École polytechnique fédérale de Lausanne

PHYS-201(d) (D. Grundler)

Final Exam 19 January 2024

Name: ______________________	Sciper: __________	Section: ___

- The exam takes place from 15:15 to 18:15.
- Please write your name, sciper number and section on the page. Put your valid ID on the table.
- Do not release the staple.
- There are 5 problems and 21 pages (incl. the front page). Each problem provides different points as indicated. The total number of points is **31**. Check that you have received all problems and pages.
- If there is not enough space for the solution on the front side of a sheet, you can use its back side.
- Allowed Materials: Calculator without internet connectivity (without wi-fi), without the ability to solve integrals with unknowns, and without access to memory by which you may have stored any material/information.

  One sheet of paper containing handwritten equations/own collection of formulas (both sides of the sheet of paper can be used).

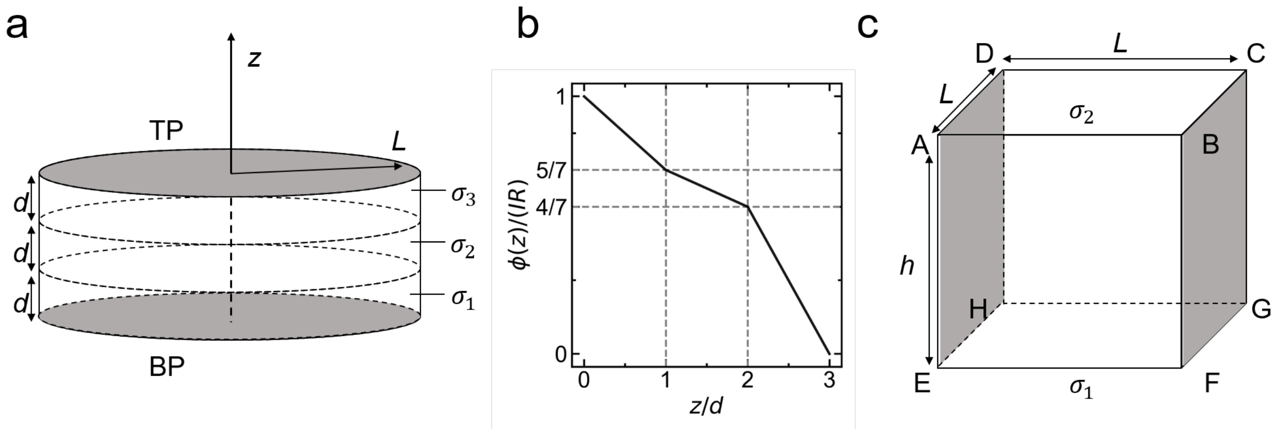
  Dictionary for English vocabulary without any added notes and without physics content.
- Respect the Honour code.
- If you want to leave before the end of the written exam, please raise your hand and wait until an invigilator arrives at your place.
- Please respect your colleagues and do not leave between 18:00 and 18:15. Use remaining time to check your solutions.
- Please remain seated while we collect the papers at the end of the exam.
- Good luck!
- Useful constants:
  $c = 3 \times 10^8 \ \mathrm{m\,s^{-1}}$
  $\mu_0 = 4\pi \times 10^{-7} \ \mathrm{T\,m\,A^{-1}}$
- The curl of a vector field $\vec{A}$ in cylindrical coordinates (unit vectors: radial direction $\hat{r}$, azimuthal direction $\hat{\theta}$, height $\hat{z}$):
  $\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{z}$
- Useful integral:
  $\int \frac{x^3}{(x^2+a^2)^{3/2}} dx = \frac{2a^2+x^2}{\sqrt{x^2+a^2}}$

