

# S-101 WEEK 15



SPACETIME  
EXPRESS

**EPFL**

Physique générale : mécanique  
(classe inversée en anglais)

Prof. Emma Tolley, 15 December 2025

# ANNOUNCEMENTS

- Updated formula sheet, check the Moodle!
- Additional exam-level exercise sets & old exams available on the Moodle

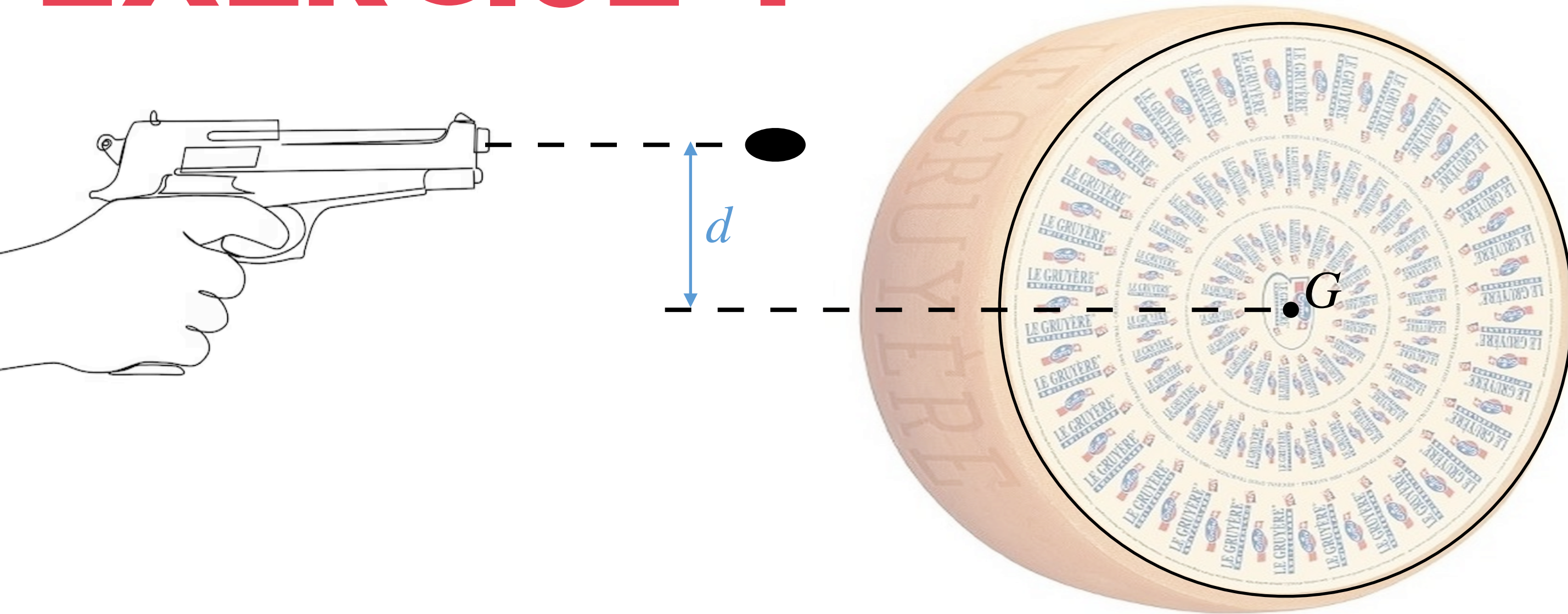
# EXAM REMINDERS

- **Final exam time: Friday January 16, 9:15-12:45**
  - We will open the room 10' before at 9:05
  - Exam is designed to be 3 hours long, but you have 3.5 hours total for the exam.
- **Grading:** 50 points total, 12 QCM/38 Exercises
  - 2 pts /QCM, no negative points
- **What you can bring:** 1-sided hand-written formula sheet, pencil/pen/eraser
  - No calculators, telephones, etc
- **What will be provided:**
  - General formula sheet
  - Problems & answer booklet
  - **Draft paper:** write your solutions on the draft paper and then copy to the answer sheet

# EXAM REMINDERS

- **General advice:**
  - **Review the entire exam** first before you choose which exercise to begin.
    - Start with topics that you are more comfortable with
  - Read the exercise from **start to finish** before beginning.
    - See which parts depend on previous sections, and which parts can be solved independently.
  - Keep in mind the **timing** & the **points per section**
    - 3 hour exam / 50 points = ~3.6 minutes per point
    - Don't get stuck on a section worth 1 point!

# EXERCISE 1



A homogenous cheese wheel with radius  $R$  and mass  $M$  has a pivot inserted through its center of mass  $G$  so that it can turn freely around its axis of symmetry.

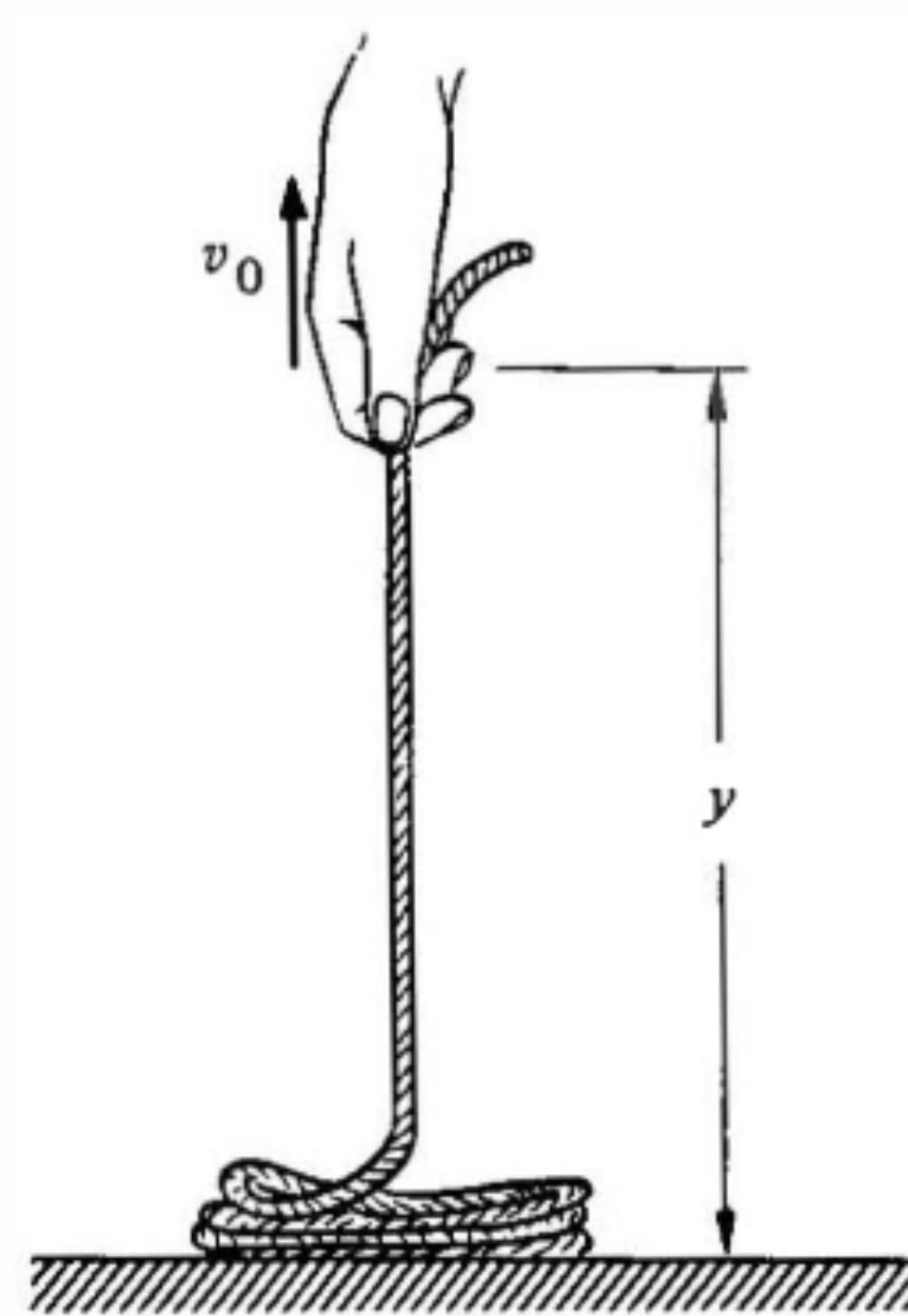
Someone fires a bullet with mass  $m$  and velocity  $v_0$ , which embeds itself in the rind of the cheese at an offset  $d$  from the CoM.

With what angular velocity does the cheese wheel turn after the bullet hits the rind?? Neglect gravity and treat the bullet as a point mass with  $m \ll M$ .

# EXERCISE 2

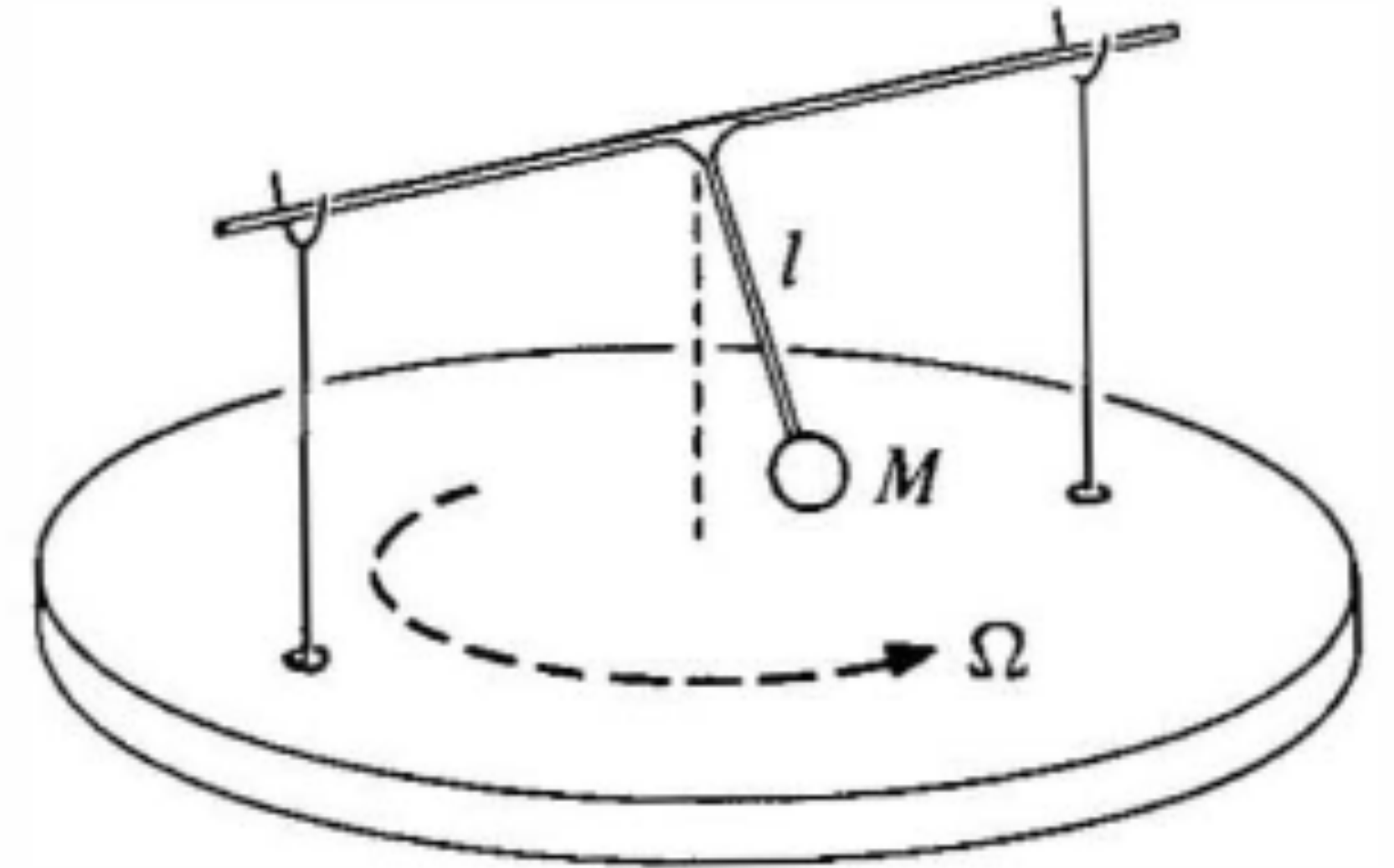
A uniform rope with linear mass density  $\lambda$  is coiled on a smooth horizontal table. One end is pulled straight up with constant speed  $V_0$ .

