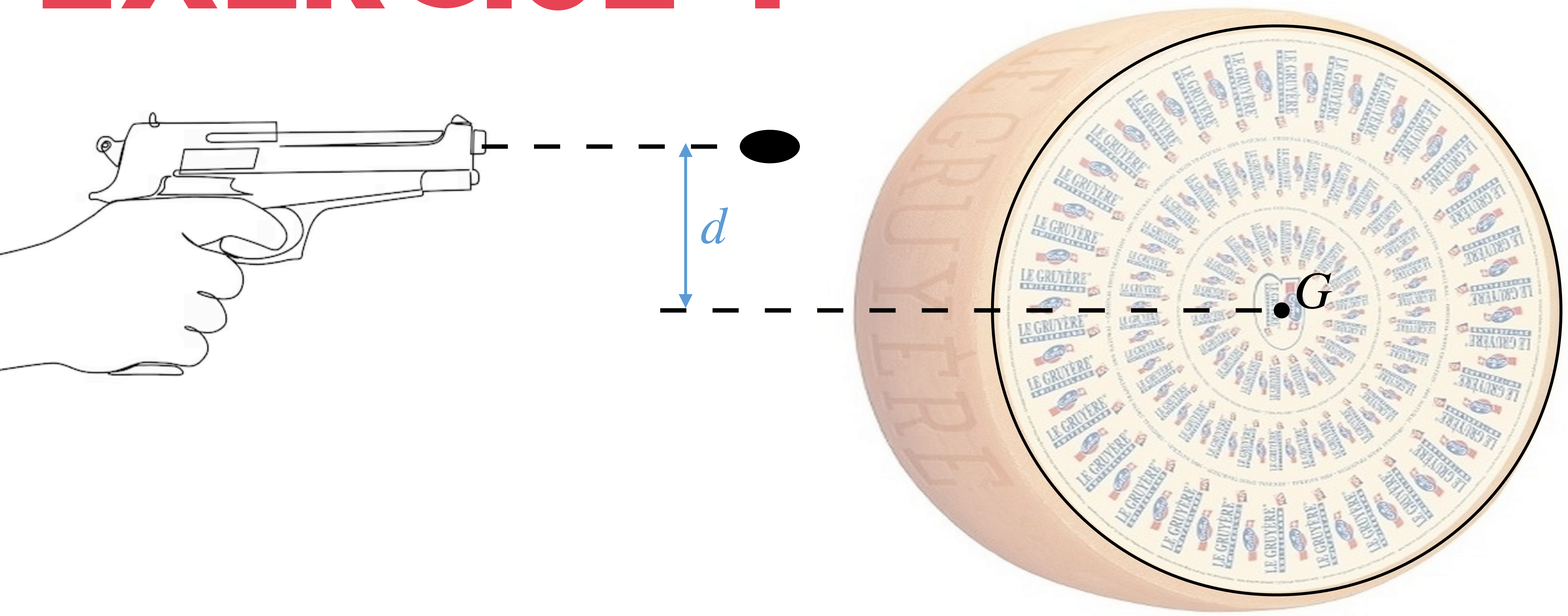
EXERCISE 1



A homogenous cheese wheel with radius R and mass M has a pivot inserted through its center of mass G so that it can turn freely around its axis of symmetry.

Someone fires a bullet with mass m and velocity $v0$, which embeds itself in the rind of the cheese at an offset d from the CoM.

With what angular velocity does the cheese wheel turn after the bullet hits the rind?? Neglect gravity and treat the bullet as a point mass with $m \ll M$.

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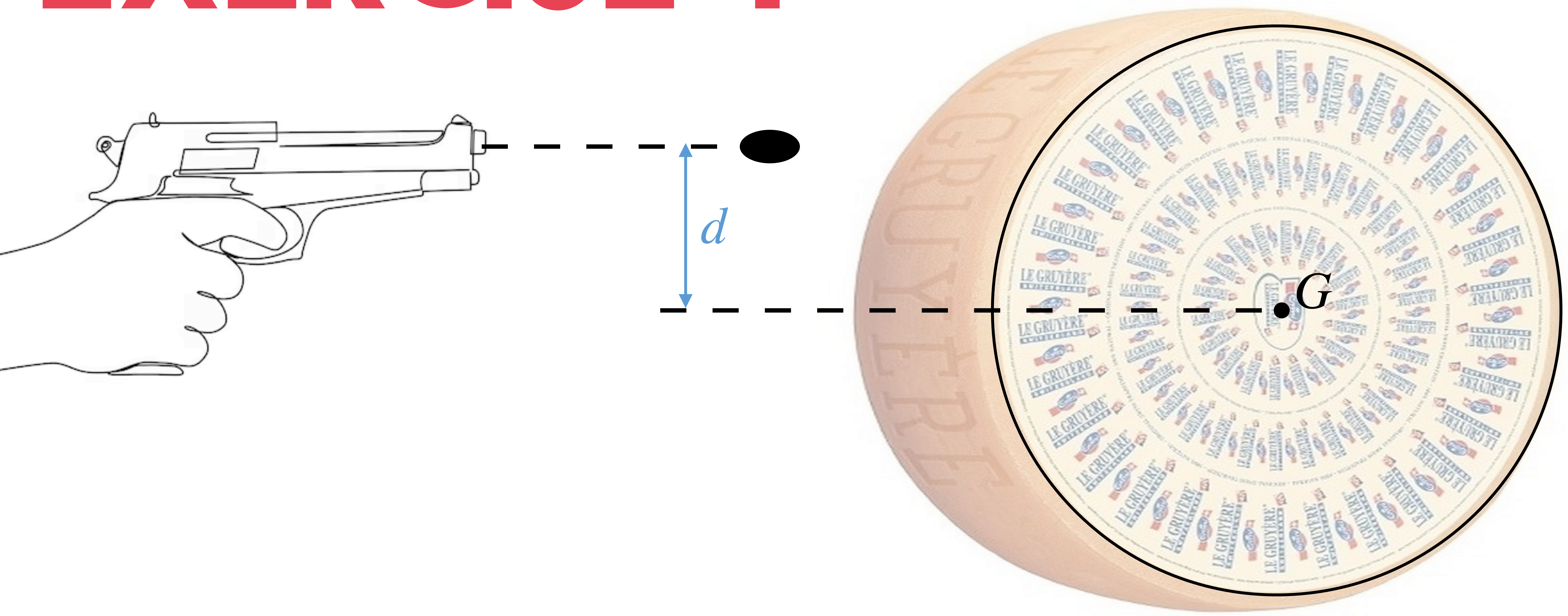

