5.12 Do the Maxwell's equations explain all magnetic phenomena?

For the lecture, problems, written exam: yes!

In general: No! Why? Maxwell's equations created in 1860's.
Relativity was discovered later.

5.12.1 Beyond Maxwell's equations: Lorentz transformation and relativity

There are subjects in electrodynamics which are not covered by Maxwell's equations, for example, spin-orbit coupling and fields generated by ultrafast charges: in such cases a **relativistic** approach is needed based on

Lorentz transformation of inertial frames $\tilde{F}$

Result in qualitative terms:

Electrical and magnetic fields are different aspects of the same electromagnetic field.

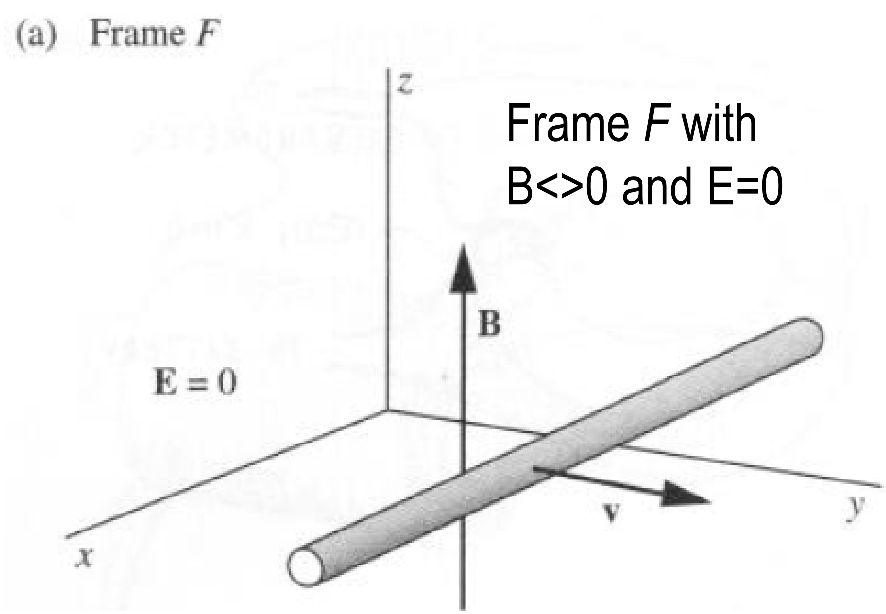
In moving from one inertial frame $\tilde{F}$ to another, the field $\vec{E}$ can change in whole or in part

to a field $\vec{B}$ or vice versa.

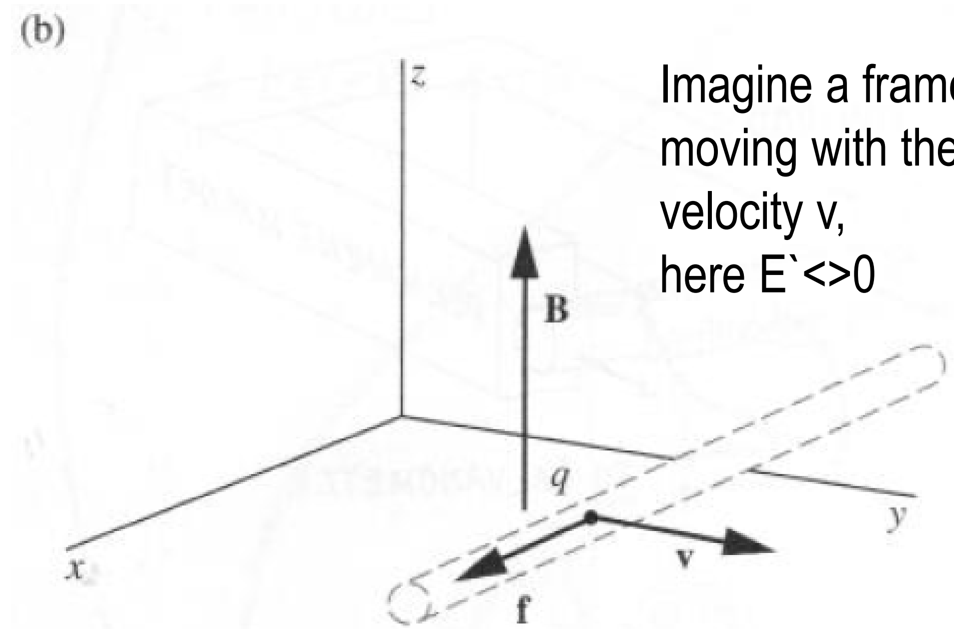
On the next slide
the basic idea is sketched
(taken from Purcell).

5.12.2 How fields transform

Illustration:



Frame F with B<>0 and E=0



Imagine a frame F' moving with the velocity v, here E`<>0

Formulas for transformation into frame F':

$$\overrightarrow{E'}_{\parallel} = \overrightarrow{E}_{\parallel} \qquad \overrightarrow{E'}_{\perp} = \gamma(\overrightarrow{E}_{\perp} + \vec{v} \times \overrightarrow{B}_{\perp})$$

$$\overrightarrow{B'}_{\parallel} = \overrightarrow{B}_{\parallel} \qquad \overrightarrow{B'}_{\perp} = \gamma(\overrightarrow{B}_{\perp} - (\frac{\vec{v}}{c^2}) \times \overrightarrow{E}_{\perp})$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c}$$

The term $\overrightarrow{E'}_{\perp} = \gamma \vec{v} \times \overrightarrow{B}_{\perp}$ provides the magnetic force

$$\vec{F} = q\vec{v} \times \vec{B}$$

if $\beta \ll 1$ (i.e. charges are slow).

$\parallel$: component parallel to $\vec{v}$ of F'

$\perp$: component perpendicular to $\vec{v}$ of F'

Summary 1: Maxwell equations in matter (2 scenarios)

in matter with polarization effects

$$\vec{\nabla} \cdot \vec{D} = \rho_{free} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E} \qquad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

in matter without any polarization effect

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Fields E and B enter the **Lorentz force law**:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Coulomb force (first term); magnetic Force (second term)

Relativistic effects are not described by these equations.

Summary 2: Maxwell's equations in vacuum without free charges

Maxwell equations **in empty space without charges and currents**:

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

The induction law and "displacement current" exist in vacuum, without considering any matter or body with charges.

These two observables led to the **theory of light.**