- A) Find the force exerted on the end of the rope as a function of height  $y$ .
- B) Compare the power delivered to the rope with the rate of change of the rope's total mechanical energy



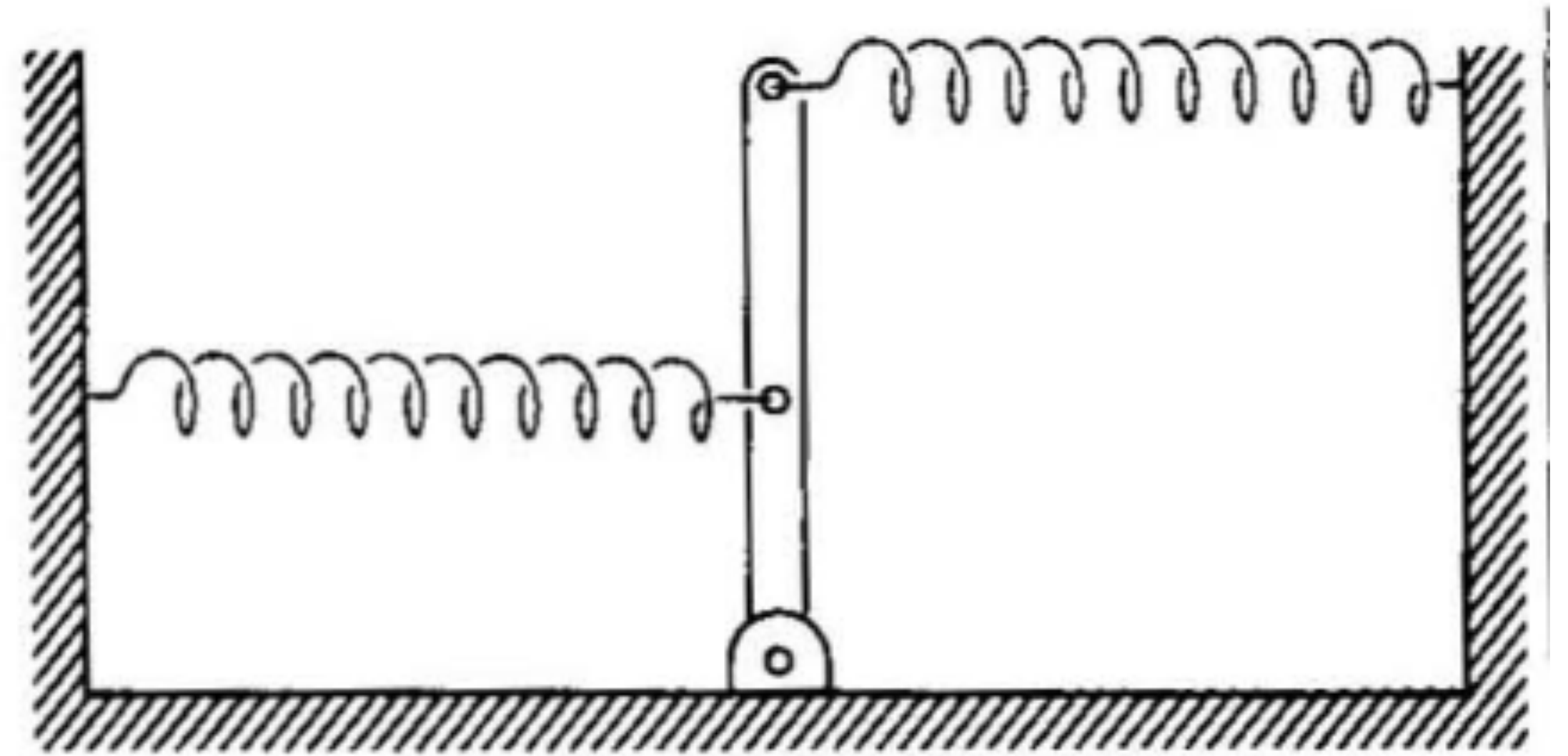
# EXERCISE 3

A pendulum is rigidly fixed to an axle held by two supports so that it can swing only in a plane perpendicular to the axle. The pendulum consists of a mass  $M$  attached to a massless rod of length  $l$ . The supports are mounted on a platform which rotates with constant angular velocity  $\Omega$ . Find the pendulum's angular frequency assuming that the amplitude is small.

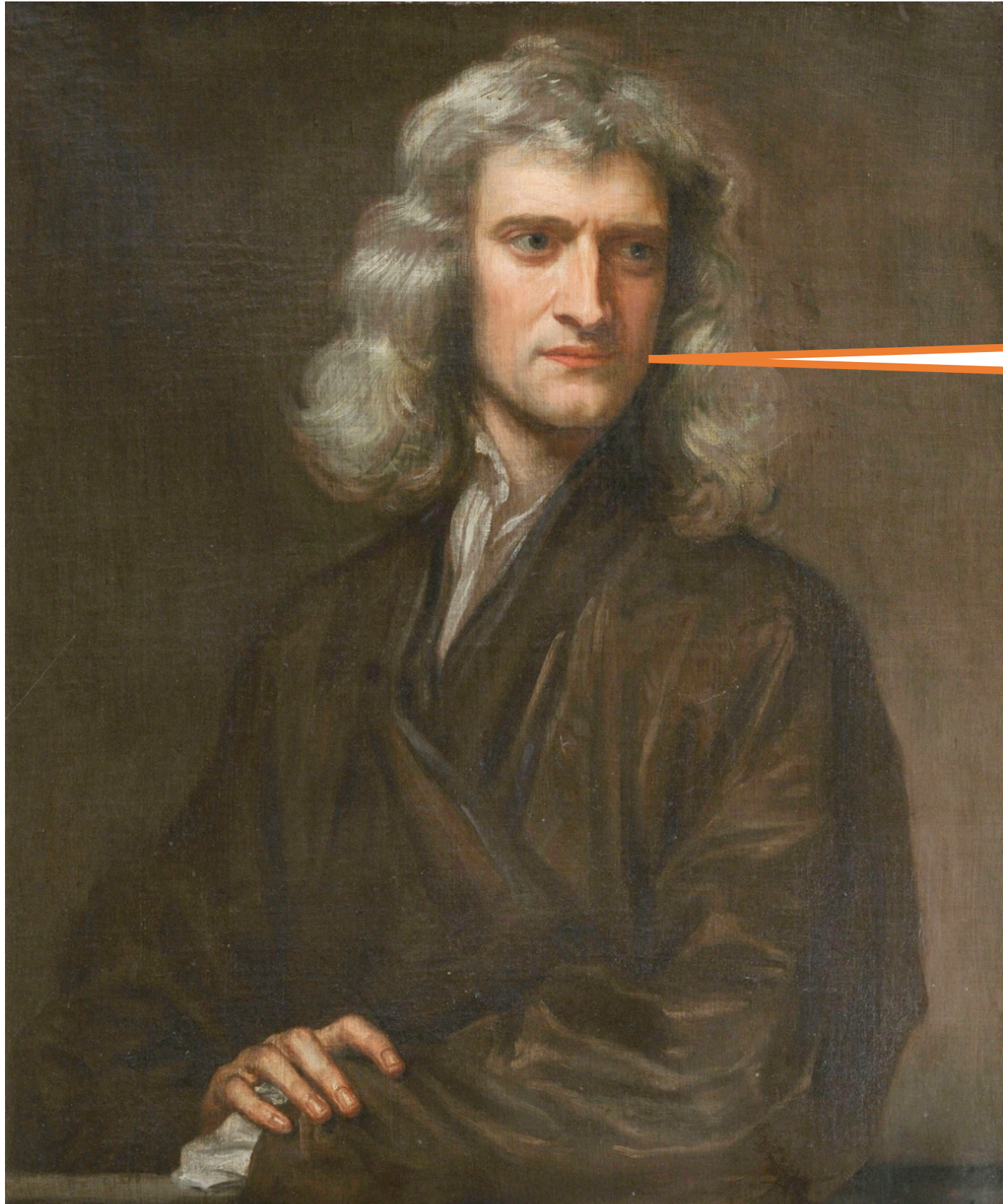


# EXERCISE 4

A rod of length  $l$  and mass  $m$ , pivoted at one end, is held by a spring at its midpoint and a spring at its far end, both pulling in opposite directions. The springs have spring constant  $k$ , and at equilibrium their pull is perpendicular to the rod. Find the frequency of small oscillations about the equilibrium position.



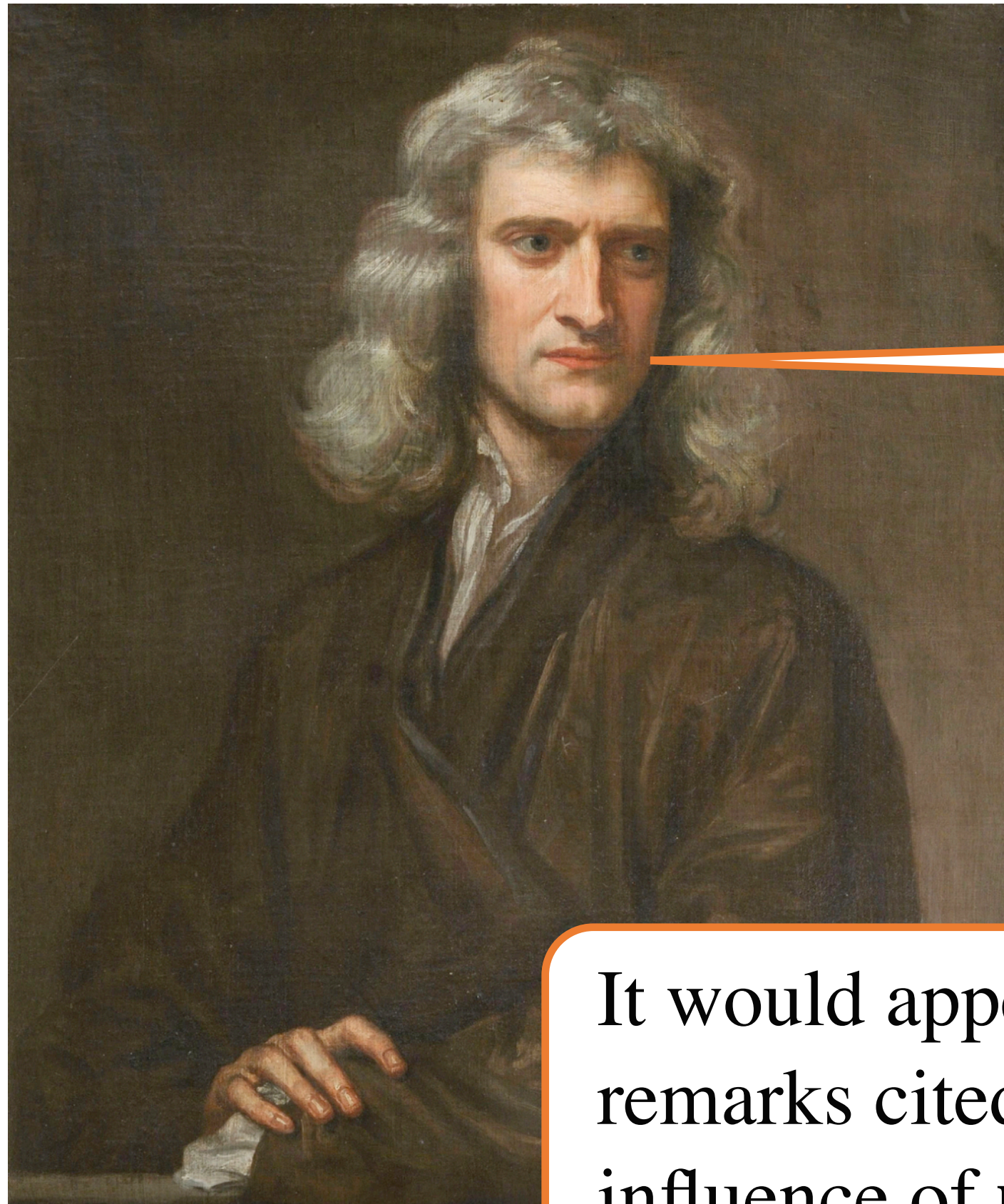
# ON THE NATURE OF TIME



Absolute, **true and mathematical time**, of itself and by its own true nature, flows uniformly on, without regard to anything external. Relative, apparent and common time is some sensible and external measure of absolute time estimated by the motions of bodies, whether accurate or inequable, and is commonly employed in place of true time; as an hour, a day, a month, a year.