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This is an inelastic collision. Just as we use conservation of momentum for collisions, we can also use **conservation of angular momentum**.

Before collision, the angular momentum of the bullet is: $\vec{L}_G = -dmv_0\vec{e}_z$ (z-axis coming out of the screen)
& the cheese wheel isn't moving so its angular momentum is zero.

After collision, the angular momentum of the cheese wheel is: $\vec{L}_G = I_\Delta \vec{\omega} = \frac{1}{2}MR^2\omega\ \vec{e}_z$ (Neglecting the contribution from the mass of the bullet)

Conservation of angular momentum:

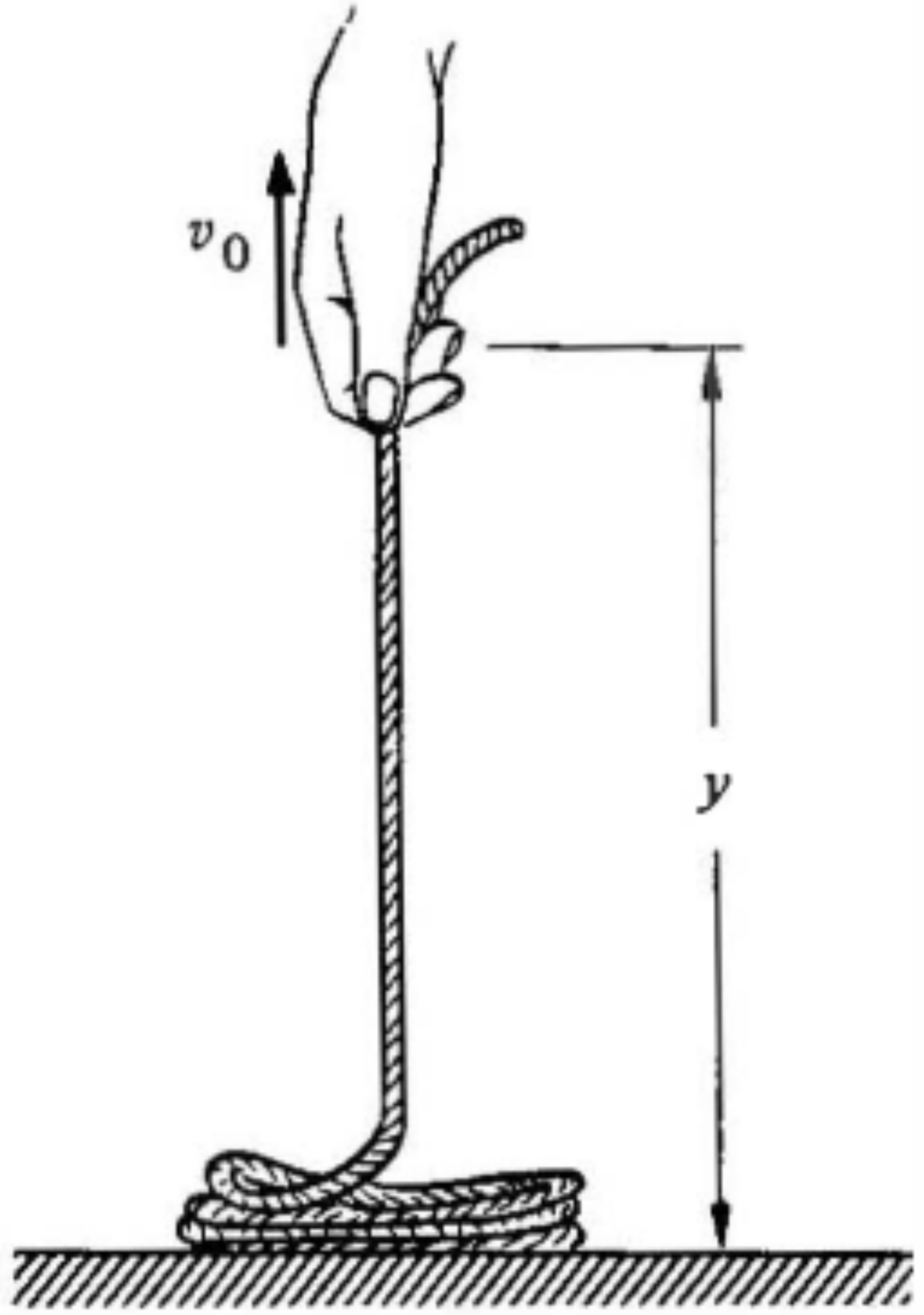
$$\omega = -2\frac{dmv_0}{MR^2}$$

EXERCISE 2

A uniform rope with linear mass density λ is coiled on a smooth horizontal table. One end is pulled straight up with constant speed Vo.

A) Find the force exerted on the end of the rope as a function of height y.

