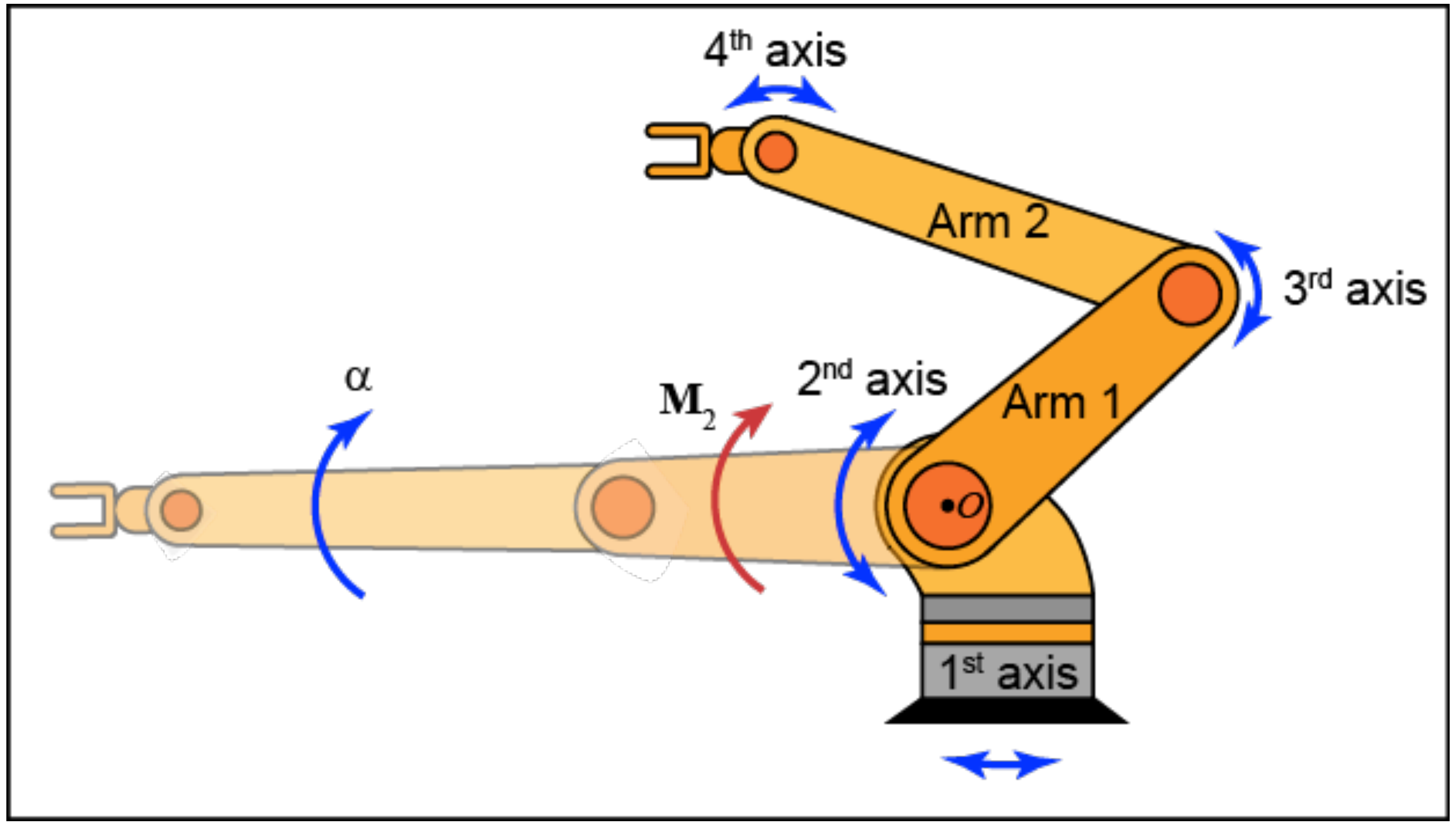
PHYS-101 WEEK 13



EPFL

Physique générale : mécanique (classe inversée en anglais)

Prof. Emma Tolley, 1 December 2025

FINAL CHAPTER

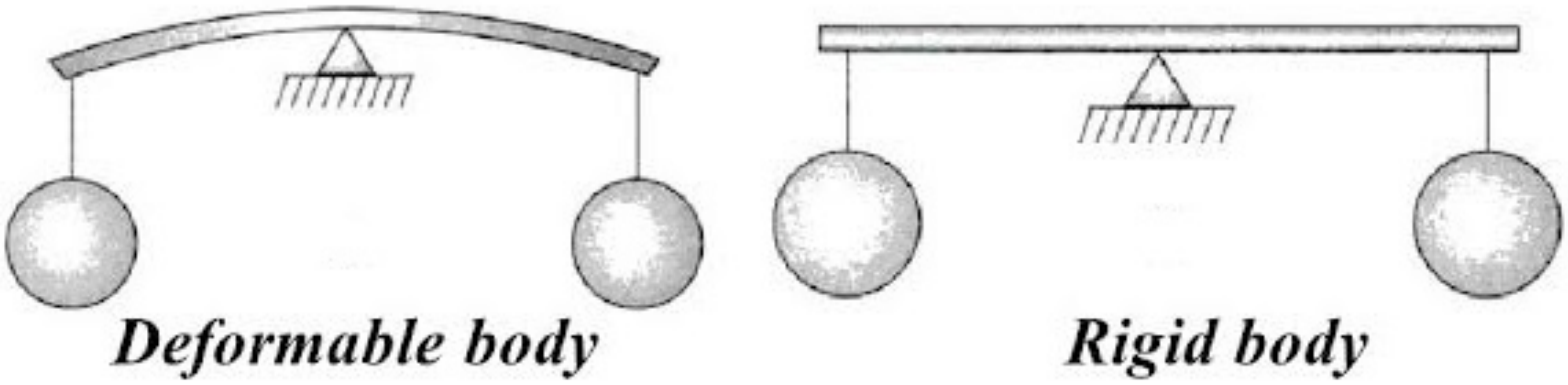
In week 15, I would like the prof and TA to review...