*Newton's Principia, 1687*

# ON THE NATURE OF TIME

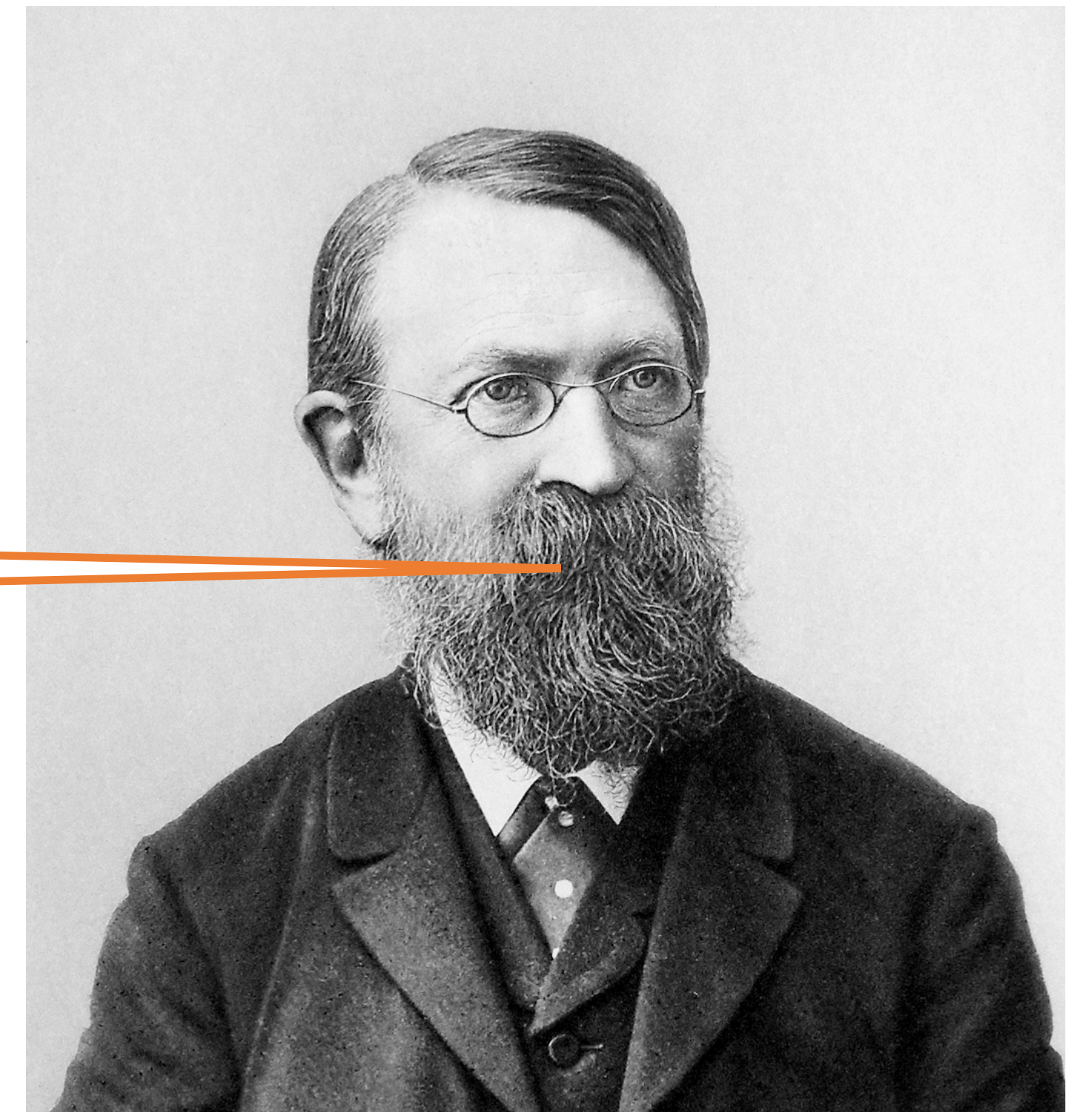


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It would appear as though Newton in the remarks cited here still stood under the influence of **medieval philosophy**, as though he had grown unfaithful to his resolve to investigate only actual facts.

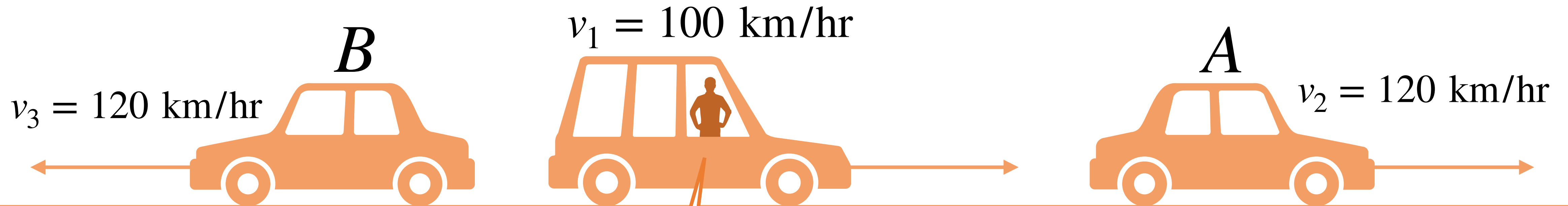
*Mach's The Science of Mechanics, 1883*



# ON THE NATURE OF TIME

The Michelson-Morley Experiment (1887) aimed to determine the effect of the Earth's motion on the velocity of light. Michelson and Morley found no significant difference between the speed of light measured in different directions, in different parts of the Earth's orbit.

# ON THE NATURE OF TIME

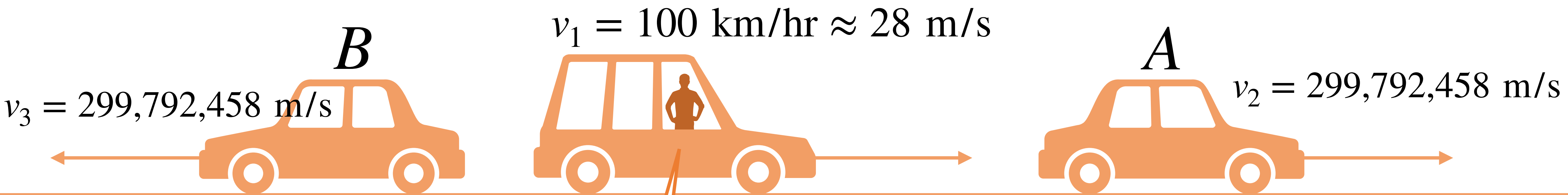


I see car A moving at a speed of  
 $|\vec{v}_2 - \vec{v}_1| = 20$  km/hr  
and car B moving at a speed of  
 $|\vec{v}_3 - \vec{v}_1| = 220$  km/hr

I see car A moving at speed  
 $|\vec{v}_2| = 120$  km/hr  
and car B moving at speed  
 $|\vec{v}_3| = 120$  km/hr



# ON THE NATURE OF TIME



I see car A moving at a speed of 299,792,458 m/s and car B moving at a speed of 299,792,458 m/s

I **also** see car A moving at speed 299,792,458 m/s and car B moving at speed 299,792,458 m/s



# ON THE NATURE OF TIME

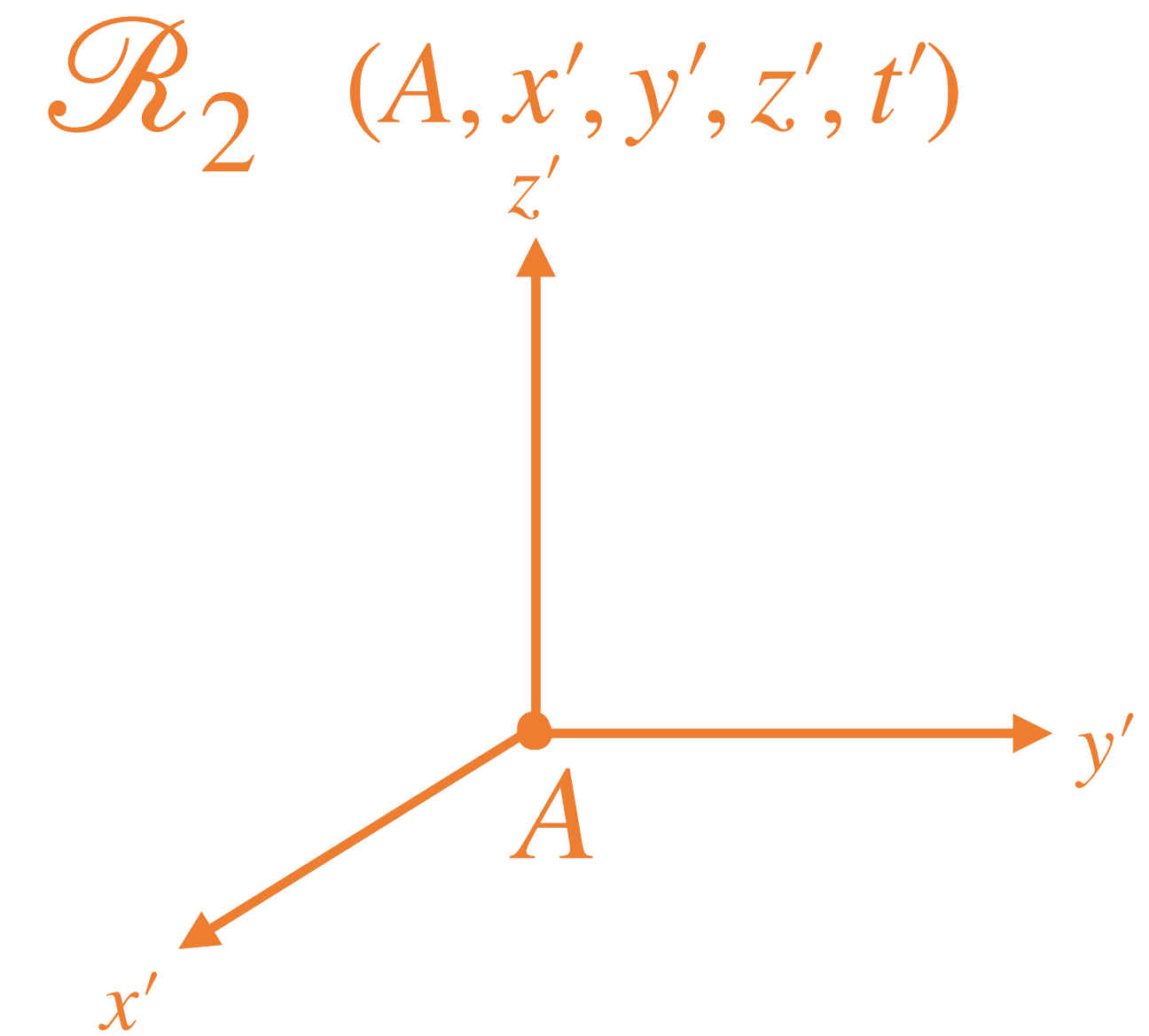
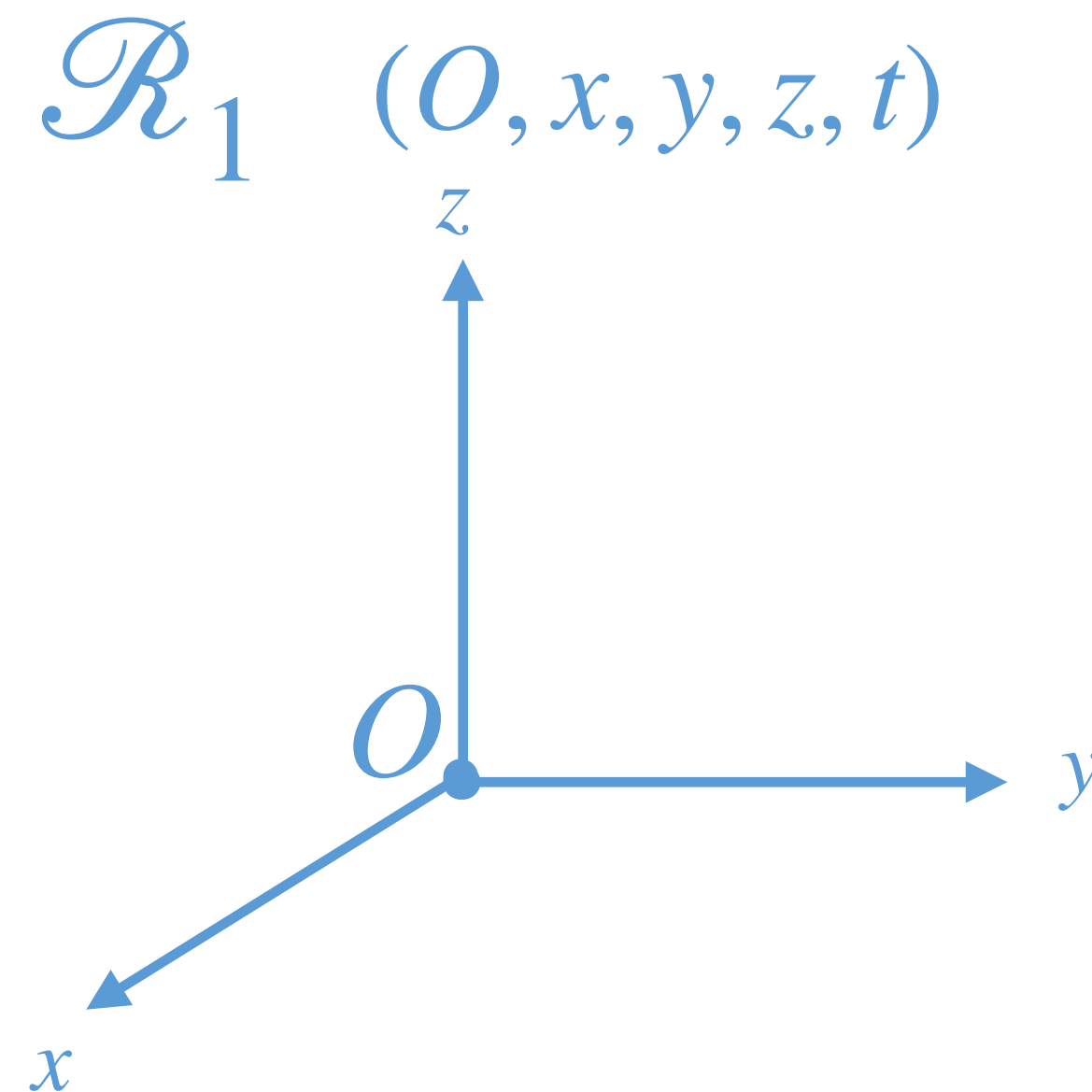
We [introduce a postulate] that light is always propagated in empty space with a definite velocity  $c$  which is independent of the state of motion of the emitting body.

