

### Exercise Sheet 3

Discussion 24.09.2025

#### Exercise 1 - Airplane take-off

- Estimate what speed an Airbus 380 needs to take off if the wings are designed such that the air velocity above the wing is 20% larger as below the wing. The area of each wing is  $A = 425 \text{ m}^2$  and the maximum take off weight  $m = 560 \text{ tons}$ . Take  $\rho = 1.3 \text{ kg/m}^3$  for the air density.
- Do you think the real take-off speed is lower or higher, and why?

#### Discussion 1 - Streamlines

Sketch possible streamlines around and in a house when there is a slight wind that causes the door to slam when the windows at opposite sides are open. How do you interpret the streamline density and its spatial variations?

#### Exercise 2 - Streamlines in 2D

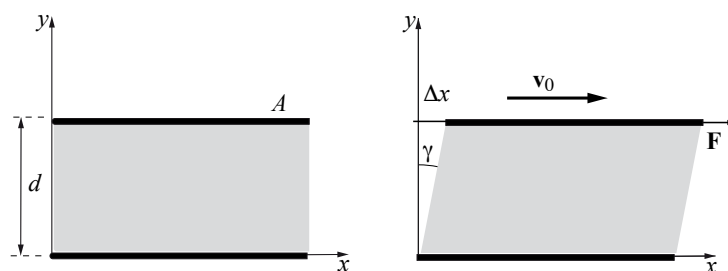
Determine the analytical expression for the streamlines and the acceleration  $\vec{a}(x, y)$  for a stationary bidimensional flow described by the velocity field :

$$\vec{v} = \left(\frac{v_0}{l}\right)(x\vec{e}_x - y\vec{e}_y) .$$

Make a sketch of the streamlines and the lines of constant acceleration.

#### Exercise 3 - Viscosity, shear stress, and shear strain

Consider a fluid with viscosity  $\eta$  between two plates (distance between the plates  $d$ , in the  $y$  direction). A force  $F$  is applied to the upper plate, which moves at constant velocity  $v_0$  in the  $x$  direction. We observe a constant velocity gradient in the fluid between the bottom stationary plate and the moving plate at the top.



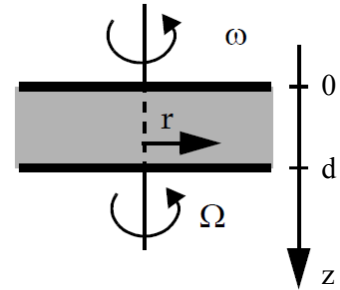
- Show that the shear stress  $S_{yx}$  is proportional to the gradient of the velocity in the  $y$  direction.
- The shear strain is given by  $e_{yx} = \frac{\Delta x}{h}$ . Rewrite the expression for  $S_{yx}$  in terms of  $\dot{\epsilon}$ , i.e. the shear rate.

#### Exercise 4 - Viscous drag

Two identical disks of radius  $R$  are able to rotate without friction around their axis: see figure. They are separated by a small distance  $d$  and the fluid between the disks has a viscosity  $\eta$ .

The top disk rotates at constant angular velocity  $\omega$  and the bottom one is initially at rest.

Determine the temporal dependency  $\Omega(t)$  describing the rotation of the second disk if we assume that the shear rate  $\frac{\partial v}{\partial z}$  depends only on  $r$  (distance from the axis) and possibly on  $t$ .



*Hints:*

- the non-slip boundary condition is satisfied: the velocity of the fluid in contact with each disk is equal to that of the disk;
- the moment of inertia of each disk with respect to its axis is  $I$ .
- note that here  $\Omega$  is not the vorticity.