B) Compare the power delivered to the rope with the rate of change of the rope's total mechanical energy



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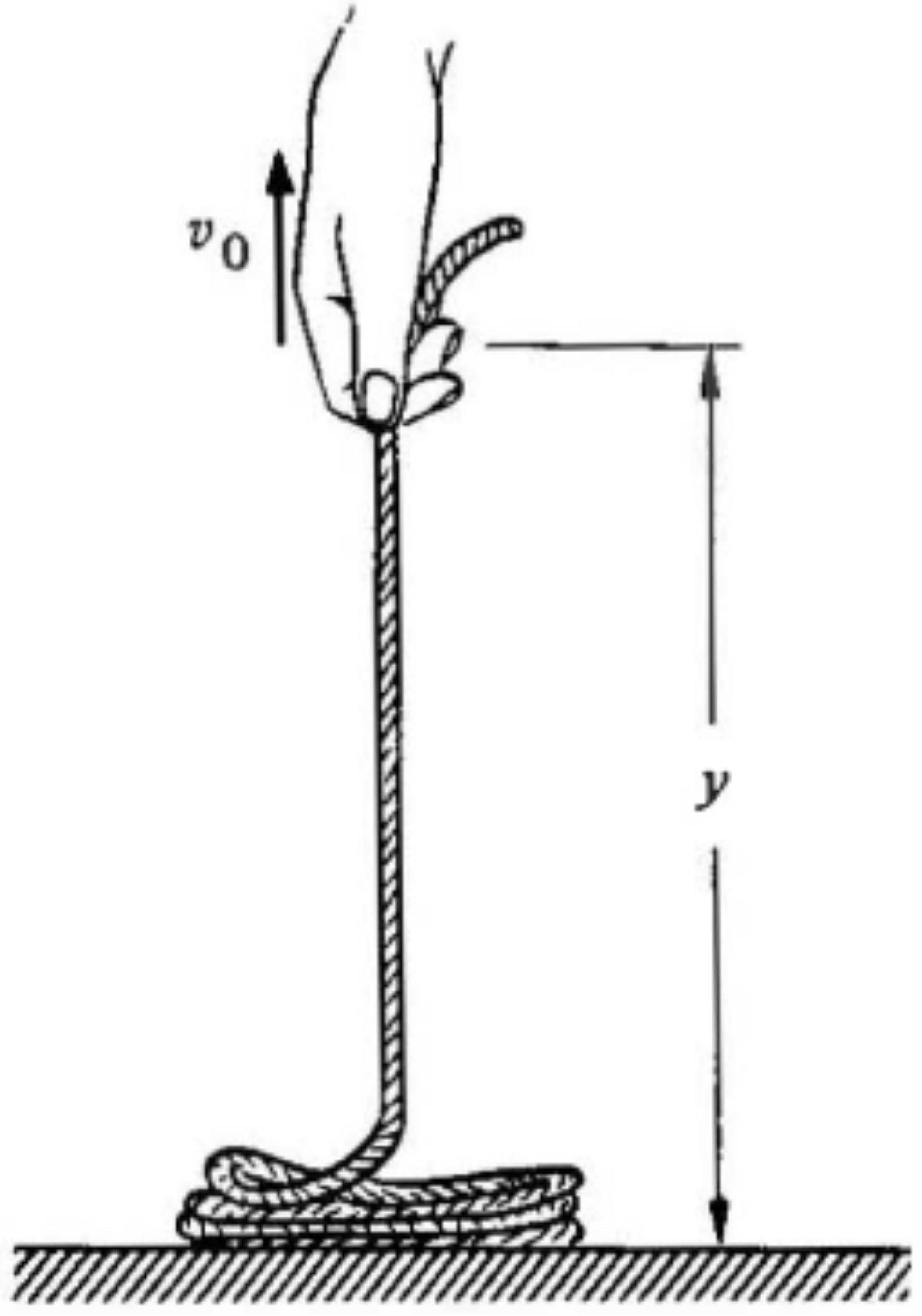

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Start by writing out quantities of the problem:

$$\vec{v} = v_0 \vec{e}_y \qquad \dot{\vec{v}} = 0 \qquad \frac{dy}{dt} = v_0 \qquad m(y) = \lambda y \qquad \frac{dm}{dt} = \lambda \frac{dy}{dt} = v_0 \lambda \qquad \vec{F}_G = -mg\vec{e}_y = -\lambda g y \vec{e}_y$$

Newton's 2nd Law:

$$\vec{F}_{ext} + \vec{F}_G = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = \dot{m}\vec{v} + m\dot{\vec{v}}$$