Einstein, *On the Electrodynamics of Moving Bodies*, 1905



# ON THE NATURE OF TIME

Both are Galilean frames.  
R2 moves with velocity  
 $\vec{v} = v\vec{e}_y$  along the y-axis  
compared to R1.

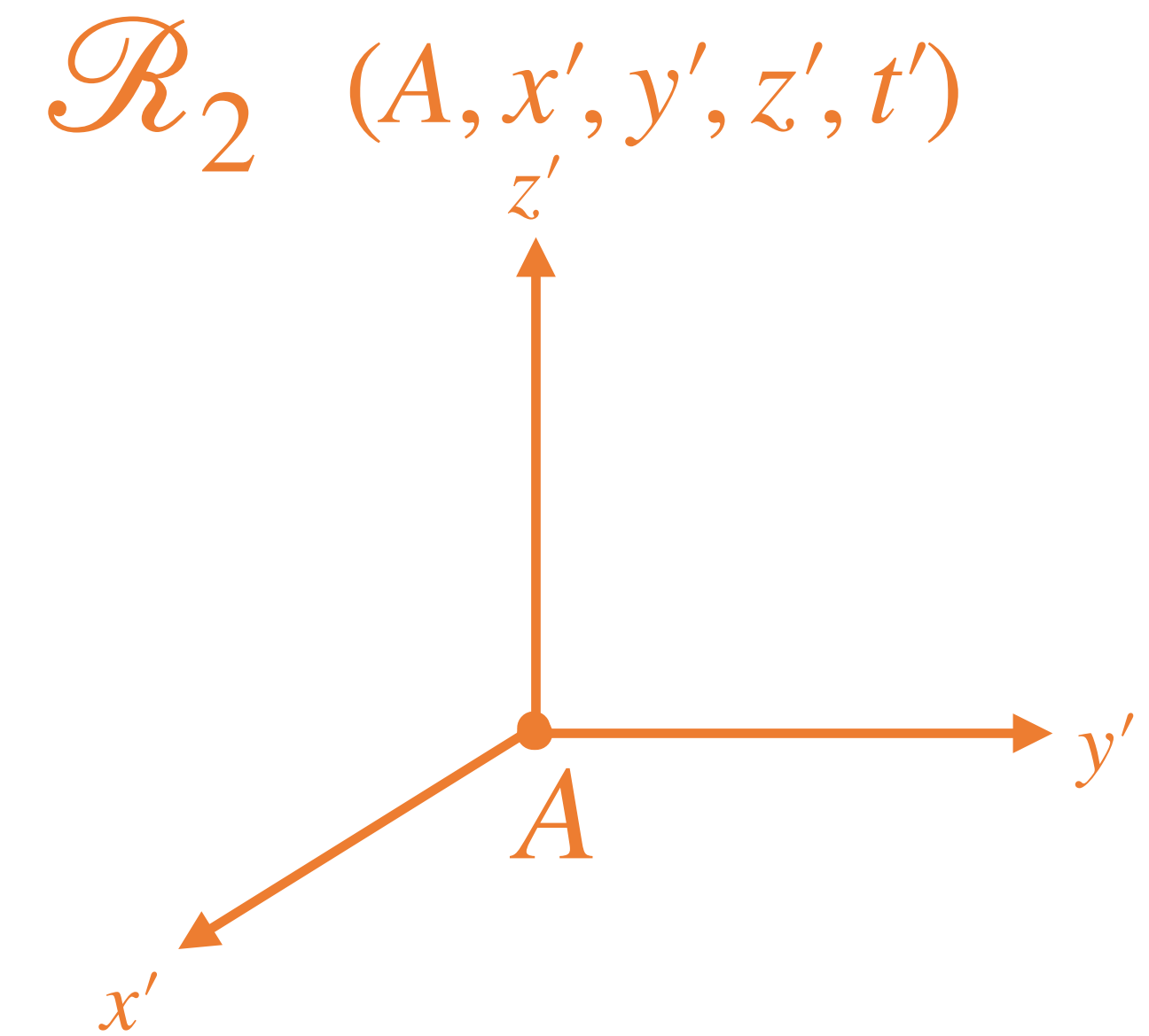
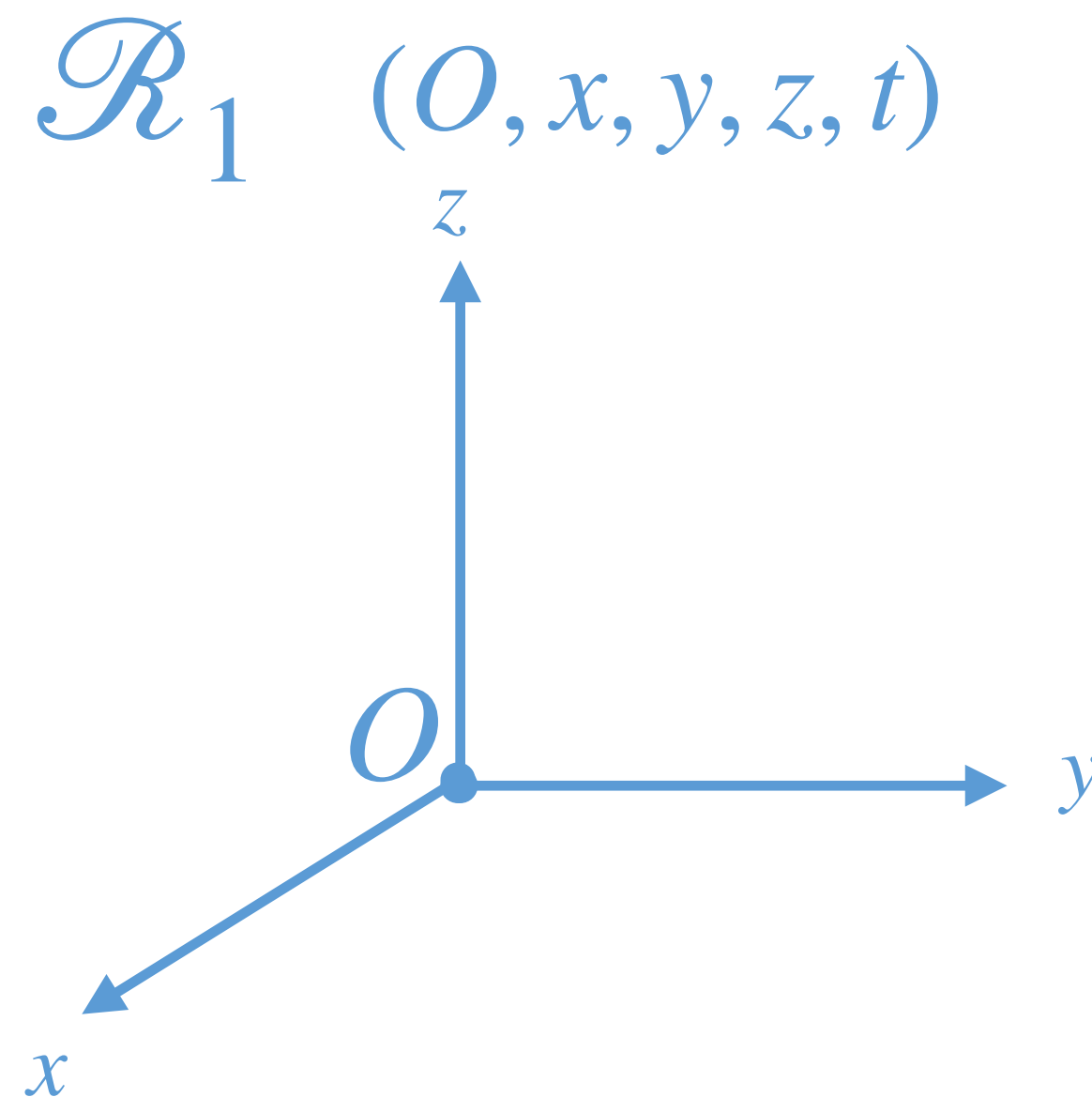


# ON THE NATURE OF TIME

**Galilean transform:**  
assumption of  
Newtonian mechanics

$$\begin{cases} x = x' & x' = x \\ y = y' + vt & y' = y - vt \\ z = z' & z' = z \\ t = t' & t' = t \end{cases}$$

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R2 moves with velocity  
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# ON THE NATURE OF TIME