- Non-inertial reference frames 0%
- Friction 0%
- Tension & Pulleys 0%
- Work & Power 0%
- Conservation Laws (Energy & Momentum) 0%
- Problems with variable mass 0%
- Harmonic oscillators (free/damped/driven) 0%
- Gravity & Orbits 0%
- Normal force, circular motion 0%
- Center of mass / moment of inertia 0%
- Torque & angular momentum for solid objects 0%

No votes

Vote

RIGID SOLIDS

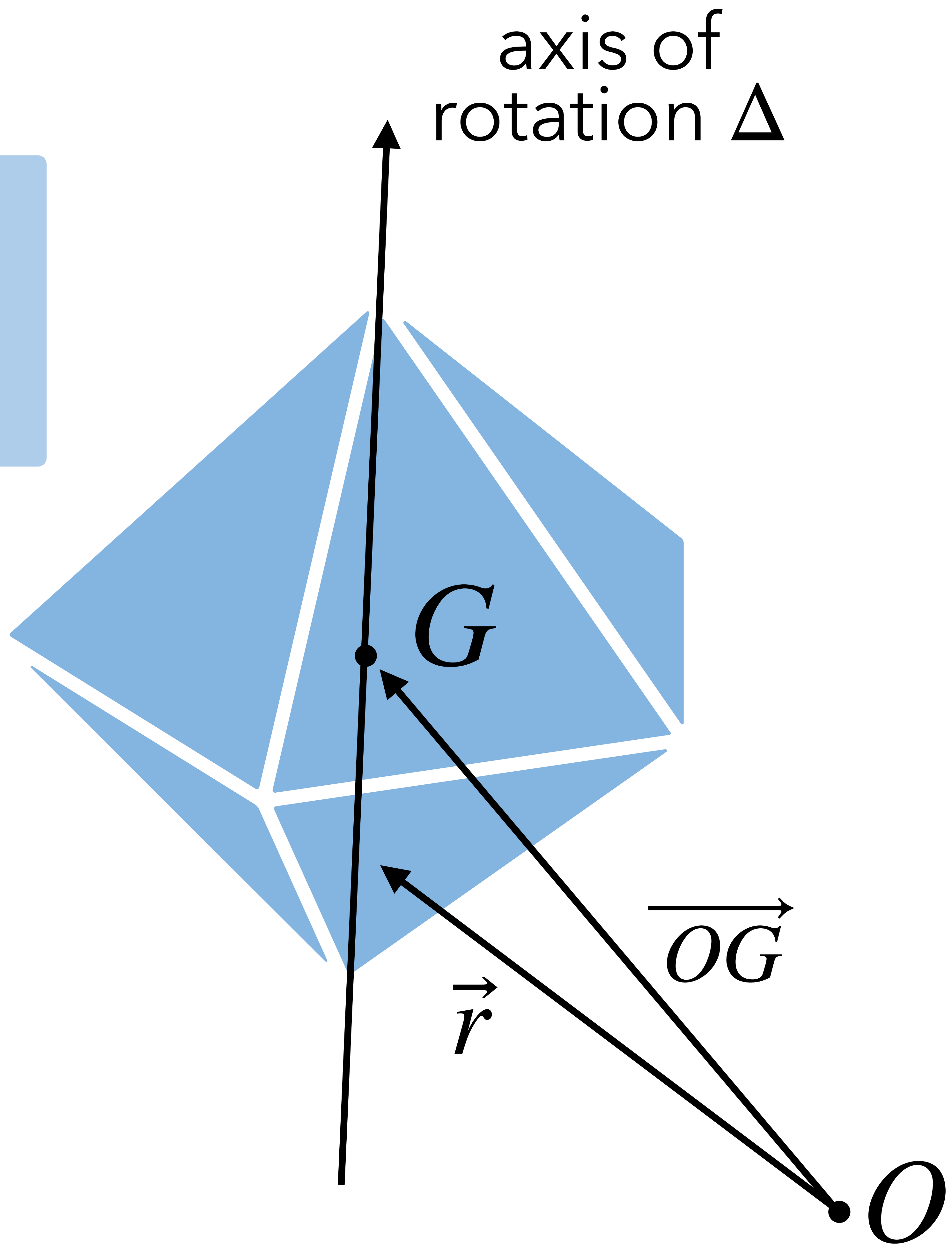


MAIN CONCEPTS

Center of mass (a position!)

$$\overrightarrow{OG} = \frac{1}{M}\iiint \vec{r}\, dm = \frac{1}{M}\iiint \vec{r}\rho(\vec{r})dV$$