Problem 1

a) Two parallel plates TP and BP of perfect conductors (dark gray) are in contact with a composite material consisting of three layers of equal thickness d with conductivity $\sigma_1, \sigma_2, \sigma_3$ as indicated in Fig. a. Here, $\sigma_1 = 2\sigma$, $\sigma_2 = 4\sigma$, $\sigma_3 = \sigma$, with σ a constant conductivity. Each resistive layer has isotropic properties. The plates have circular cross-section with radius L. Determine the resistance R between the plates depending on the given parameters. (2p)

b) Assume a steady current I is flowing between the plates from bottom to top of Fig. a (along the z-axis). The material extends from $z = 0$ to $z = 3d$. Figure b shows the potential function $\phi(z)$ in the material. Find the charge distribution in the structure shown in Fig. a that is consistent with current I. Fringe fields are neglected. (3p)

c) Now consider the case where perfectly conducting parallel plates are applied to the side faces ADHE and BCGF (Fig. c) of another material. Between the plates is a material for which the conductivity σ decreases linearly from a value σ_2 at the top face ABCD to σ_1 at bottom face EFGH. The edges of the top and bottom sides have lengths L. Compute the resistance between the plates assuming a height of the inhomogeneous material of h. (3p)



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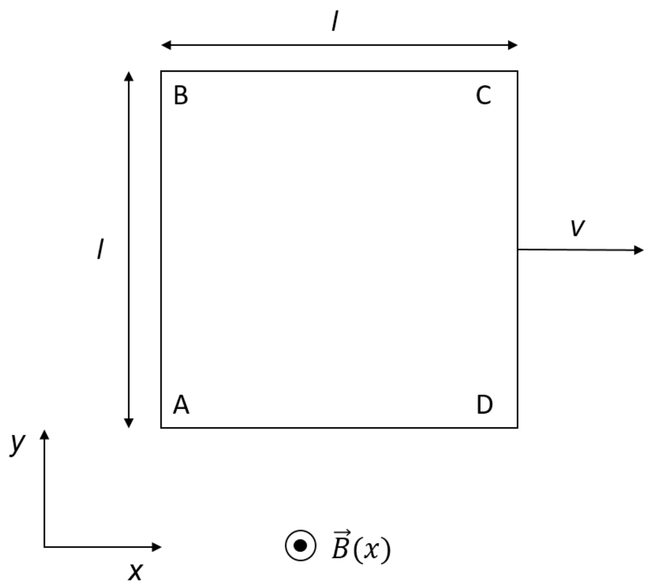
Problem 2

Consider a square loop with sides of length $l = 8$ cm placed in the x, y-plane (Fig. a). The loop is placed in an inhomogeneous magnetic field $\vec{B}(x)$. The field $\vec{B} = (0, 0, B(x))$ is along the z-direction, and varies in strength $B(x)$ along the x-axis according to the relation $dB/dx = 10^{-1}\ \mathrm{T\,m^{-1}}$. For $x = 0$, the magnetic field is zero. The loop moves at a fixed velocity $v = 15\ \mathrm{m\,s^{-1}}$ along the positive x-axis as indicated. The loop is rigid. In this problem, consider the conductive wire forming the loop to be infinitesimally thin.

a) Compute the electromotive force (emf) induced in the loop. Provide its value. (2p)

b) In the closed loop, an induced DC current I is flowing. In the loop, the segments BC and DA exhibit a resistance of 5 Ω each. Segments AB and CD are perfect conductors. Calculate all the force vectors which act on the four segments of the loop. Which force is needed to produce the motion? Provide its direction and value. (3p)

c) How much distance does the loop move to dissipate an electrical energy of 2×10^{-9} J? (2p)