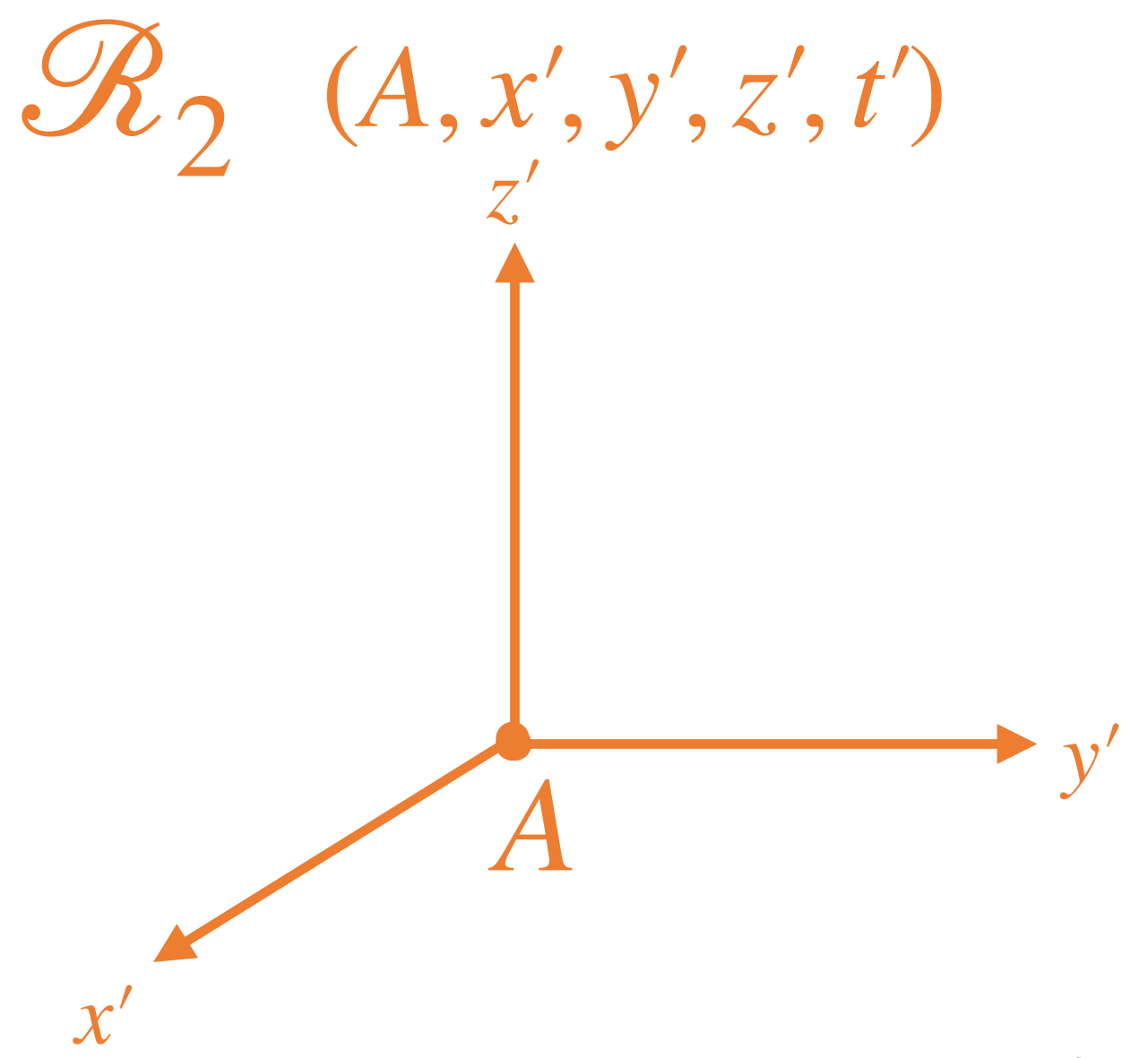
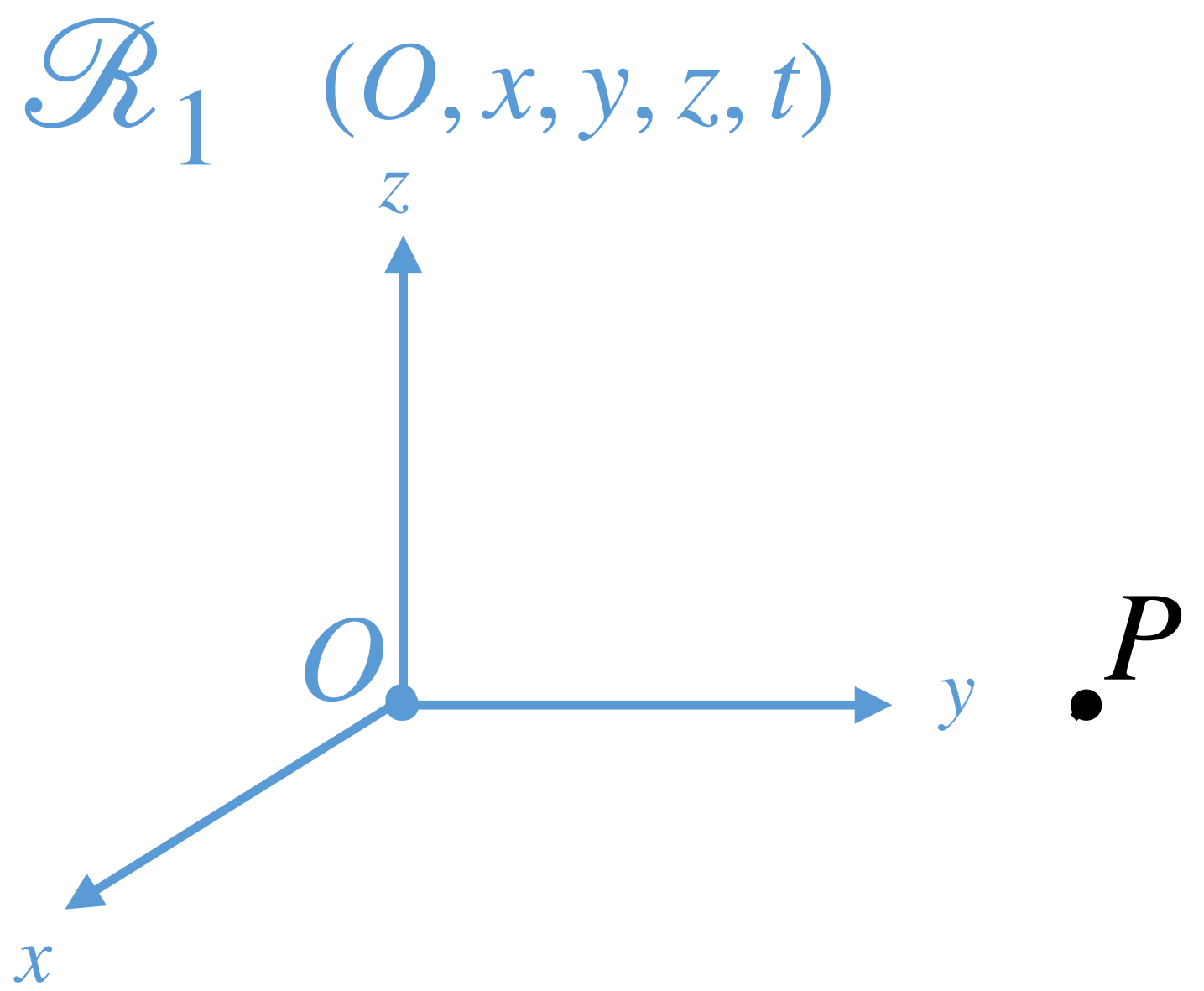
**Galilean transform:**  
 assumption of Newtonian mechanics

$$\begin{cases} x = x' & x' = x \\ y = y' + vt & y' = y - vt \\ z = z' & z' = z \\ t = t' & t' = t \end{cases}$$

P is moving with velocity  $\vec{u} = u\vec{e}_y$  in frame R1:

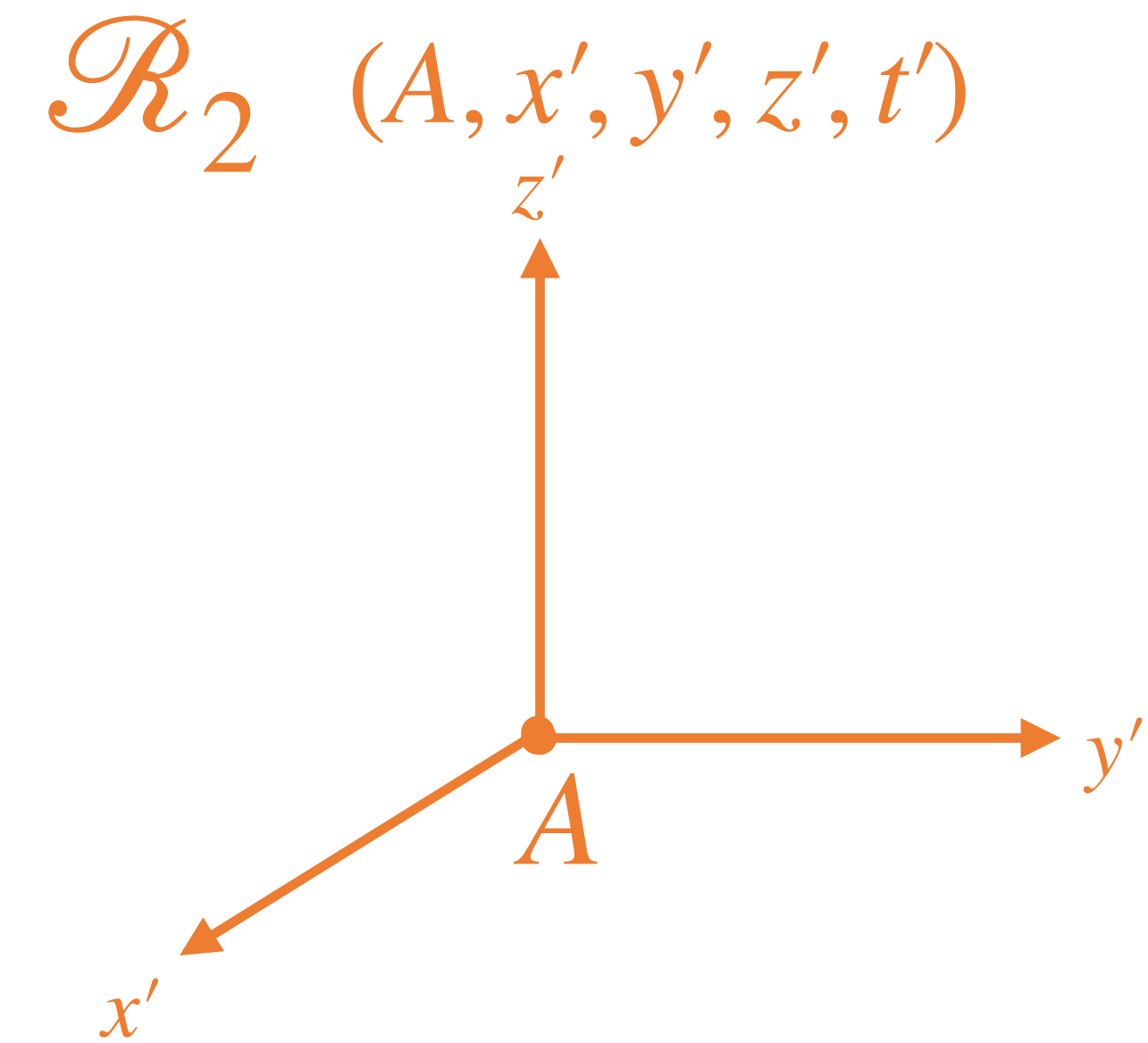
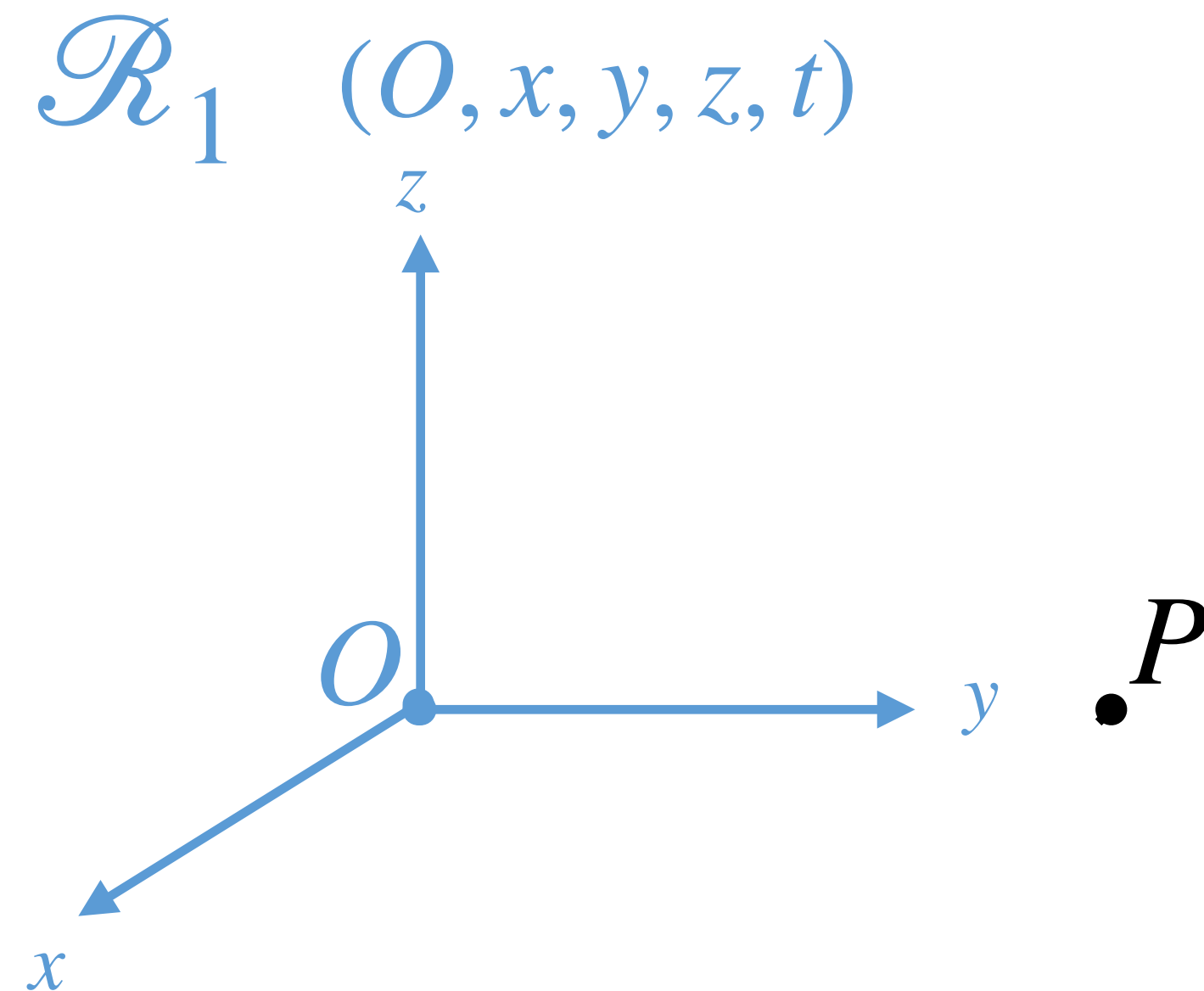
$$y = ut$$