$$\boxed{\vec{F}_{ext} = \lambda v_0^2 \vec{e}_y + \lambda g y \vec{e}_y}$$

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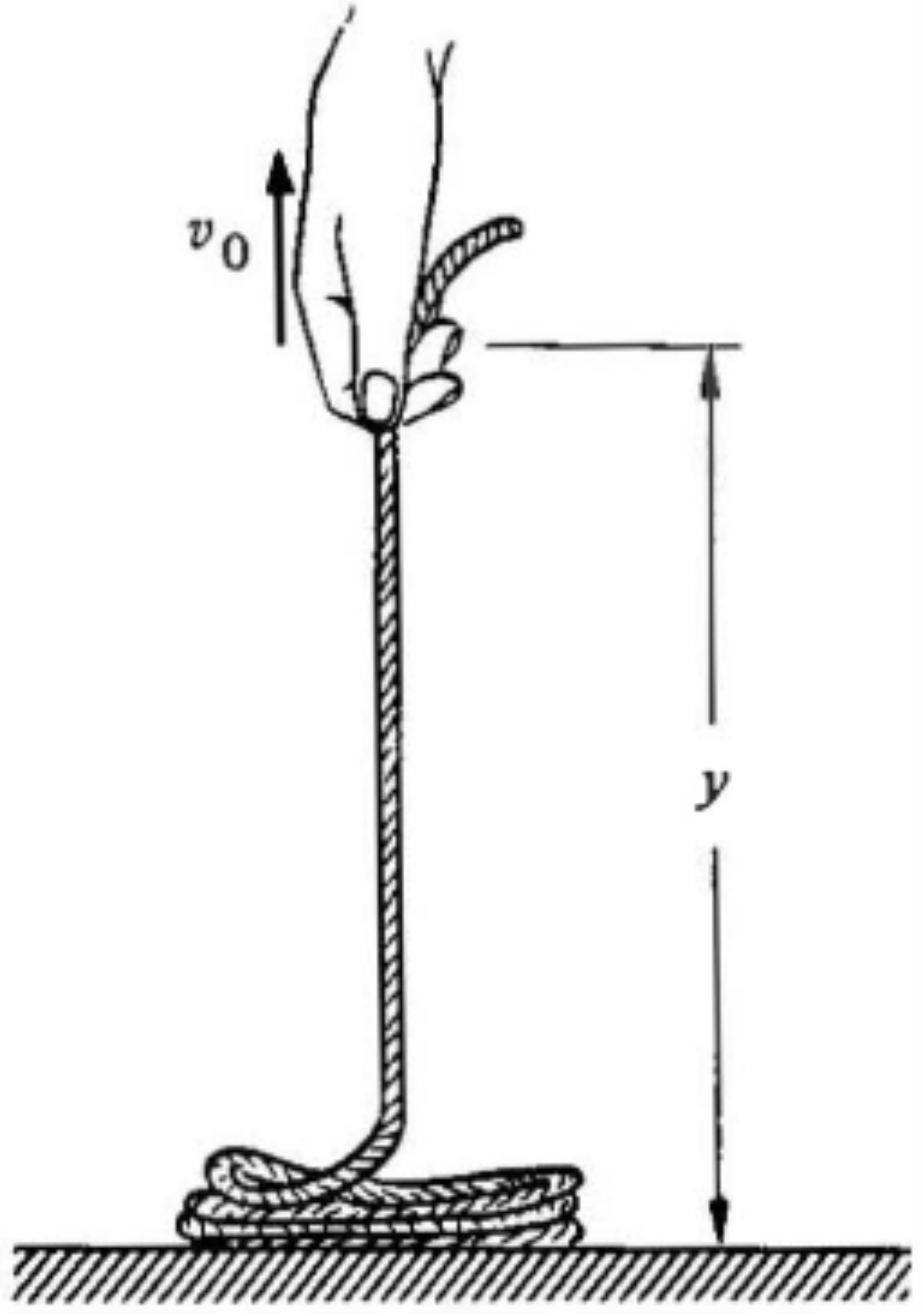

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Power is $P = \overrightarrow{F} \cdot \vec{v} = \lambda v_0^3 + \lambda g v_0 y$

Kinetic energy of the rope: $E_c = \frac{1}{2}mv^2 = \frac{1}{2}\lambda v_0^2 y$

Potential energy due to gravity (applies at CoM): $E_p = mgh = (\lambda y)g(y/2) = \frac{1}{2}\lambda g y^2$

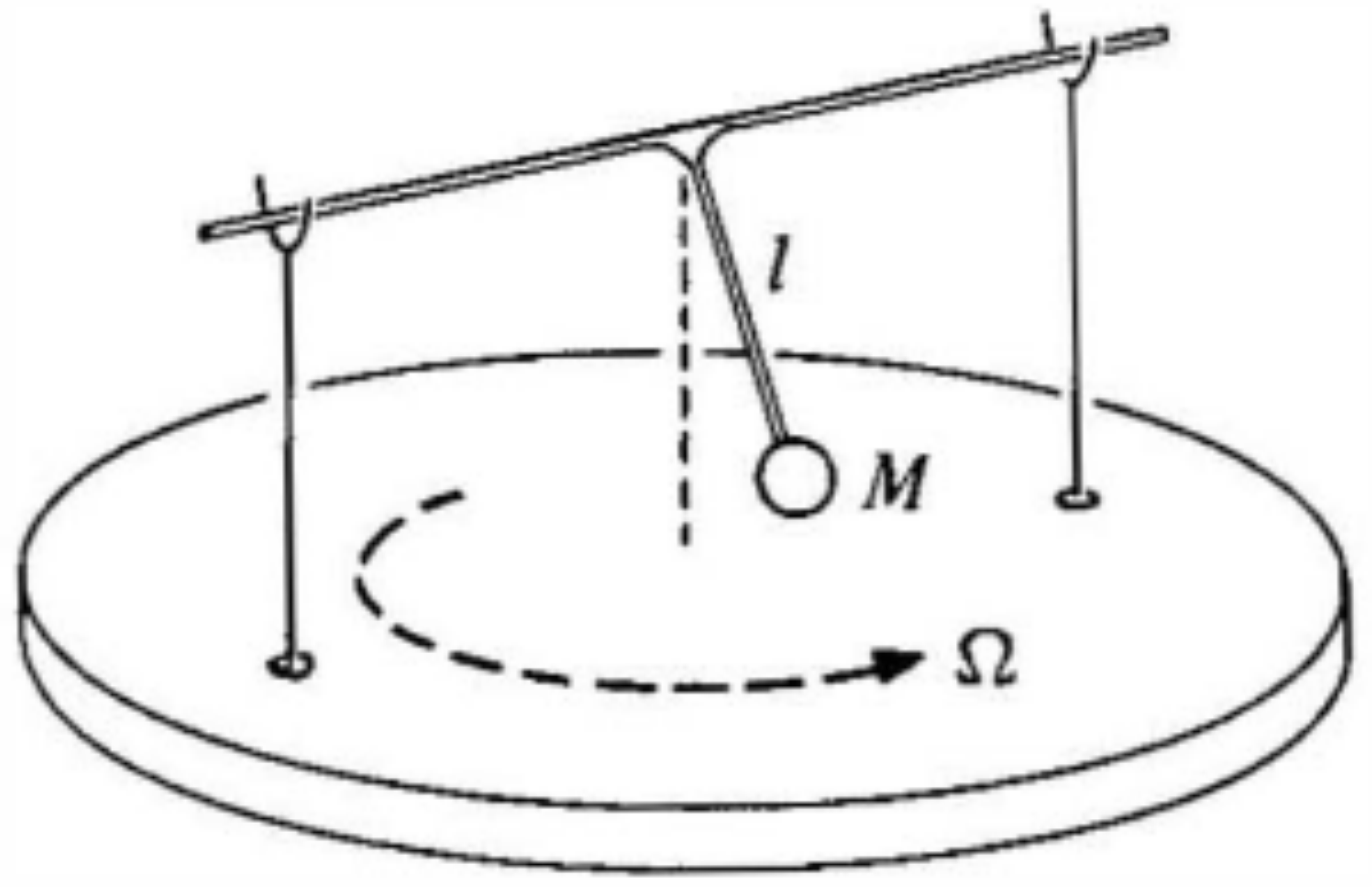
Rate of change of total mechanical energy:

