Exercise Sheet 8

Discussion 05.11.2025

Exercise 1 - Image charge

Two large grounded conducting planes intersect orthogonal to each other. A point charge Q is now placed in the corner of the two planes with a perpendicular distance of d from both planes.

a) Make a sketch of the situation and place image charges in order to make both planes zero equipotentials and draw the electric field lines.

b) What is the magnitude and direction of the force acting on Q?

The concept of image charges can also be used for a dipole (image dipole). Consider a dipole placed at distance d from a grounded conducting flat surface with its dipole moment $\vec{p}$ perpendicular to the surface, pointing away from it.

c) From the formulas derived in the lecture, evaluate the electric field $\vec{E}(r)$ for this system along the axis of the dipole.

d) Evaluate the potential energy of the dipole with respect to its image dipole.

Discussion 1 - Dipole moment

In the previous question the electric dipole moment was oriented perpendicular to the plane and you should have been able to determine the orientation of the image dipole moment. Now consider an electric dipole with the dipole moment parallel to a conducting plane. How will the image dipole moment be oriented in this case? What do these orientations of the mirror dipole moment tell you about the dipole moment as a vector? What do you think is the reason for this?

Exercise 2 - Capacitance of wires

Two wires with uniform charge distribution $+\lambda$ and $-\lambda$ are hanging parallel to each other at a distance d (wire center to wire center). The radius of the wires is $a << d$.

a) Evaluate the capacitance per unit length between the two wires.

Consider now a single wire with uniform charge distribution λ hanging parallel to the ground at distance d. The ground can be considered as a grounded conductive plane.

b) By using the result of the previous point and the concept of image charge, evaluate the capacitance per unit length between the wire and the ground.

Exercise 3 - Polarization of Barium titanate

The perovskite $BaTiO_3$ is a widely used dielectric and is a ferroelectric material for temperatures lower than 120 °C. At this temperature the crystal structure changes and the Ti^{4+} ion (i.e. with a charge of $+4e$) in the middle of the unit cell is shifted 0.012 nm from the centre as shown in fig. 1 a) and b). Here two methods to obtain the dipole moment will be compared.

a) Replace the charge of the Ba^{2+} and O^{2-} ions by a virtual ion in the centre of the unit cell. What is the charge of this ion? (Hint: if ions are shared between adjacent unit cells also their charge is shared)

b) Determine the size of the dipole moment $\vec{p}$ per $BaTiO_3$ unit cell.

c) Now determine the magnitude of the moment $\vec{p}$ at room temperature using the size of the unit cell and the value of the polarization $\vec{P}$ given in fig. 1 a) and c).

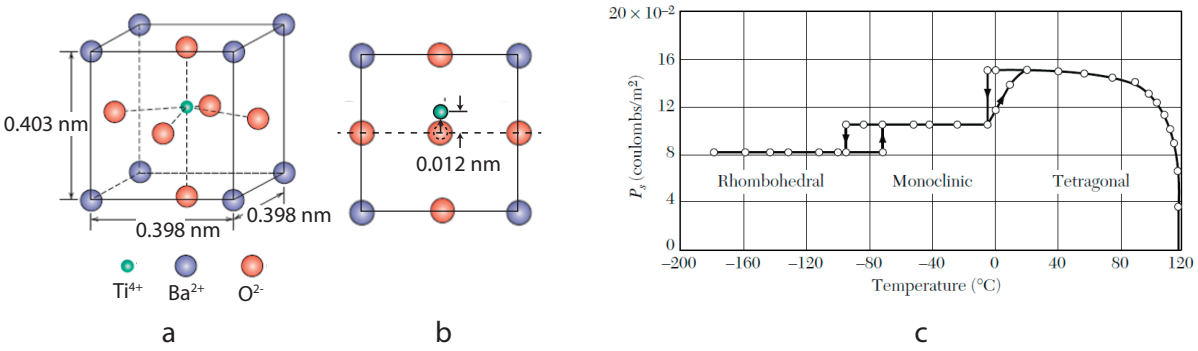

Figure 1: a) Unit cell of $BaTiO_3$ at room temperature. b) Simplified side view of the unit cell. c) Polarization of $BaTiO_3$ as a function of temperature for the ferroelectric phases.