In R2:  $y' = ut - vt$   
 $= (u - v)t$   
 $= (u - v)t'$



# ON THE NATURE OF TIME

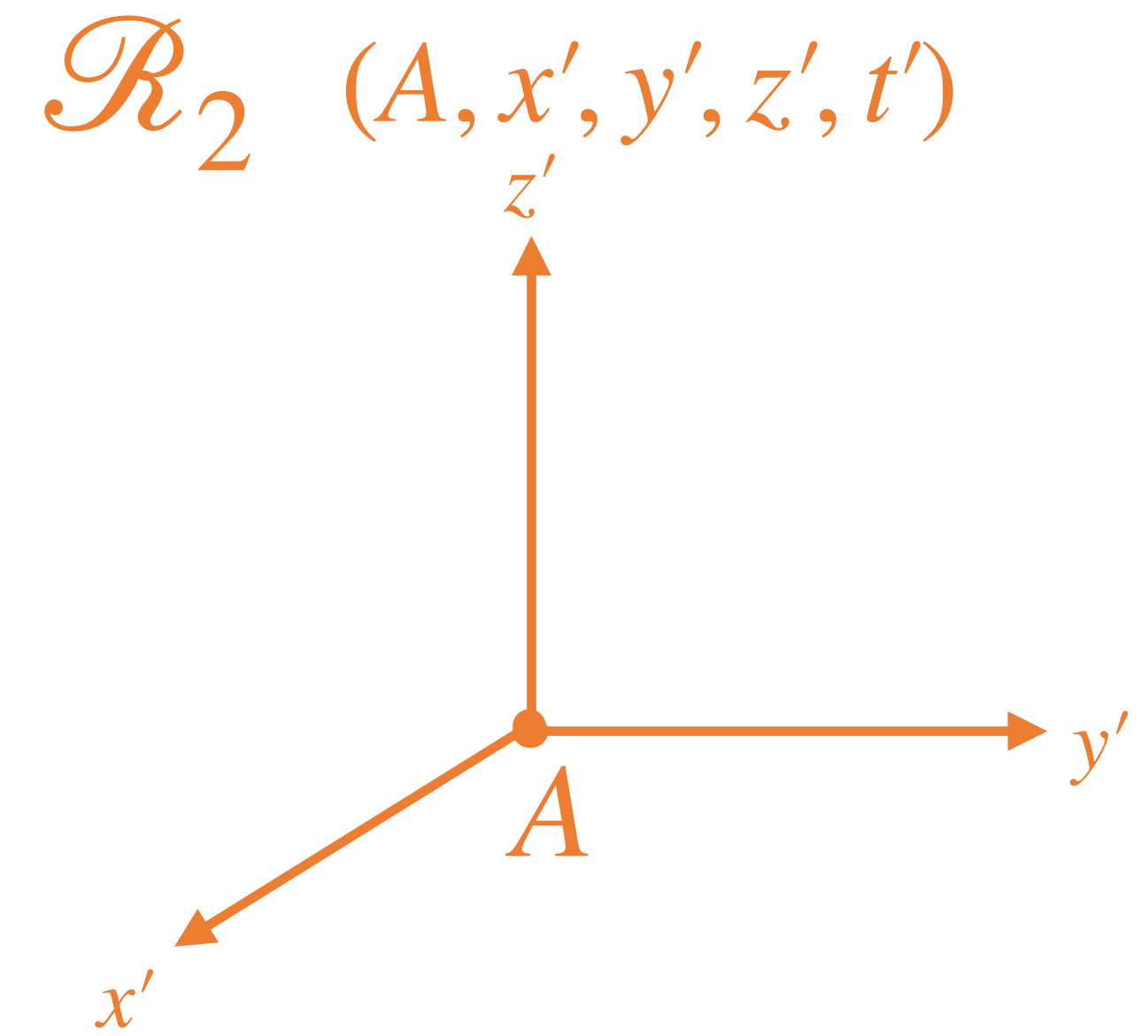
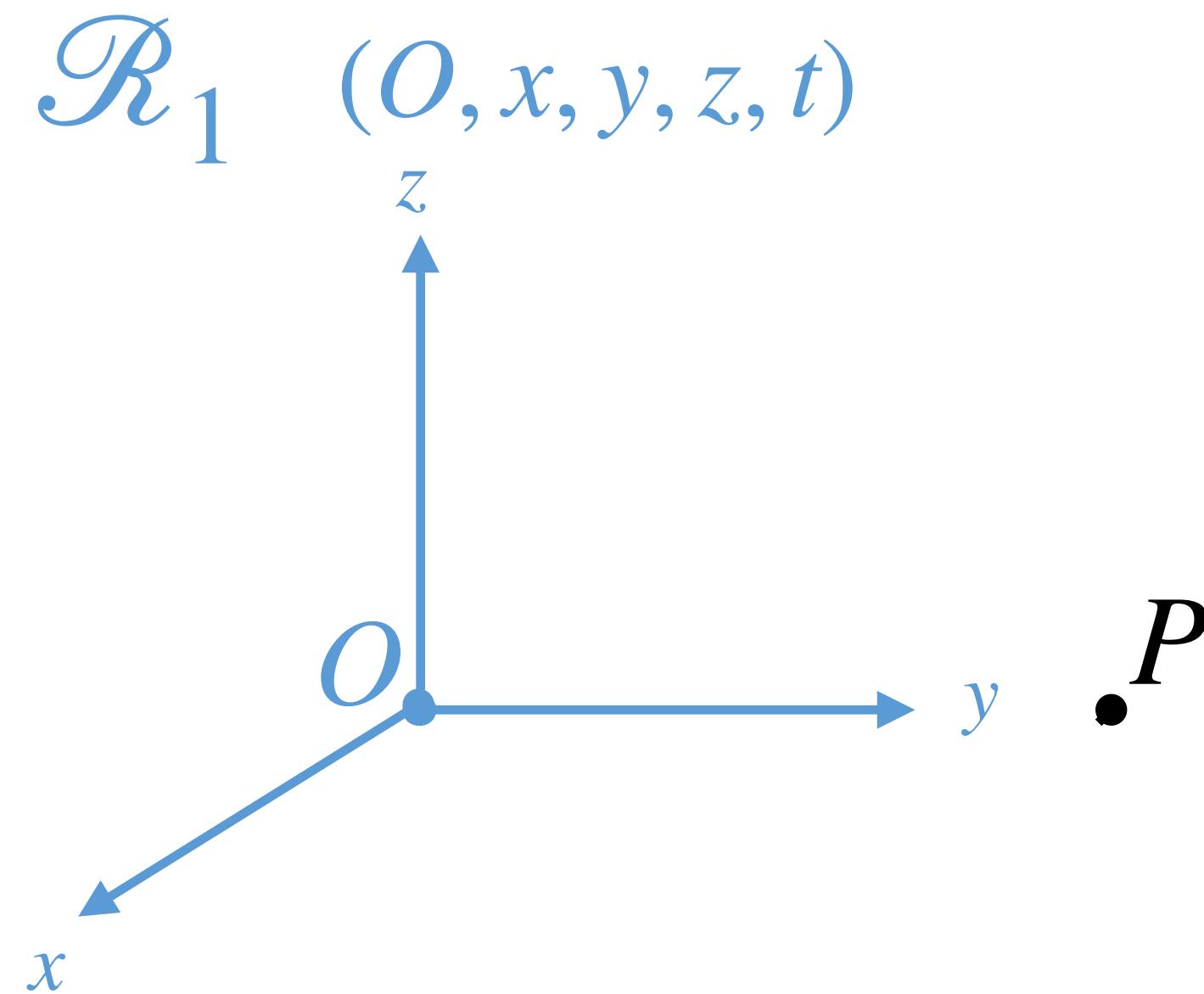
What if  $P$  is moving at the speed of light  $c$  along the  $y$ -axis? How can  $P$  be measured with the same speed in  $R_1$  and  $R_2$ ? **How does kinematics change?**



# ON THE NATURE OF TIME

$$\text{Lorentz transform} \begin{cases} x = x' \\ y = \gamma y' + \gamma v t' \\ z = z' \\ t = \gamma t' + \frac{\gamma v}{c^2} y' \end{cases} \quad \begin{cases} x' = x \\ y' = \gamma y - \gamma v t \\ z' = z \\ t' = \gamma t - \frac{\gamma v}{c^2} y \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



# ON THE NATURE OF TIME

$$\text{Lorentz transform} \left\{ \begin{array}{l} x = x' \\ y = \gamma y' + \gamma v t' \\ z = z' \\ t = \gamma t' + \frac{\gamma v}{c^2} y' \end{array} \right.$$

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$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

P is moving with velocity  
 $\vec{u} = c\vec{e}_y$  in frame R1:

$$y = ct$$

$$\begin{aligned} \text{In R2: } y' &= \gamma y - \gamma v t \\ &= \gamma ct - \gamma v t \end{aligned}$$

$$\begin{aligned} t' &= \gamma t - \frac{\gamma v}{c^2} y \\ &= \gamma t - \frac{\gamma v}{c} t \end{aligned}$$

$$y' = ct'$$

# ON THE NATURE OF TIME

**Lorentz transform**

$$\begin{cases} x = x' \\ y = \gamma y' + \gamma vt' \\ z = z' \\ t = \gamma t' + \frac{\gamma v}{c^2} y' \end{cases} \quad \begin{cases} x' = x \\ y' = \gamma y - \gamma vt \\ z' = z \\ t' = \gamma t - \frac{\gamma v}{c^2} y \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



I am standing still  $\frac{dy}{dt} = 0$ , but moving through time at  $\frac{dt}{dt} = 1$  s/s.