Can usually calculate this using symmetry



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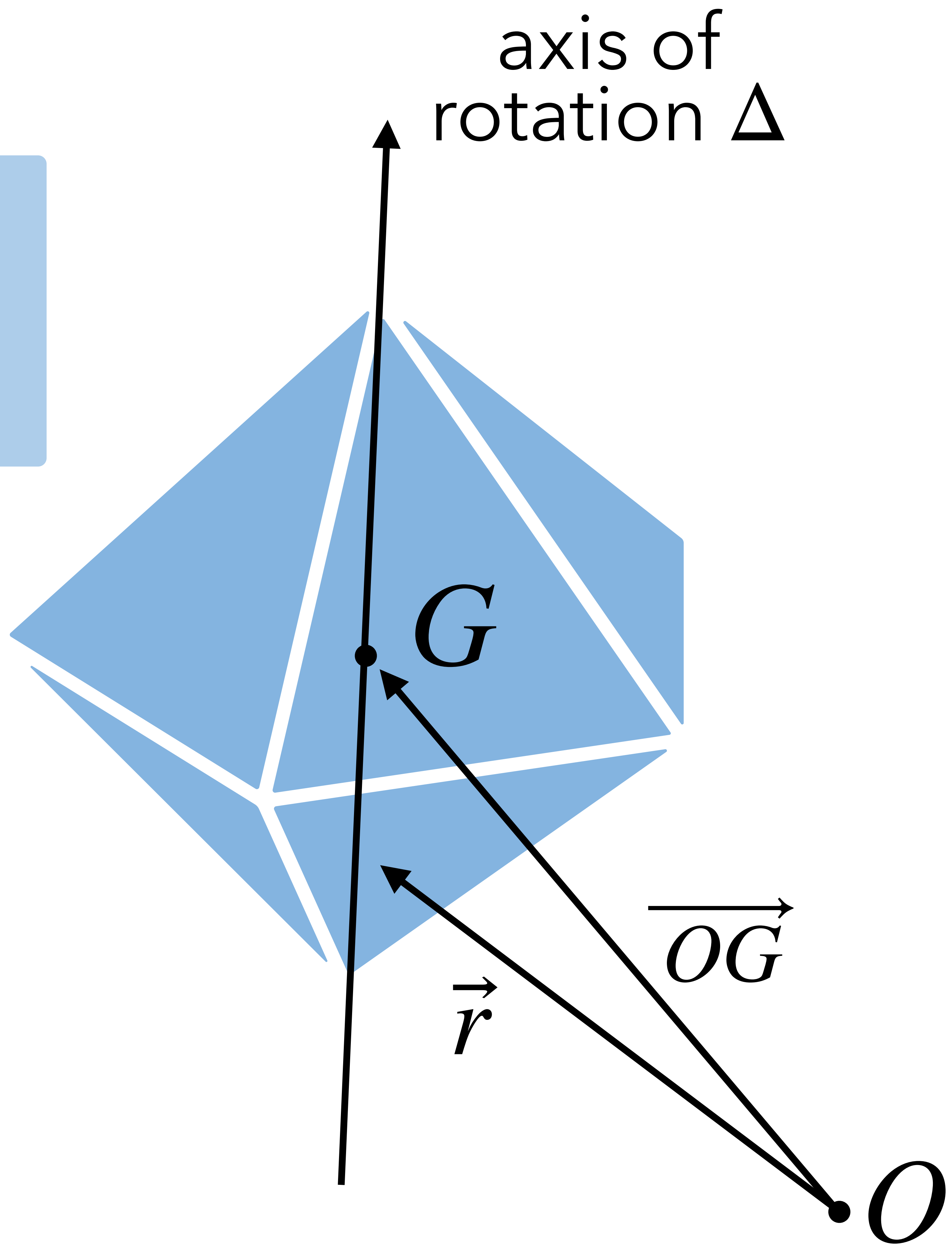
Refining Newton's 2nd Law:

$$\sum \overrightarrow{F}^{ext} = m\vec{a}_G$$

Acceleration of the center of mass



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Acceleration of the center of mass

Moment of inertia I_Δ with respect to axis of rotation Δ

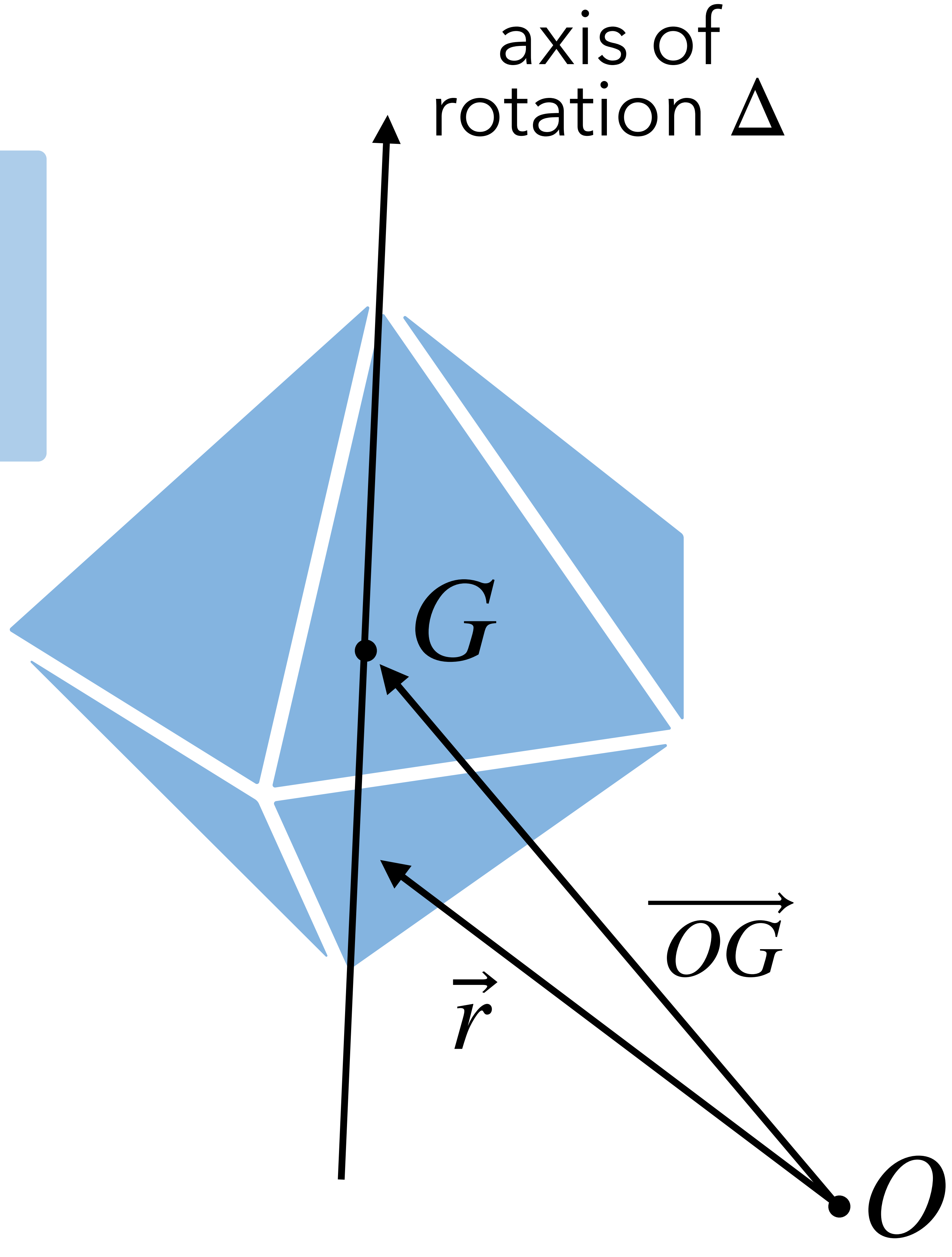
$$I_\Delta = \iiint \left(R_\Delta(\vec{r})\right)^2 \rho(\vec{r})dV$$

