École polytechnique fédérale de Lausanne

PHYS-201(d) (D. Grundler)

Mock Exam December 2023

Name: ______________________	Sciper: __________	Section: ___

- The exam takes place from 15:15 to 18:15.
- Please write your name, sciper number and section on the page. Put your valid ID on the table.
- Do not release the staple.
- There are 4 problems and 17 pages (incl. the front page). Each problem provides different points as indicated. The total number of points is 33. Check that you have received all problems and pages.
- If there is not enough space for the solution on the front side of a sheet, you can use its back side.
- Allowed Materials: Calculator without internet connectivity (without wi-fi), without the ability to solve integrals with unknowns, and without access to memory by which you may have stored any material/information.

  One sheet of paper containing handwritten equations/own collection of formulas (both sides of the sheet of paper can be used).

  Dictionary for English vocabulary without any added notes and without physics content.
- Respect the Honour code.
- If you want to leave before the end of the written exam, please raise your hand and wait until an invigilator arrives at your place.
- Please respect your colleagues and do not leave between 18:00 and 18:15. Use remaining time to check your solutions.
- Please remain seated while we collect the papers at the end of the exam.
- Good luck!

Problem 1

Assume two concentric conducting spherical shells of radii a, b where $a < b$. The three-dimensional shells are sketched as black lines in the cross sections shown below. The shells are assumed to be perfect conductors. Neglect their thicknesses in the calculations below.

a) For the problem a) assume that the material (gray in Fig. a) between the two conducting spherical shells is an insulating dielectric. Its dielectric constant is inhomogeneous and given by

$$\epsilon_{\mathrm{r}} = \frac{1}{1 - Gr}.$$

Here, r is the distance from the center. G is a positive constant in units of 1/m with $G < 1/b$. An amount of free charges $+Q$ is placed on the inner spherical shell. The outer shell is connected to the ground with an infinitely thin conducting wire. Derive the expressions asked below in terms of the given parameters.

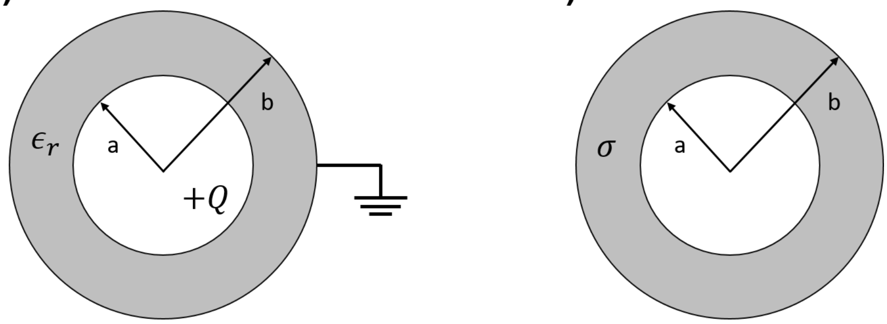
a1. Find the expressions for the electric displacement vector $\vec{D}$ for $a < r < b$ and $r > b$. (2 pts)

a2. Find the expression for the capacitance C of this device. (3 pts)

a3. In the dielectric, find the densities of bound charges σ_{b} at its inner ($r = a$) and outer surface ($r = b$). (2 pts)

b) For the problem b) assume the space between the shells to be filled with a different material which is conducting (gray in Fig. b). Its specific conductivity σ varies according to $\sigma = KE$, with K a constant and E the local electric field. The spherical shells are kept at a constant electric potential difference ΔV and a dc current I flows between the shells. Find an expression for the current I as a function of K, ΔV, a, and b. (3 pts)

a) **b)**



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Problem 2

An infinitely long cylindrical solenoid of radius b has n turns per unit length. The current I in the wire of the solenoid depends on the time t according to $I(t) = I_0 \sin \omega t$, where ω is the angular frequency and I_0 the current amplitude. Additionally, a non-uniform magnetic field $\vec{B}_{\text{ex}}(x) = \alpha_0 x \hat{z}$ with α_0 a positive constant and x the x-coordinate is applied. The origin $(0, 0, 0)$ of the coordinate system is considered to be on the central axis (dashed line) inside the infinitely long solenoid.

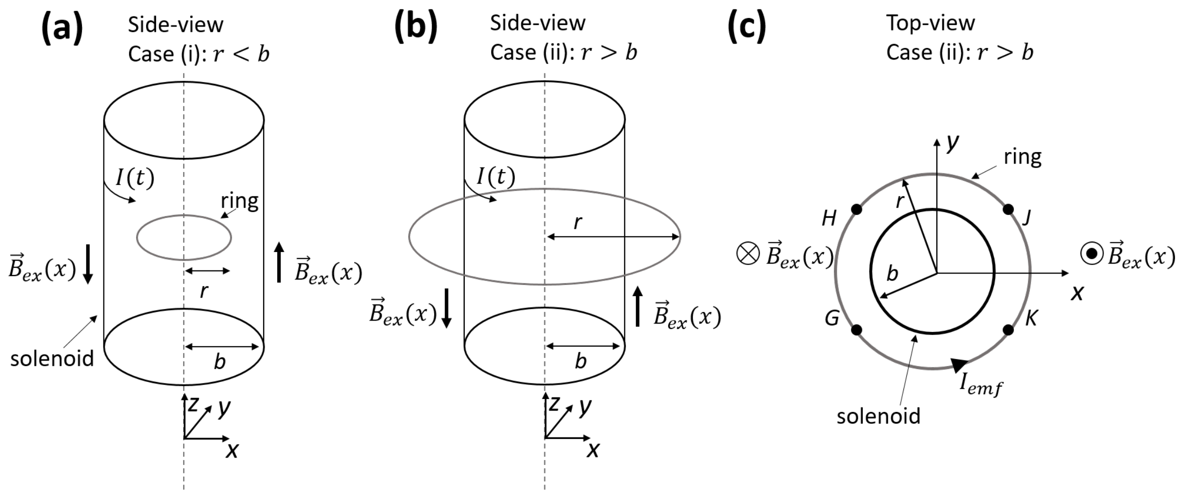
A conducting ring (gray) of radius r is centered on the solenoid's axis. The ring is in the x, y plane and held in place. In this problem, neglect the self-inductance. The thicknesses of the solenoid and the ring are negligible.