# ON THE NATURE OF TIME

**Lorentz transform**

$$\begin{cases} x = x' & x' = x \\ y = \gamma y' + \gamma vt' & y' = \gamma y - \gamma vt \\ z = z' & z' = z \\ t = \gamma t' + \frac{\gamma v}{c^2} y' & t' = \gamma t - \frac{\gamma v}{c^2} y \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

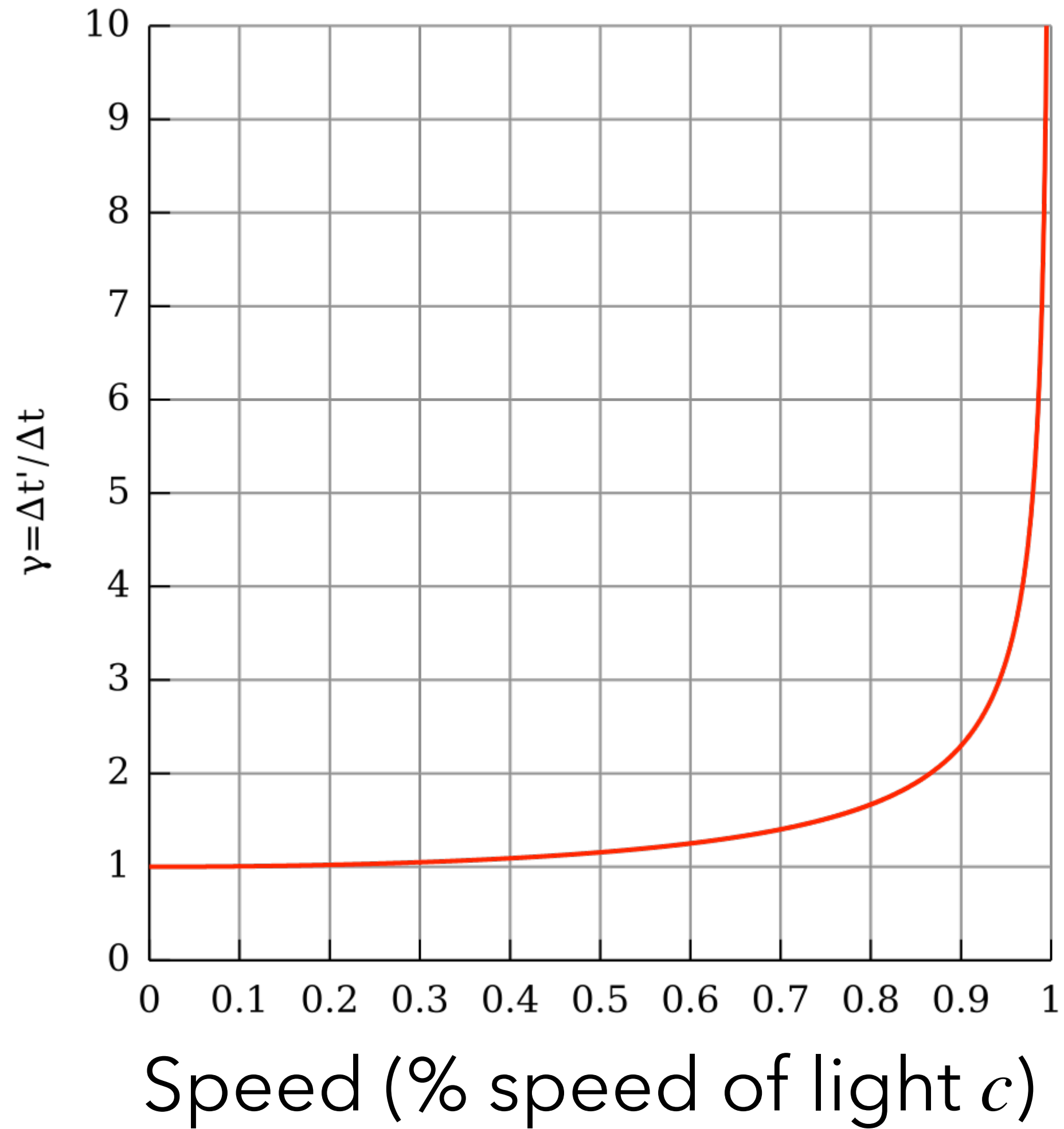


I am standing still  $\frac{dy}{dt} = 0$ , but moving through time at  $\frac{dt}{dt} = 1$  s/s.

I see a car moving at a speed of  $\frac{dy}{dt} = 0.5c$ . From my perspective, they are moving through time at  $\frac{dt'}{dt} = \gamma \frac{dt}{dt} - \frac{\gamma 0.5c}{c^2} \frac{dy}{dt} = \gamma - 0.5^2 \gamma = 0.86$  s/s.



# ON THE NATURE OF TIME



$$c = 299,792,458 \text{ m/s} = 1,079,252,849 \text{ km/hr}$$

# ON THE NATURE OF TIME

Things moving quickly will have slower clocks (*special relativity*)

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Things moving quickly will have slower clocks (*special relativity*)

Things at higher elevation will have quicker clocks (*general relativity*)

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<http://www.leapsecond.com/great2005/>

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# ON THE NATURE OF TIME

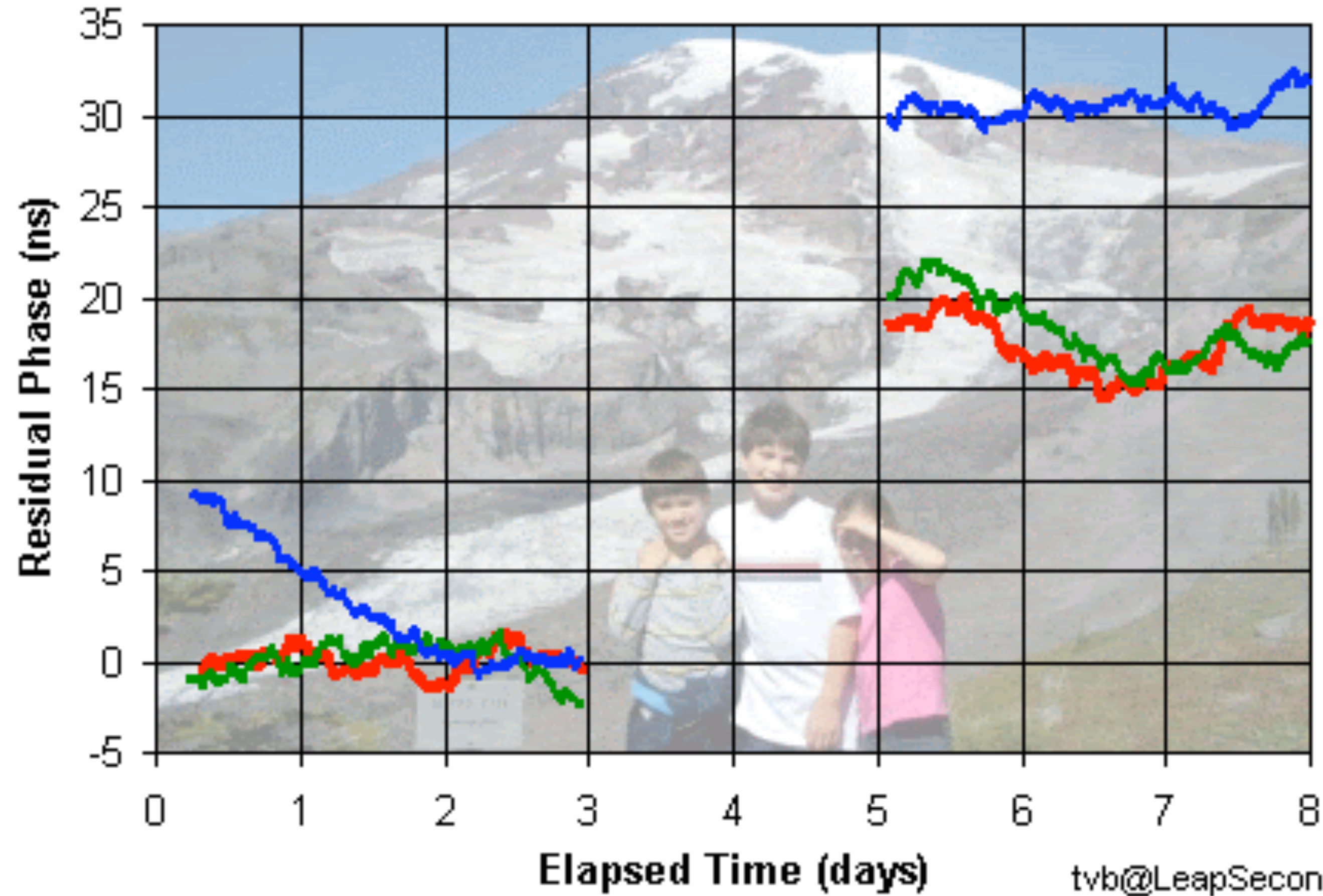
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**Project GREAT - Relativity on Mt Rainier**  
Three Cesium Clocks: Red Green Blue



