ?



Question 4a

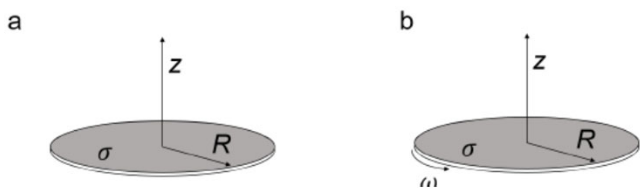
Problem 3

An ultrathin disk with radius R contains a uniform surface charge density σ of fixed positive charges. Its thickness is negligible in the following.

a) The disk is at rest (Fig. a). Provide the expression for the electrical potential function $\phi(z)$ along the central axis of the disk (z-axis) using the given parameters. (4p)

b) Now the disk spins with constant angular frequency ω (Fig. b). What is the direction of the magnetic field on the central axis (above and below the disk in Fig. b)? Explain your answer. (2p)

c) Provide the expression for the magnetic field vector $\vec{B}(z)$ along the central axis of the disk (z-axis) shown in Fig. b using the given parameters. (2p)



Question 4a

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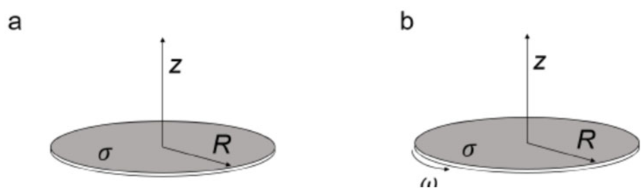


c) To compute the magnetic field along the central axis of the disk we use the Biot-Savart law. For a 2D charge distribution, the Biot-Savart law for a line element dl is $d\vec{B}(\vec{x}) = \frac{\mu_0 I}{4\pi} \frac{\vec{u}_t \times \vec{u}_r}{r^2} dl$. To approach the problem, the disk is subdivided into rings of width dr and the field produced by one ring is calculated. Then, the contribution of all rings are summed up.

Consider a ring of radius r and width dr. We define a linear charge density $d\lambda = \sigma dr$. The current inside this ring of width dr is given by $dI = d\lambda v = \sigma\omega r dr$, where $\omega = vr$. A small length element along the ring $dl = rd\theta$ with current dI will produce a magnetic field $d\vec{B}$ whose direction can be determined from Biot-Savart. The magnetic field due to the full ring can be found by integrating over the length dl. Finally, the magnetic field produced by the disk is found by integrating over dI.

By symmetry, all off-axis components must cancel out in the end. Therefore, only the z-component of the field needs to be considered: $\vec{B}(z) = \int dB \cos\varphi \hat{z} = \int dB \frac{r}{\sqrt{r^2+z^2}} \hat{z}$, where φ is the angle between the position vector pointing from the origin to the point r on the disk and the vector pointing from the point r to the z-axis. Proceeding with the integration in cylindrical coordinates:

$$\vec{B}(z) = \iint \frac{\mu_0}{4\pi} \cos\varphi dl dI \hat{z} = \int_0^R \int_0^{2\pi} \frac{\mu_0 \sigma \omega}{4\pi} \frac{r^3}{(r^2+h^2)^{3/2}} d\theta dr \hat{z} = \int_0^R \frac{\mu_0 \sigma \omega}{2} \frac{r^3}{(r^2+h^2)^{3/2}} dr \hat{z}. \quad (10)$$

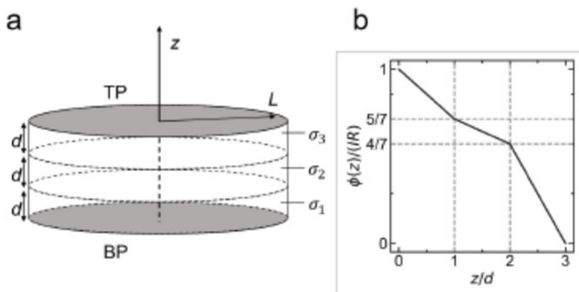
To perform the integration, the following hint given on the first page can be used:

$$\int \frac{x^3}{(x^2+a^2)^{3/2}} dx = \frac{2a^2+x^2}{\sqrt{x^2+a^2}}. \quad (11)$$

Then, the integral above can be evaluated as:

$$\vec{B} = \frac{\mu_0 \sigma \omega}{2} \left(\frac{2h^2+R^2}{\sqrt{R^2+h^2}} - 2|h| \right) \hat{z}. \quad (12)$$

Question 4b: I also have a question on exercise 1 of [the test blanc No. 1]:
What it means to give the charge distribution and why it is different from 0 at the interface of two dielectric. I would have say that minus charges from one dielectric counterbalance plus charges from the other dielectric ?