a) Find the electromotive force (emf) induced in the ring as a function of I_0, b, r, n, t, and ω. Consider the two cases separately where (i) $r < b$ (Fig. a) and (ii) $r > b$ (Fig. b). (2 pts)

b) Assume a ring that has $r > b$ and a resistance R. Sketch both the time-dependent induced current $I_{\text{emf}}(t)$ in the ring and the current flowing through the solenoid $I(t)$ as a function of t for one full period T with respect to the same time axis t. (1 pt)

c) Sketch the directions of magnetic force vectors acting at the positions G, H, J, and K on the ring with $r > b$ (see Fig. c). Assume the induced current I_{emf} to flow as sketched. (1 pt)

d) Find the expression for the total time-dependent magnetic force vector acting on the whole ring based on the given parameters. Consider the ring for which $r > b$. (4 pts)



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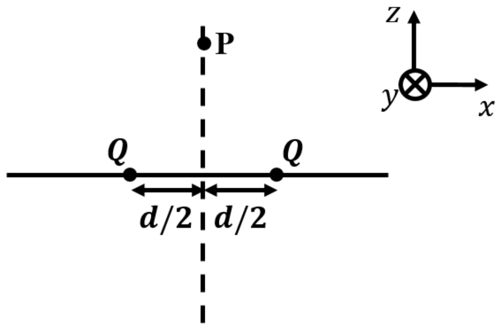
Problem 3

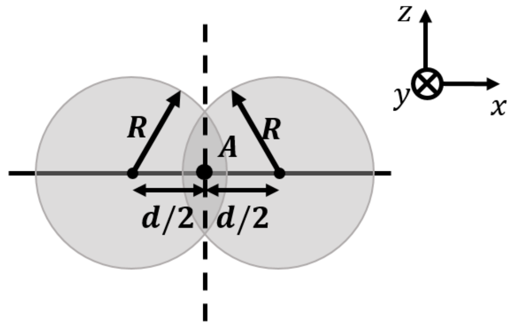
a) Consider two positive point charges Q separated by a distance d as shown in the sketch below on the left. Find the electric field $\vec{E}$ (magnitude and direction) at point P situated at a distance z above the midpoint between the charges, as a function of Q, z and d. The midpoint is indicated by the broken line. (3 P)

b) Now consider that there are two three-dimensional solid spheres of radius R. Positive electrical charges are uniformly distributed in the volume of each sphere. The total amount of charges in each sphere is Q. These two spheres are brought closer and made to penetrate each other. The cross-sectional view is shown in the sketch below on the right. This means that in the overlapping region, the charge density is twice that of the rest. The centres of the spheres are separated by a distance $d = \frac{8}{5}R$.

   (I) Find the expression of the electric field $\vec{E}$ at a point P situated at a distance z above the midpoint between the two spheres in terms of Q, z, and R. Consider the two cases for which the point P is either inside or outside the two spheres. (4 P)

   (II) Calculate the electrical potential at the mid-point between the centres of the two spheres (point A in the sketch on the right) in terms of Q and R. (2 P)





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Problem 4

We consider a plane-wave electromagnetic wave whose electric field vector is given by

$$\vec{E}(z,t) = E_0\left(\cos(kz-\omega t)\,\hat{x} + \sin(kz-\omega t)\,\hat{y}\right),$$

with E_0 the amplitude, k the wavevector, t the time and ω the angular frequency. We assume a Cartesian coordinate system and k to be positive.

a) What is the propagation direction of the wave? Compute the electric field vector in terms of E_0 at position $z = \lambda/2$ and time $t = \frac{\pi}{4\omega}$. Here, λ is the wavelength associated with the plane wave. What is the corresponding magnetic field vector $\vec{B}$ at the same position and time? (3 pts)

b) Now assume an electron of charge q and mass m which moves in the x, y plane in response to the electric field vector given above. Show that the following formula can describe the motion of the electron in response to the electric field vector given above:

$$\vec{v}(z,t) = \Omega\left(-\sin(kz-\omega t)\,\hat{x} + \cos(kz-\omega t)\,\hat{y}\right).$$

One assumes that the time-averaged velocity of the electron is zero and that there is no effect of the magnetic field of the wave. Determine the expression for Ω in the given variables. (2 pts)

c) Does the electron emit an electromagnetic wave? If so, give the assumed propagation direction of the wave. Explain your answer. (1 pt)

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