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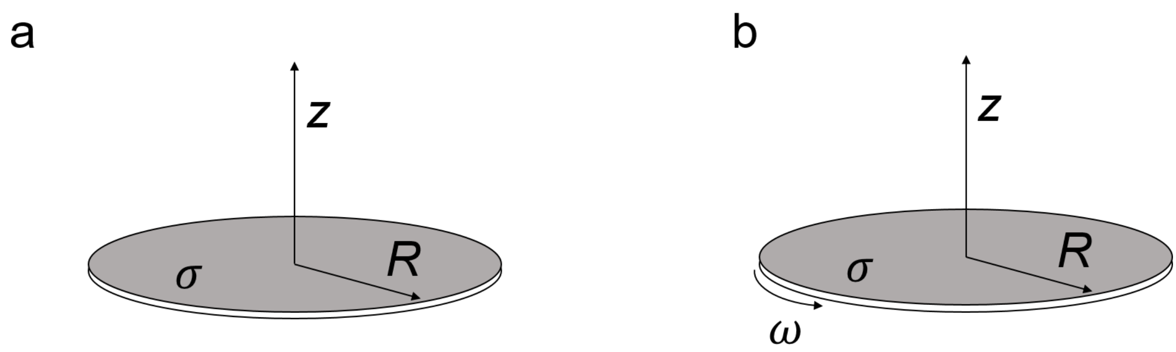
Problem 3

An ultrathin disk with radius R contains a uniform surface charge density σ of fixed positive charges. Its thickness is negligible in the following.

a) The disk is at rest (Fig. a). Provide the expression for the electrical potential function $\phi(z)$ along the central axis of the disk (z-axis) using the given parameters. (4p)

b) Now the disk spins with constant angular frequency ω (Fig. b). What is the direction of the magnetic field on the central axis (above and below the disk in Fig. b)? Explain your answer. (2p)

c) Provide the expression for the magnetic field vector $\vec{B}(z)$ along the central axis of the disk (z-axis) shown in Fig. b using the given parameters. (2p)



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Problem 4

Consider an infinitely long cylindrical wire along the z-axis with a radius R. A DC current flows along the solid wire in positive z-direction and produces a magnetic field $\vec{B}$ of constant magnitude B_0 inside the non-magnetic material of the wire.
Make a sketch of the wire defining the coordinate system. Provide the vector description of $\vec{B}$ inside the wire. How does the current density $\vec{J}$ depend on the radial coordinate r? Find the expression for $\vec{J}$ as a function of radial coordinate r and parameter B_0. (3p)

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Problem 5

A linearly polarized electromagnetic wave $\vec{E}_\mathrm{i}$ is traveling in vacuum in positive y-direction with frequency $f = 500 \times 10^{12}$ Hz and electric field amplitude $E_{0\mathrm{i}} = 2\ \mathrm{V\,m^{-1}}$. It is reflected under normal incidence at the surface of a lossless glass with an index of refraction $n = 1.50$. Due to the mismatch in the index of refraction, 4% of the energy is reflected. The superposition of the incident and reflected wave gives rise to the following electric field $\vec{E}$ in vacuum in front of the glass:

$$\vec{E} = (E_{0\mathrm{i}} + E_{0\mathrm{r'}}) \sin\frac{2\pi y}{\lambda} \sin\frac{2\pi ct}{\lambda}\hat{z} + (E_{0\mathrm{i}} - E_{0\mathrm{r'}}) \cos\frac{2\pi y}{\lambda} \cos\frac{2\pi ct}{\lambda}\hat{z},$$

with $E_{0\mathrm{i}}$ the incident electric field amplitude, $E_{0\mathrm{r'}}$ the reflected electric field amplitude, λ the wavelength of the incident wave, y the position coordinate, and t the time. c is the velocity of light.

a) Sketch the electric field E at points (i) $y = -\lambda/4$ and (ii) $y = -\lambda/2$ as a function of time t. Provide two separate figures (i) and (ii) for which you consider one period starting from $t = 0$. Label the axes. The points y are in vacuum. (2p)

b) Find the value of the electric field amplitude $E_{0\mathrm{r'}}$ of the reflected wave $\vec{E}_{\mathrm{r'}}$. (1p)

c) Assume that the electromagnetic wave $\vec{E}_\mathrm{r}$ (electric field component) inside the glass has a maximum value $E_{0\mathrm{r}}$. Provide an expression for $\vec{E}_\mathrm{r}$ using the given values which describes its dependence on y and t in quantitative terms in the lossless glass. The glass extends to plus infinity from $y = 0$. (2p)

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