$$\frac{d}{dt}(E_c + E_p) = \frac{1}{2}\lambda v_0^3 + \lambda g v_0 y$$

Mechanical energy is smaller than the power

EXERCISE 3

A pendulum is rigidly fixed to an axle held by two supports so that it can swing only in a plane perpendicular to the axle. The pendulum consists of a mass M attached to a massless rod of length l. The supports are mounted on a platform which rotates with constant angular velocity Ω. Find the pendulum's angular frequency assuming that the amplitude is small.



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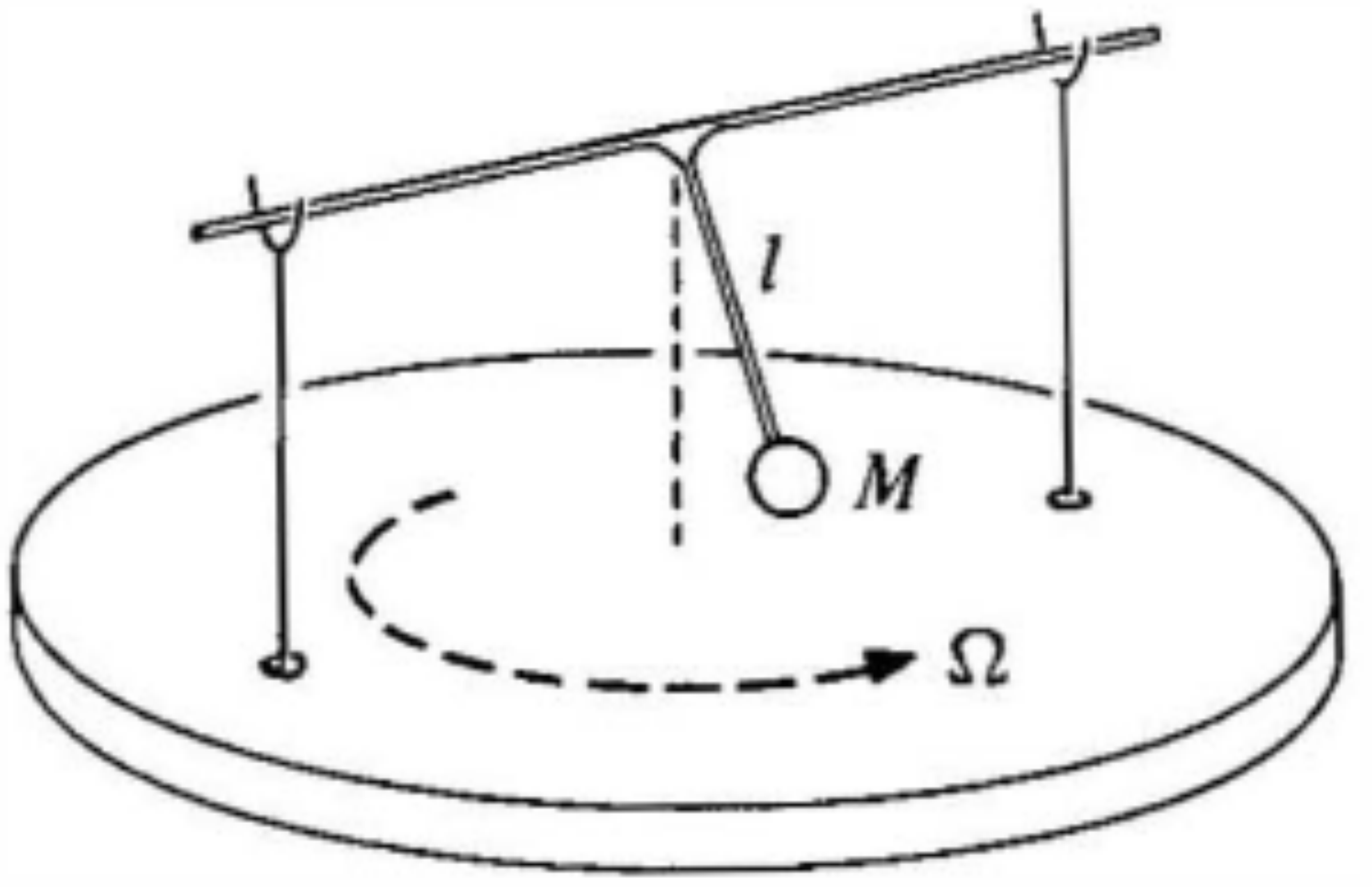

Hint: Can solve this problem in the non inertial rotating frame, the pendulum will only rotate in the (xy) plane.