Given for certain solids

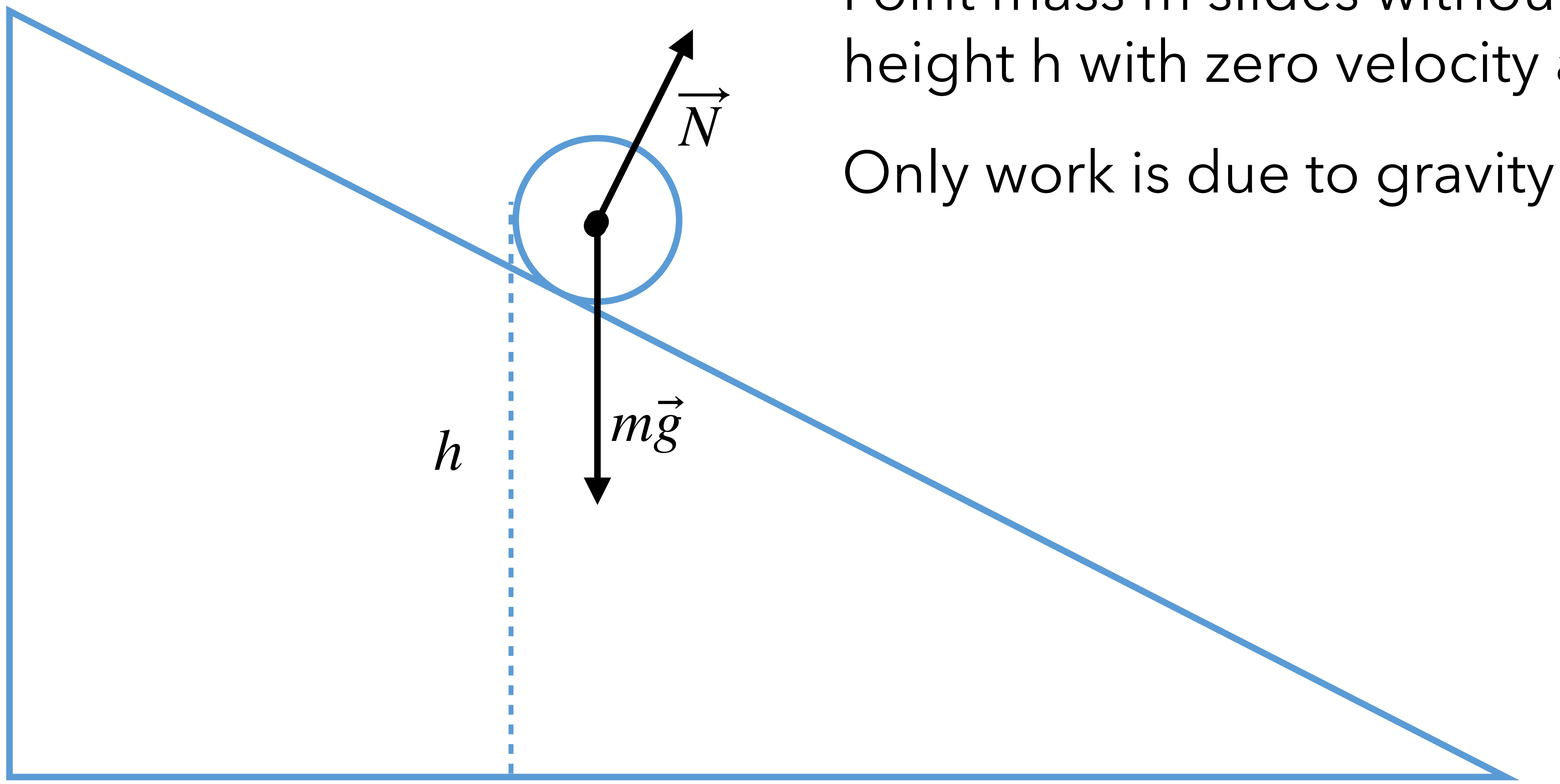
Calculate offsets with Steiner's theorem $I_{\Delta'} = I_\Delta + Ma^2$

