Exercise Sheet 14

Discussion 17.12.2025

Exercise 1 - Refraction of Electromagnetic Waves

Consider a large glass aquarium filled with water. The thickness of the glass is 10mm and the basin is 20mm. A laser beam ($\lambda = 589.3$ nm) is aimed from underneath with an angle of 20° with respect to the bottom surface of the aquarium. For this wavelength, relative permittivity ϵ_r of glass and water are 4.7 and 1.77 respectively. In the following we consider ϵ_r as a frequency dependent constant.

a) Plot the speed of the laser beam and the wavelength as a function of distance.

b) Sketch the trajectory of the laser beam trough the aquarium.

Now the laser is replaced by a collimated microwave emitter with a wavelength of 10mm. In this energy range the ϵ_r of glass and water are 3.8 and 33.6 respectively.

c) Plot the speed, wavelength, and trajectory also for the microwave beam.

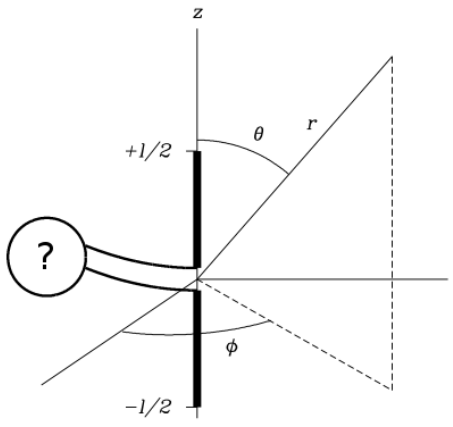
d) What is the minimum kinetic energy of an electron in the water to emit Cherenkov radiation? Take the relative permittivity for the laser wavelength as reference and ignore relativistic effects.

Exercise 2 - Oscillating dipoles: antennas

The E field created by a oscillating dipole can be found to be:

$$\bar{E}_{\text{rad}} = \frac{\ddot{p}(t - r/c)\sin(\theta)}{4\pi\epsilon_o rc^2}\hat{\theta} \tag{1}$$

a) Letting $\vec{p}(t) = q\vec{l}\sin(\omega t)$, confirm the E field pattern seen in the lectures. How could one practically realize this oscillating dipole with an antenna as seen in the figure?



b) Using $\vec{B} = 1/\text{c}\ \hat{r} \times \vec{E}$, find the B field of this simple antenna

c) Find the Poynting vector: the energy radiated per unit area and per unit time.