NB: In the Galilean frame, you would need to use spherical coordinates. Very difficult because if we use phi to represent the rotation with angular velocity Ω, we can't use the small angle approximation for theta. Vice-versa, theta can't describe the rotation because it's only defined on the interval $[0,\pi)$.

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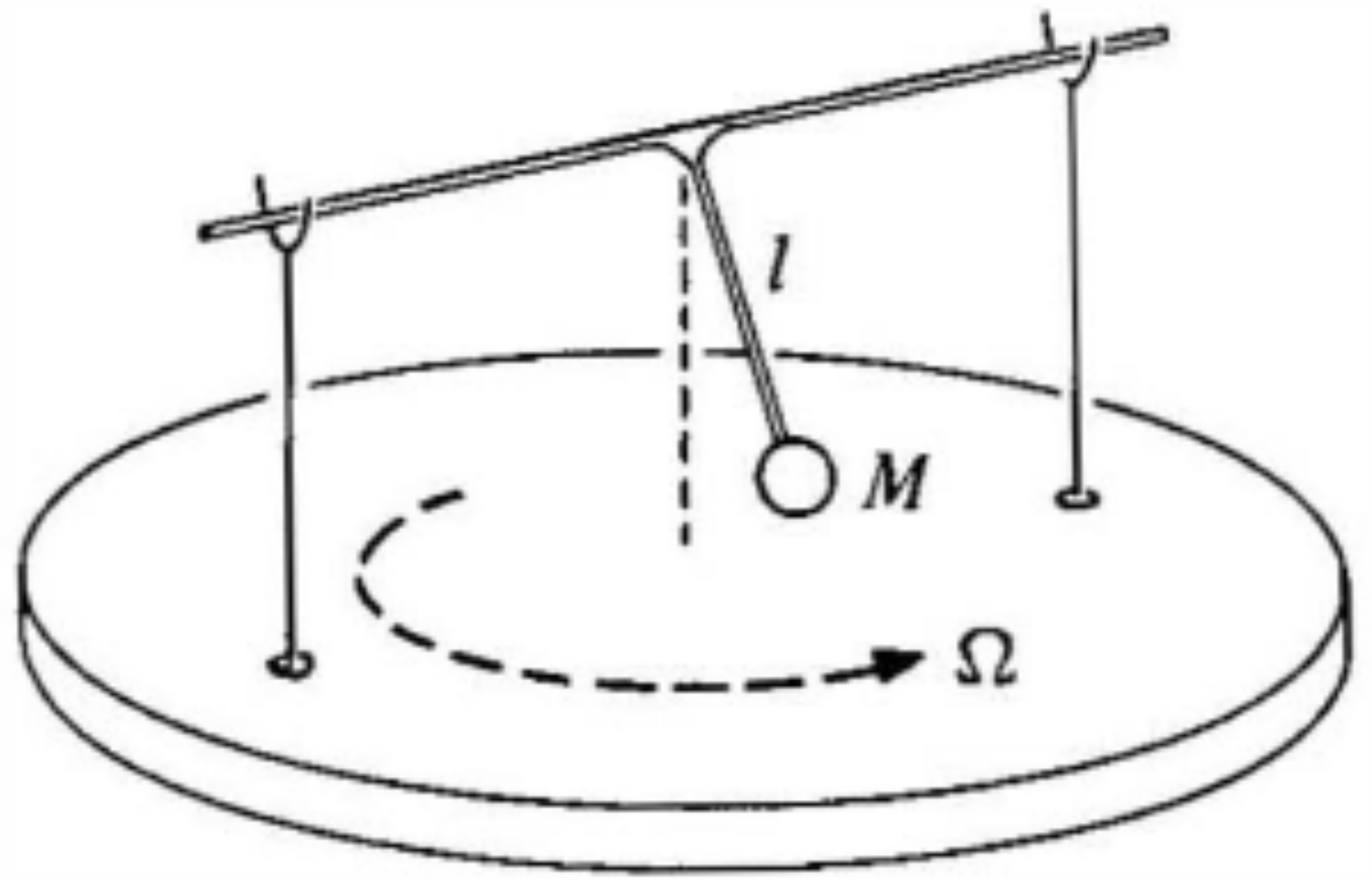