Kinetic energy of rotation

$$E_{c,rot} = \frac{1}{2} I_\Delta \omega^2$$



MOTION ALONG AN INCLINED PLANE



Point mass m slides without friction, starting from a height h with zero velocity and moving to a height 0

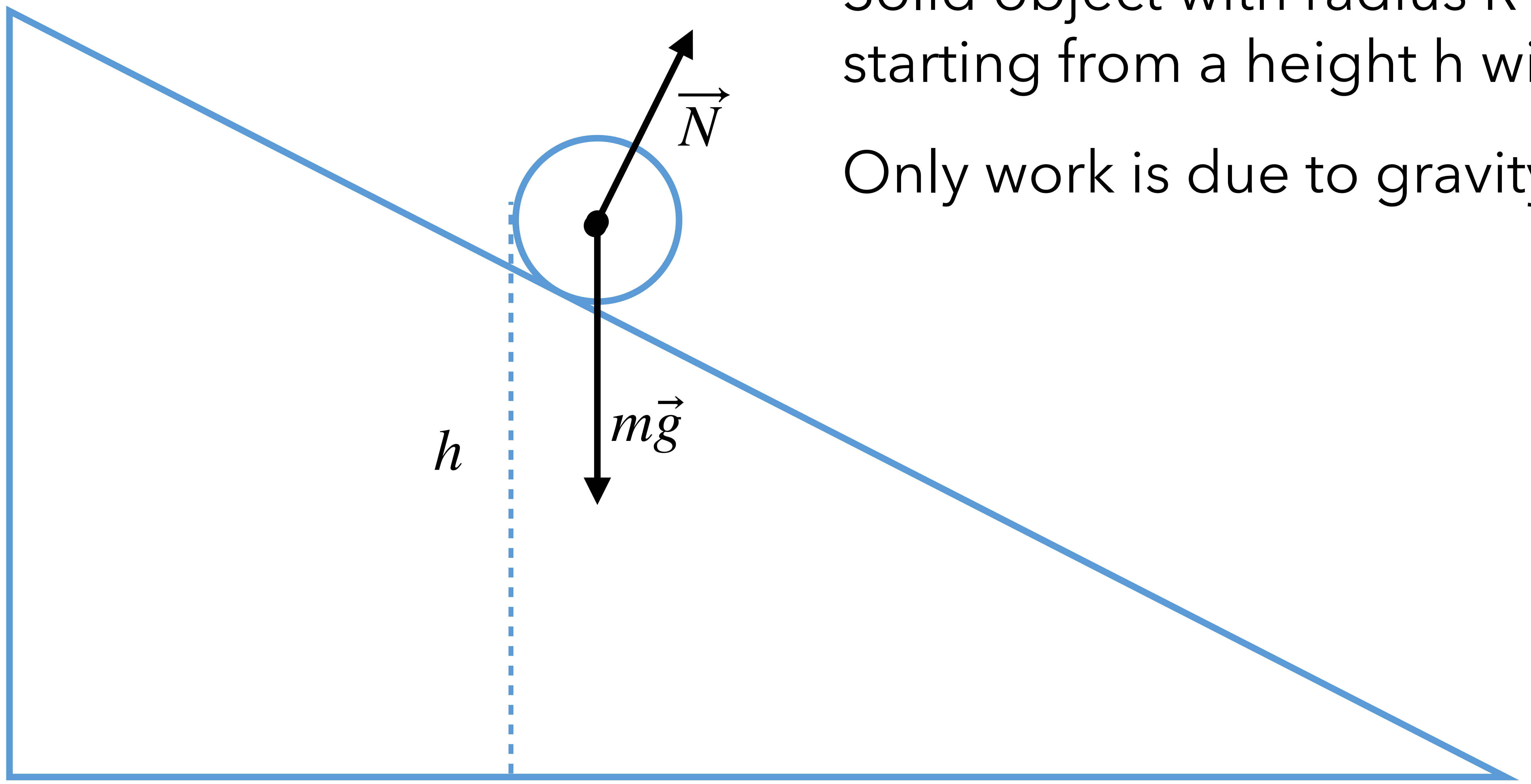
Only work is due to gravity: $\Delta E_c = Ep^G(h) - Ep^G(0)$

$$\frac{1}{2}mv^2 = mgh$$

$$\implies v = \sqrt{2gh}$$

Final velocity depends only on g and h

MOTION ALONG AN INCLINED PLANE



Solid object with radius R and mass m rolls without slipping, starting from a height h with zero velocity and moving to a height 0