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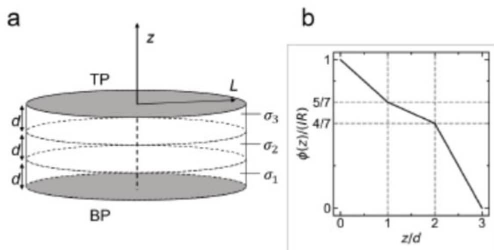

a) To start, we image a potential of ΔV is applied to the two conducting plates. The electric field will be constant in each layer. Using Ohm's law, $\vec{E} = \vec{J}/\sigma$, we can write the total voltage drop as:

$$\Delta V = \frac{J_1}{\sigma_1}d + \frac{J_2}{\sigma_2}d + \frac{J_3}{\sigma_3}d. \tag{1}$$

The current density through each layer is related to the current I flowing between the plates by:

$$J_1 = J_2 = J_3 = \frac{I}{\pi L^2}. \tag{2}$$

Substituting this in the previous relation and using $\Delta V = RI$:

$$\Delta V = \frac{I}{\pi L^2 \sigma_1}d + \frac{I}{\pi L^2 \sigma_2}d + \frac{I}{\pi L^2 \sigma_3}d = \frac{7}{4}\frac{d}{\pi L^2 \sigma}I \rightarrow R = \frac{7}{4}\frac{d}{\pi L^2 \sigma}. \tag{3}$$

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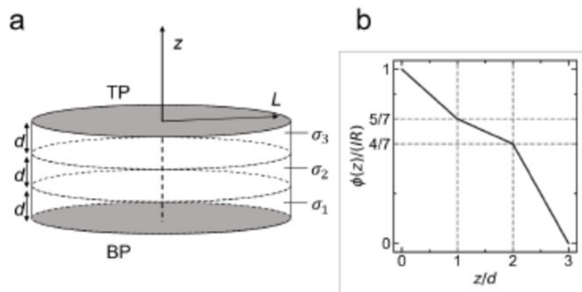

Question 4c: I also have a question on exercise 1 of [the test blanc No. 1]:
And one last thing about it How should we know if it is
1/sigma1 – 1/sigma2 or the other way around ? What gives us the information if it matters ?

b) In Figure b, we see that the potential function is a piecewise linear function. The changes in the slope of the potential are due to surface charges at the interfaces. We compute the surface charge using Gauss's law. At the interface between layers 1 and 2 we draw a small cylindrical Gaussian box with axis parallel to the z-axis. Gauss's law gives:

$$\oiint \vec{E} \cdot d\vec{a} = \oiint (E_2 - E_1)\, da = (E_2 - E_1)\, A = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{\sigma_{\text{charge},12} A}{\epsilon_0}, \tag{4}$$

where $\sigma_{\text{charge},12}$ is the surface charge at the interface between layers 1 and 2 and A the surface of the Gaussian box parallel to the interface. The electric fields in layer 1 and 2 are related to the (uniform) current I via Ohm's law: $E_i = \frac{I}{\pi L^2 \sigma_i}$, with i indicating the layer. Thus, the surface charge density at the interface between layer 1 and 2 is $\sigma_{\text{charge},12} = \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right) \frac{\epsilon_0 I}{\pi L^2} = -\frac{1}{4\sigma} \frac{\epsilon_0 I}{\pi L^2}$. Similarly, at the interface between layer 2 and 3 the surface charge density is $\sigma_{\text{charge},23} = \left(\frac{1}{\sigma_3} - \frac{1}{\sigma_2}\right) \frac{\epsilon_0 I}{\pi L^2} = \frac{3}{4\sigma} \frac{\epsilon_0 I}{\pi L^2}$. Finally, following similar reasoning the surface charge density on the bottom plate is $\sigma_{\text{bottom}} = \frac{1}{\sigma_1} \frac{\epsilon_0 I}{\pi L^2} = \frac{1}{2\sigma} \frac{\epsilon_0 I}{\pi L^2}$ and the charge on the top plate is $\sigma_{\text{top}} = -\frac{1}{\sigma_3} \frac{\epsilon_0 I}{\pi L^2} = -\frac{1}{\sigma} \frac{\epsilon_0 I}{\pi L^2}$.