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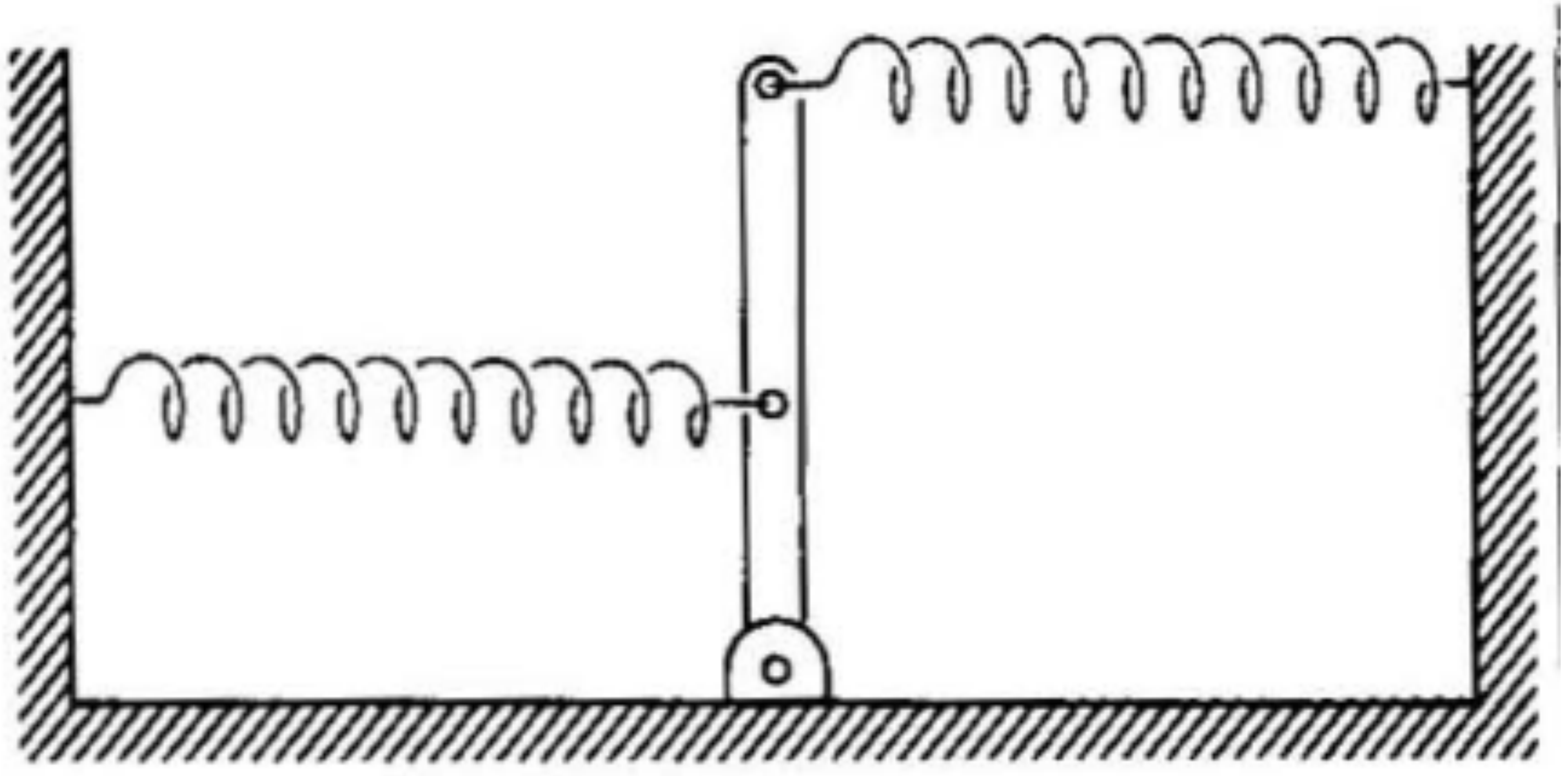
Solution: $$\Omega_0 = \sqrt{\frac{g}{l} - \Omega^2}$$

EXERCISE 4

A rod of length l and mass m, pivoted at one end, is held by a spring at its midpoint and a spring at its far end, both pulling in opposite directions. The springs have spring constant k, and at equilibrium their pull is perpendicular to the rod. Find the frequency of small oscillations about the equilibrium position.



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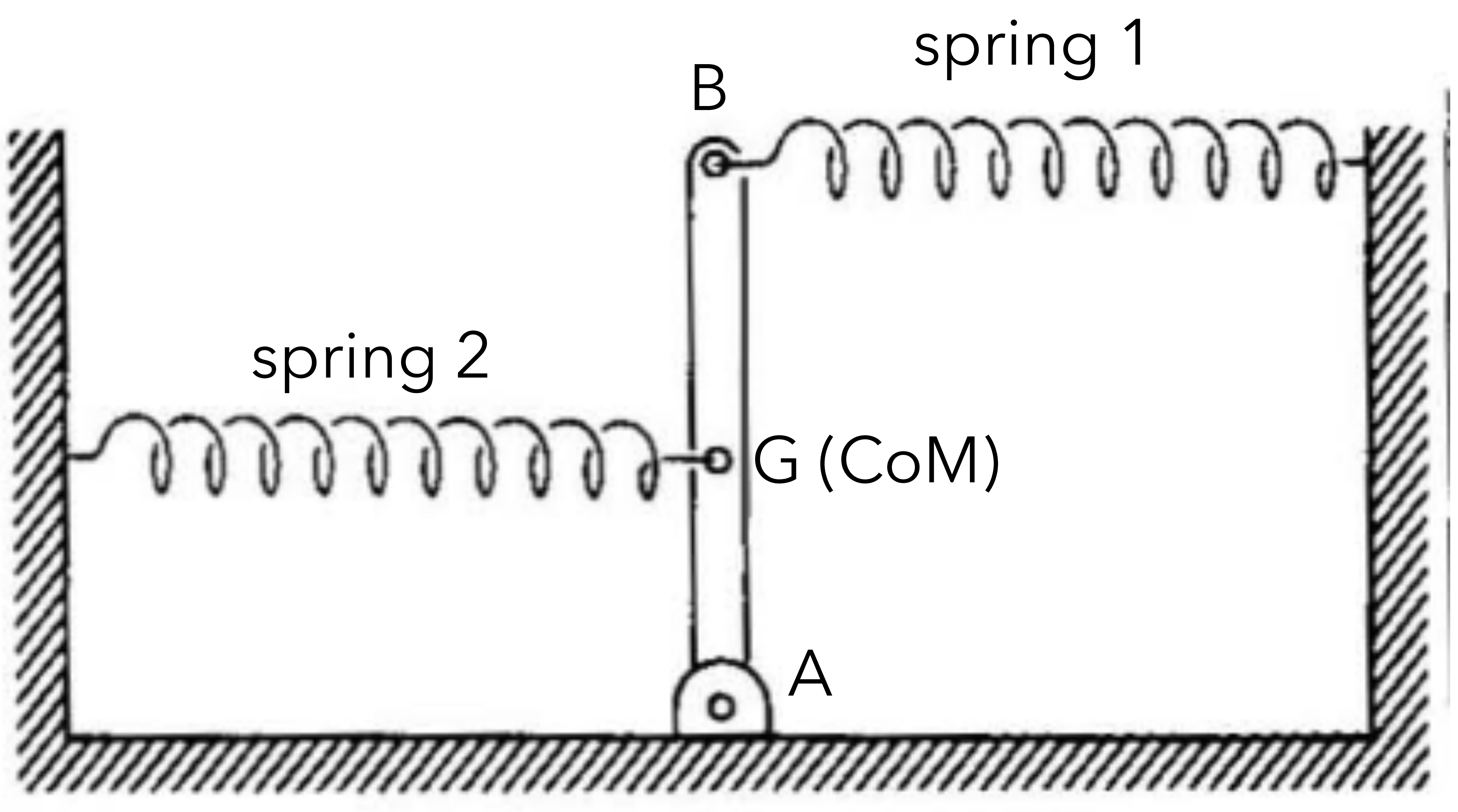