Only work is due to gravity: $\Delta E_c = Ep^G(h) - Ep^G(0)$

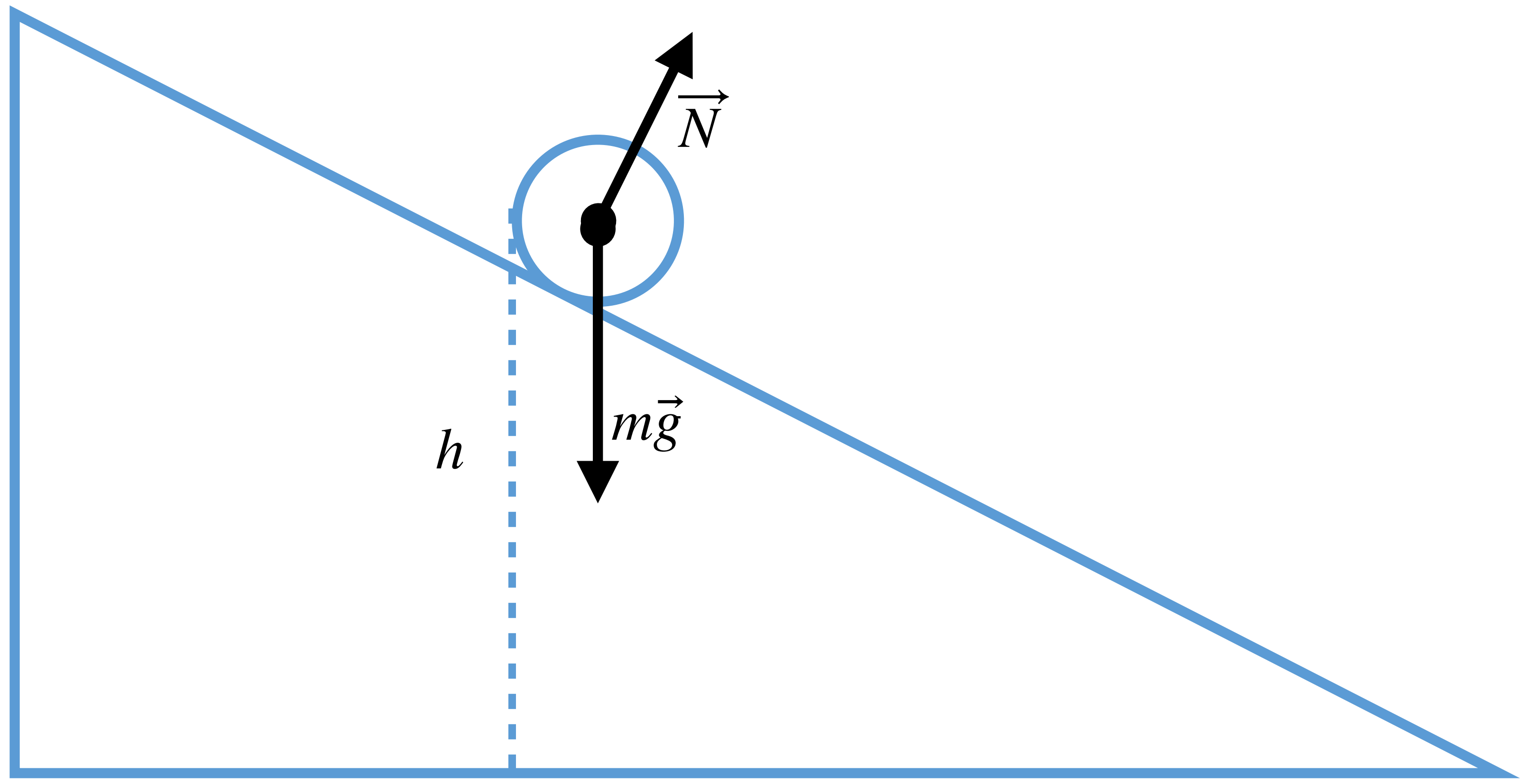
$$\frac{1}{2}mv^2 + \frac{1}{2}I_\Delta \omega^2 = mgh$$

Rolling without slipping: $\omega = v/R$

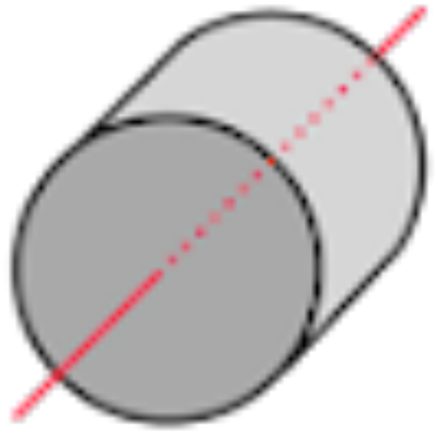
$$v^2 = \frac{2gh}{1 + \frac{I_\Delta}{mR^2}}$$

Much more complicated!

