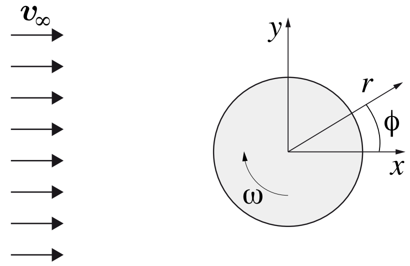
Exercise Sheet 4

Discussion 01.10.2025

Exercise 1 - Magnus effect

A cylinder (radius R, length L) of horizontal axis is immersed in a perfect, incompressible fluid of density ρ flowing horizontally at a constant velocity v_∞ far away from this obstacle. The velocity field of the flow around the cylinder is given by (polar coordinates):



$$v_r = v_\infty \cos\phi \left(1 - \frac{R^2}{r^2}\right) \quad \text{et} \quad v_\phi = -v_\infty \sin\phi \left(1 + \frac{R^2}{r^2}\right).$$

The cylinder rotates at a constant angular velocity ω. The viscosity of the fluid, although low, drags the fluid. This imposes a circular motion of the fluid around the cylinder. This additional motion of the fluid is not described by the velocity field given above.
The system is modeled by considering that, in the vicinity of the cylinder, the velocity of the fluid particles is given by the superposition of the velocity due to the flow and to the rotational speed.

Neglecting the gravitational force and considering that Bernoulli's theorem can be applied, derive (establish) the Kutta-Joukowski formula expressing the lift force on the cylinder:

$$F_{p,y} = 2\pi\omega R^2 \rho L v_\infty$$

Exercise 2 - Reynolds number

a) The air resistance of a car of length 4 m at a velocity of 30 m/s is to be investigated using a model.
Find the velocity required for the tests to be reliable using a 1:4 model in air and in water.
Air (NTP): $\rho = 1.2$ kg/m^3; $\eta = 1.8 \cdot 10^{-5}$ kg/(m s)
Water (5°C): $\rho = 1 \cdot 10^3$ kg/m^3; $\eta = 1.5 \cdot 10^{-3}$ kg/(m s)
b) What do these results tell us? Do you think it is possible to scale down the size even more? Can you figure out what are the limitations of such models? (Think about the hypothesis under which the calculations are carried out)

Discussion 1 - Drag and boundary layer

Consider the flow past a cylindrical object. Do you think that the drag force is higher in the laminar ($\mathcal{R} \approx 10$) or in the turbulent boundary layer ($\mathcal{R} \approx 10^6$) regime? Can you use this to explain the fact that golf balls have dimples?

Exercise 3 - Planar flows between two plates

Consider the flow of a viscous, incompressible fluid between two horizontal parallel plates separated by a distance $2h$ in the z direction.
Starting from the Navier-Stokes equation, determine the velocity field, sketch the velocity profile, and find the expression for the average velocity for two distinct cases:
a) the upper plate moves at a constant velocity v_0 along x and no pressure gradient is applied;
b) both planes are fixed, but a constant pressure gradient $\partial p/\partial x = -K$ is applied.
Neglect the effect of gravity and consider a steady flow with no-slip conditions.

Hints
i) Using the hypotheses and symmetry considerations, determine the non-zero components of the velocity and their dependencies.
ii) Use the hypothesis and the results of i) to simplify the Navier-Stokes equation.
iii) Solve the equation to find the expression for the velocity field.
iv) Find the expression for the average velocity.