Hint: Remember that we have 3 torques relative to the pivot point A, applied by spring 1 at the end B, spring 2 at the CoM, and gravity acting at the CoM.

Gravity will aways act along the vertical axis.Under the small angle approximation, we can assume that the springs will always act along the horizontal axis.

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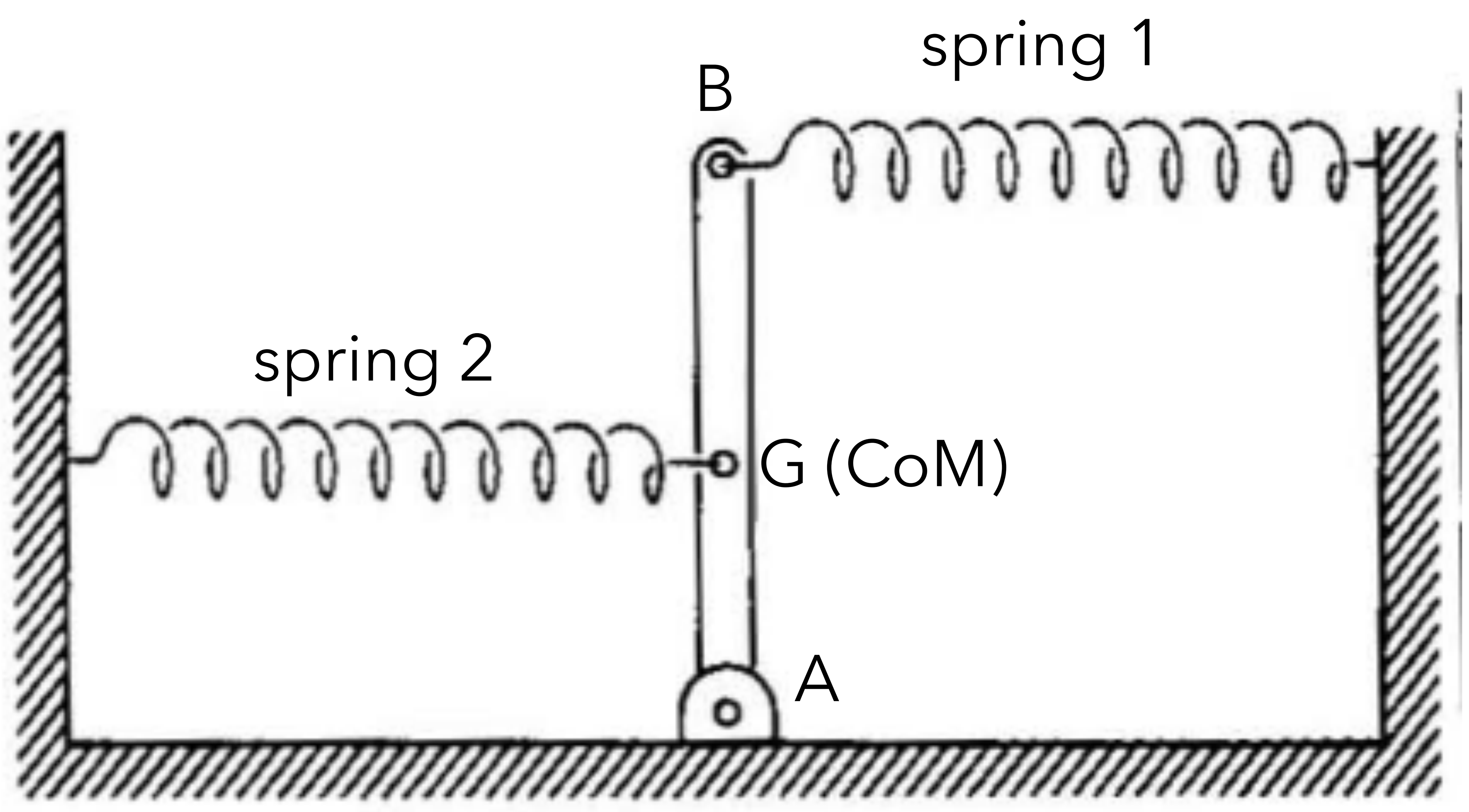

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Total torque: $\vec{M}_A = \phi \vec{e}_z \left(\frac{mgl}{2} - \frac{5}{4} kl^2 \right)$

Bar angular momentum: $\vec{L}_A = \frac{1}{3} ml^2 \dot{\phi} \vec{e}_z$

$\vec{M}_A = \dot{\vec{L}}_A \implies \phi \left(\frac{mgl}{2} - \frac{5}{4} kl^2 \right) = \frac{1}{3} ml^2 \ddot{\phi}$

$\ddot{\phi} + \phi \left(\frac{5}{4} kl^2 - \frac{mgl}{2} \right) \frac{3}{ml^2} \implies \boxed{\Omega_0 = \sqrt{\frac{15}{4} \frac{k}{m} - \frac{3}{2} \frac{g}{l}}}$