www.LeapSecond.com/great2005

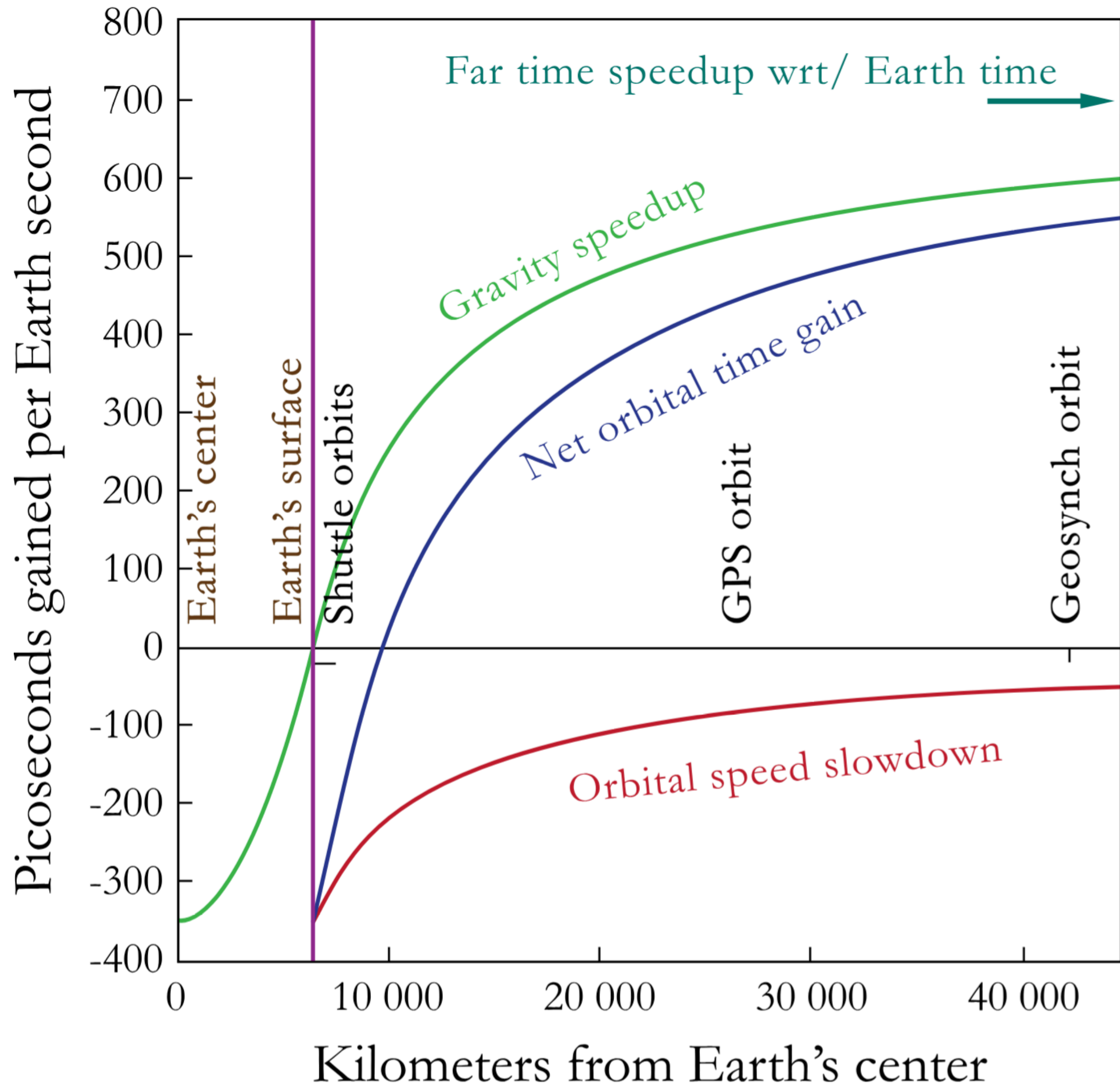
tvb@LeapSecond.com

# ON THE NATURE OF TIME

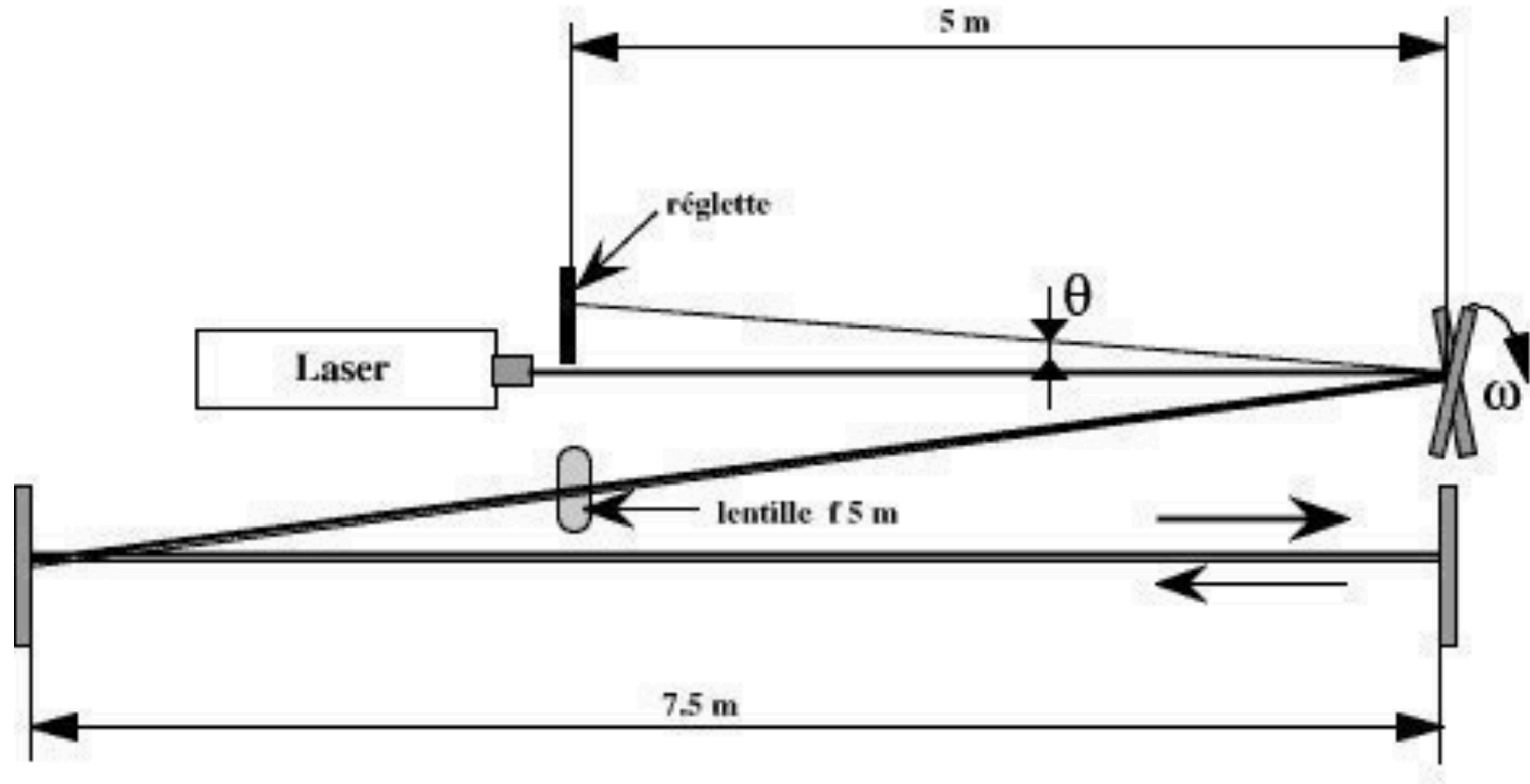
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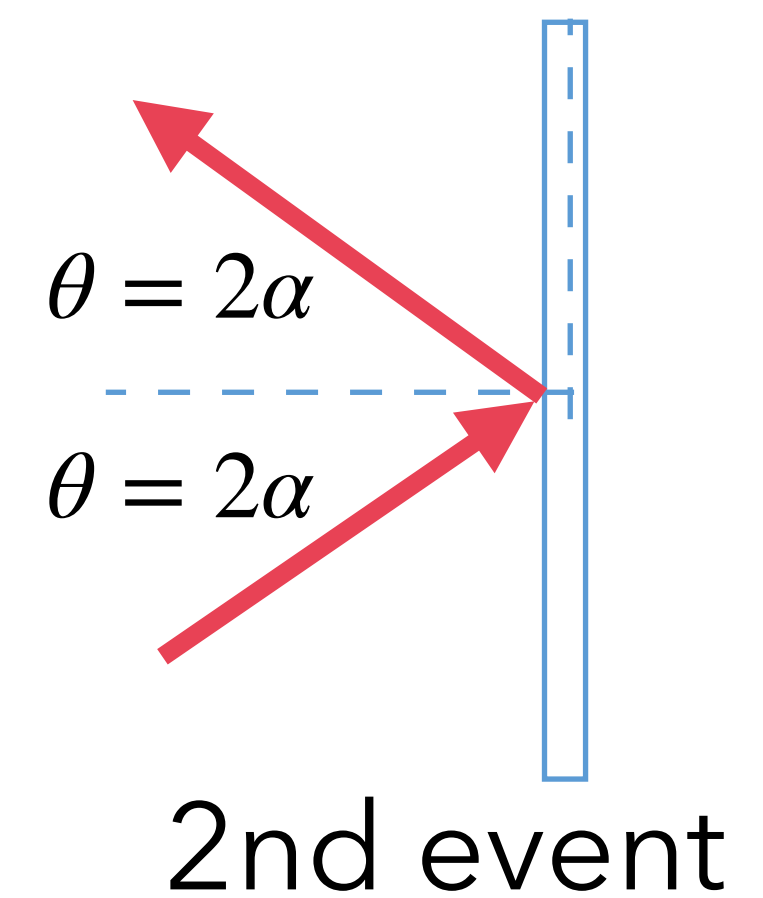
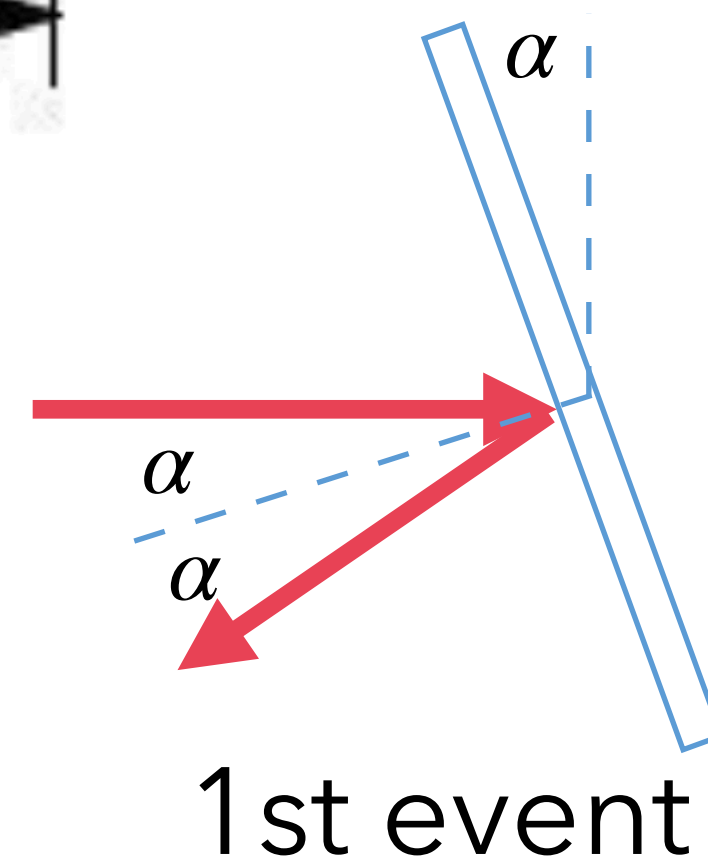
### Time Dilation Effects on Earth



# MEASURING THE SPEED OF LIGHT



The mirror turns by an angle  $\alpha = \omega \Delta t$  during the time interval  $\Delta t$ , changing the reflection angle of the returning laser



# MEASURING THE SPEED OF LIGHT

Mirror rotation frequency  $f$

Angular speed  $\omega = 2\pi f$

Laser deviation:  $\Delta y$

Length of deviation:  $R$  }  $\Delta y = R \sin \theta \implies \theta \approx \frac{\Delta y}{R}$

Flight time:  $\Delta t$ :

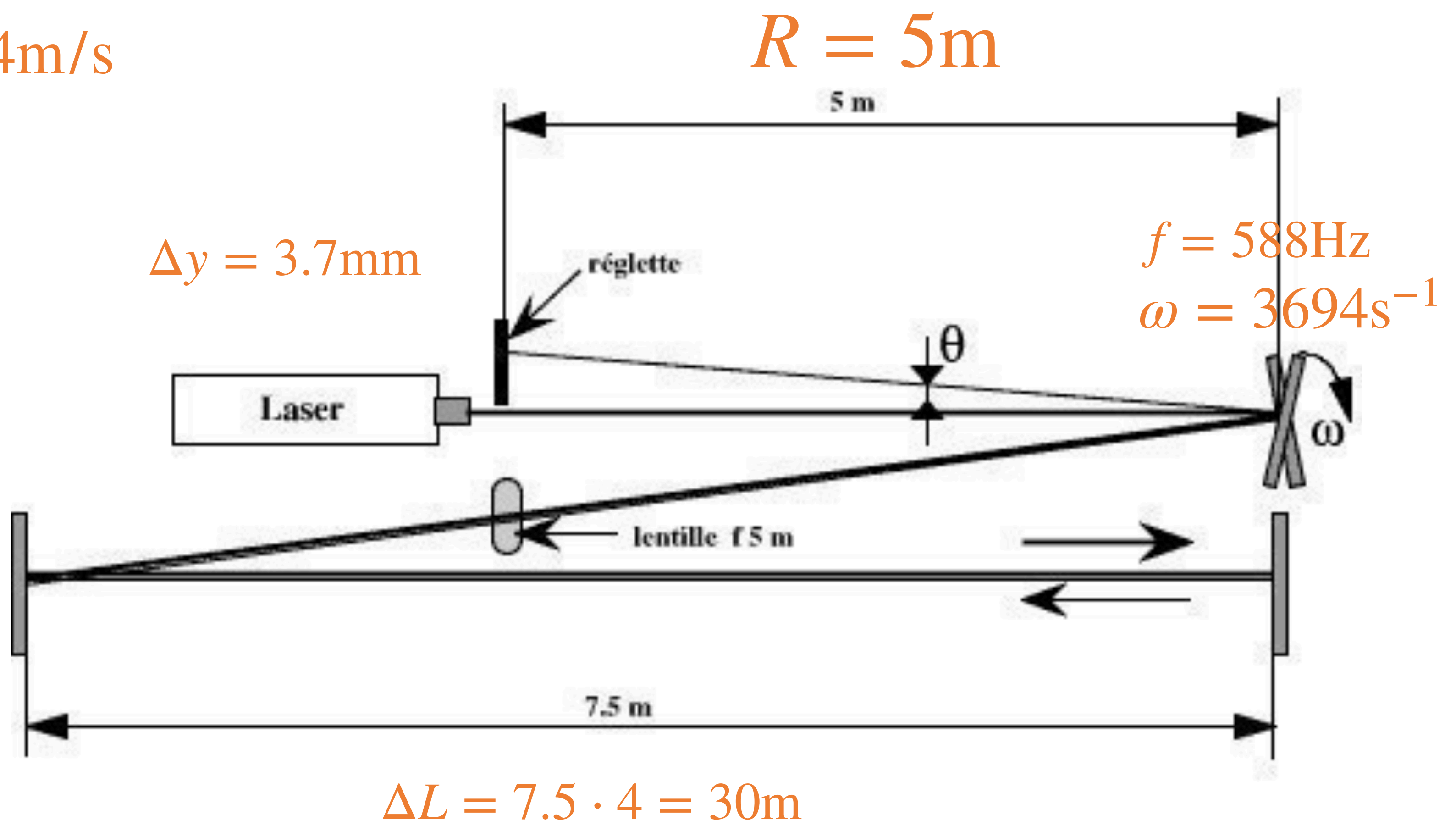
$$\omega = \frac{\alpha}{\Delta t} = \frac{\theta}{2\Delta t} \implies \Delta t = \frac{\Delta y}{2R\omega}$$

Length of trajectory:  $\Delta L$

$$\text{Speed of light: } c = \frac{\Delta L}{\Delta t} = \frac{2R\omega\Delta L}{\Delta y}$$

# MEASURING THE SPEED OF LIGHT

$$c = \frac{2R\omega\Delta L}{\Delta y} = 299,554,054\text{m/s}$$



True speed of light:  $c = 299,792,458\text{m/s}$

# A SLOWER SPEED OF LIGHT

PLAY  
CREDITS  
OPTIONS  
QUIT



[https://gamelab.mit.edu/  
games/a-slower-speed-of-light/](https://gamelab.mit.edu/games/a-slower-speed-of-light/)