MOTION ALONG AN INCLINED PLANE



Solid cylinder or disc, symmetry axis

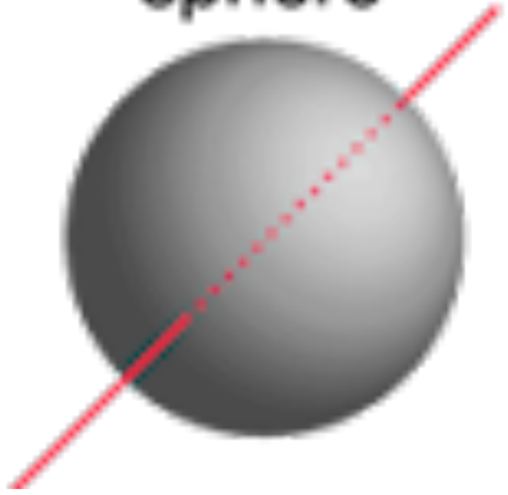


$$I_\Delta = \frac{1}{2} mR^2$$

Hoop about symmetry axis

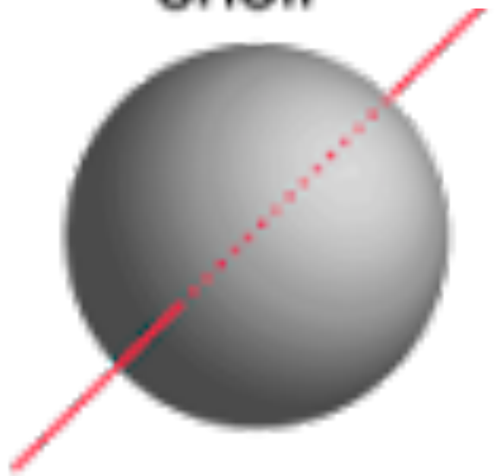
$$I_\Delta = mR^2$$

Solid sphere



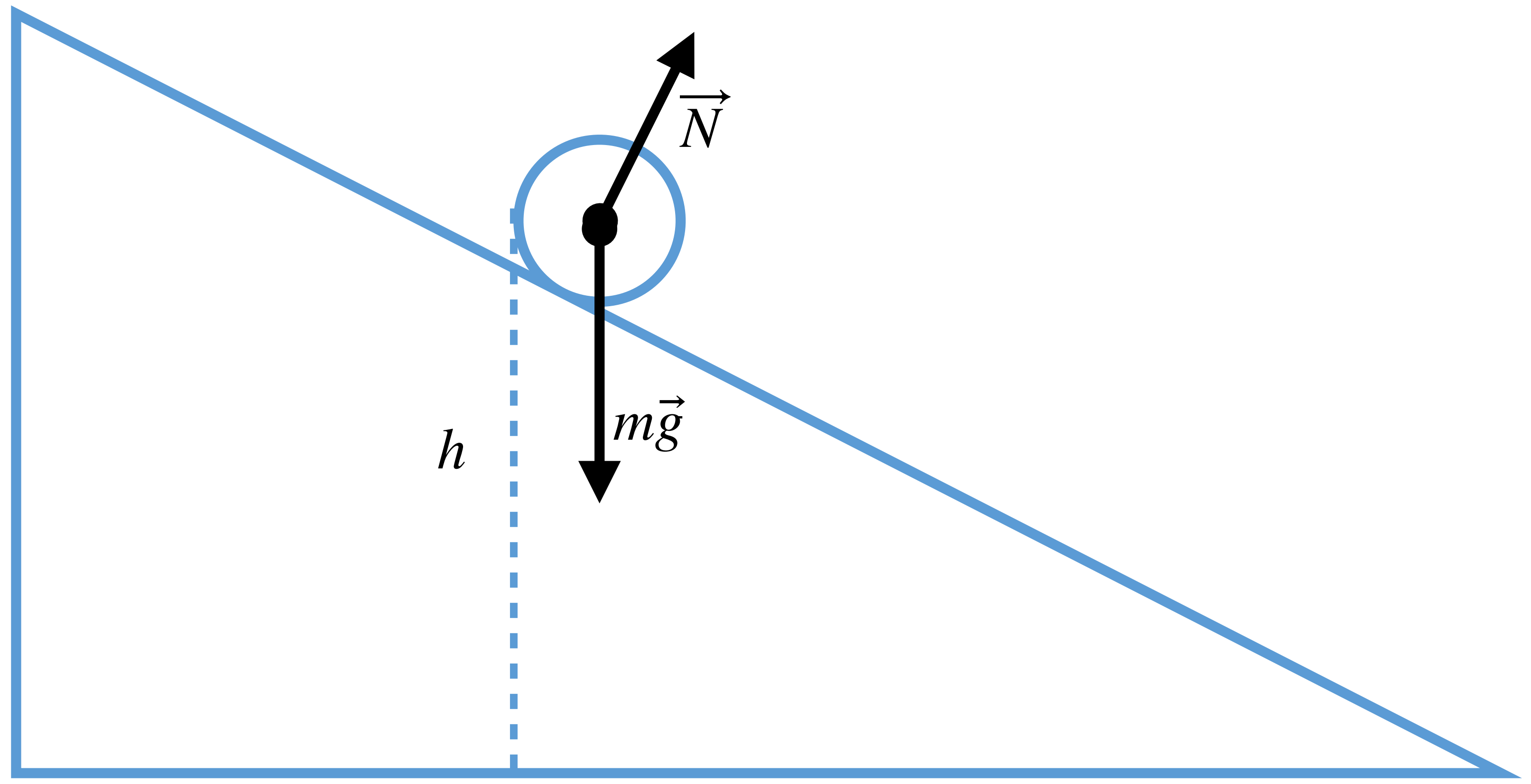
$$I_\Delta = \frac{2}{5} mR^2$$

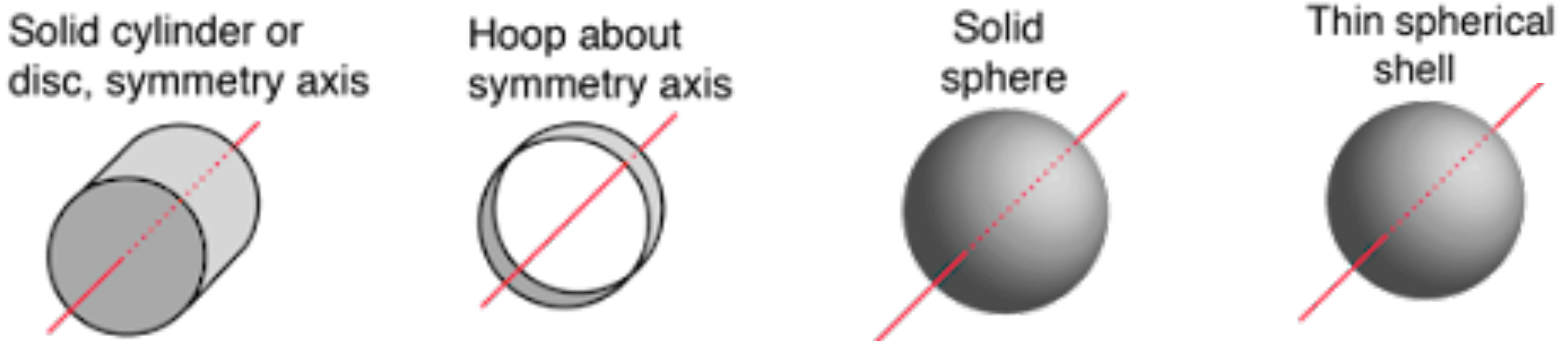
Thin spherical shell



$$I_\Delta = \frac{2}{3} mR^2$$

MOTION ALONG AN INCLINED PLANE





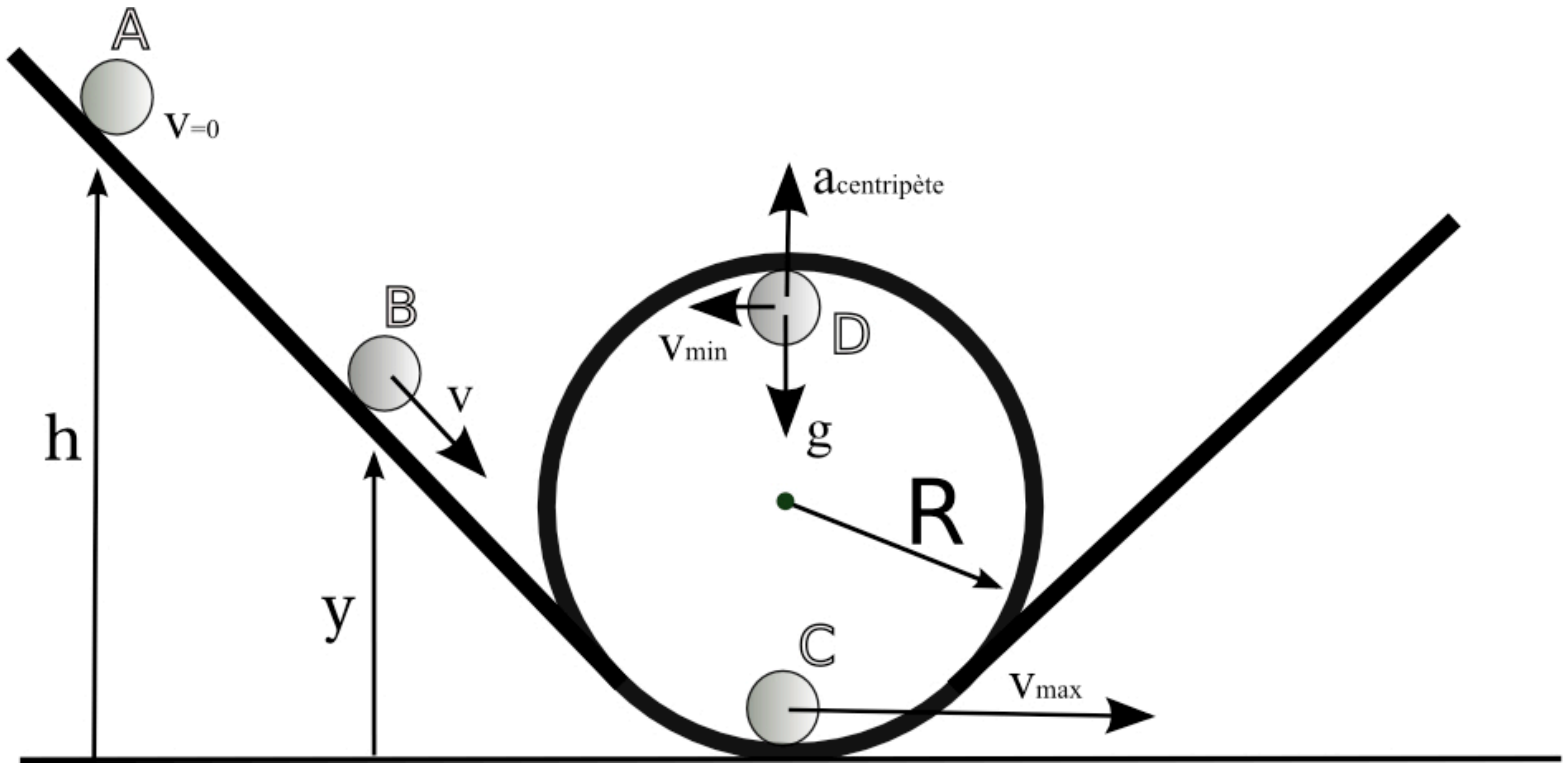
$I_\Delta = \frac{1}{2}mR^2$ $\quad$ $I_\Delta = mR^2$ $\quad$ $I_\Delta = \frac{2}{5}mR^2$ $\quad$ $I_\Delta = \frac{2}{3}mR^2$

Point mass $\quad$ **Generic solid**

$v^2 = 2gh$ $\quad$ $v^2 = \dfrac{2gh}{1 + \frac{I_\Delta}{mR^2}}$ $\quad$ $v^2 = \frac{4}{3}gh \approx 1.3gh$ $\quad$ $v^2 = gh$ $\quad$ $v^2 = \frac{10}{7}gh \approx 1.4gh$ $\quad$ $v^2 = \frac{6}{5}gh \approx 1.2gh$

Radius and mass do not matter, only geometry, g, h

REVISITING THE MARBLE IN THE LOOP



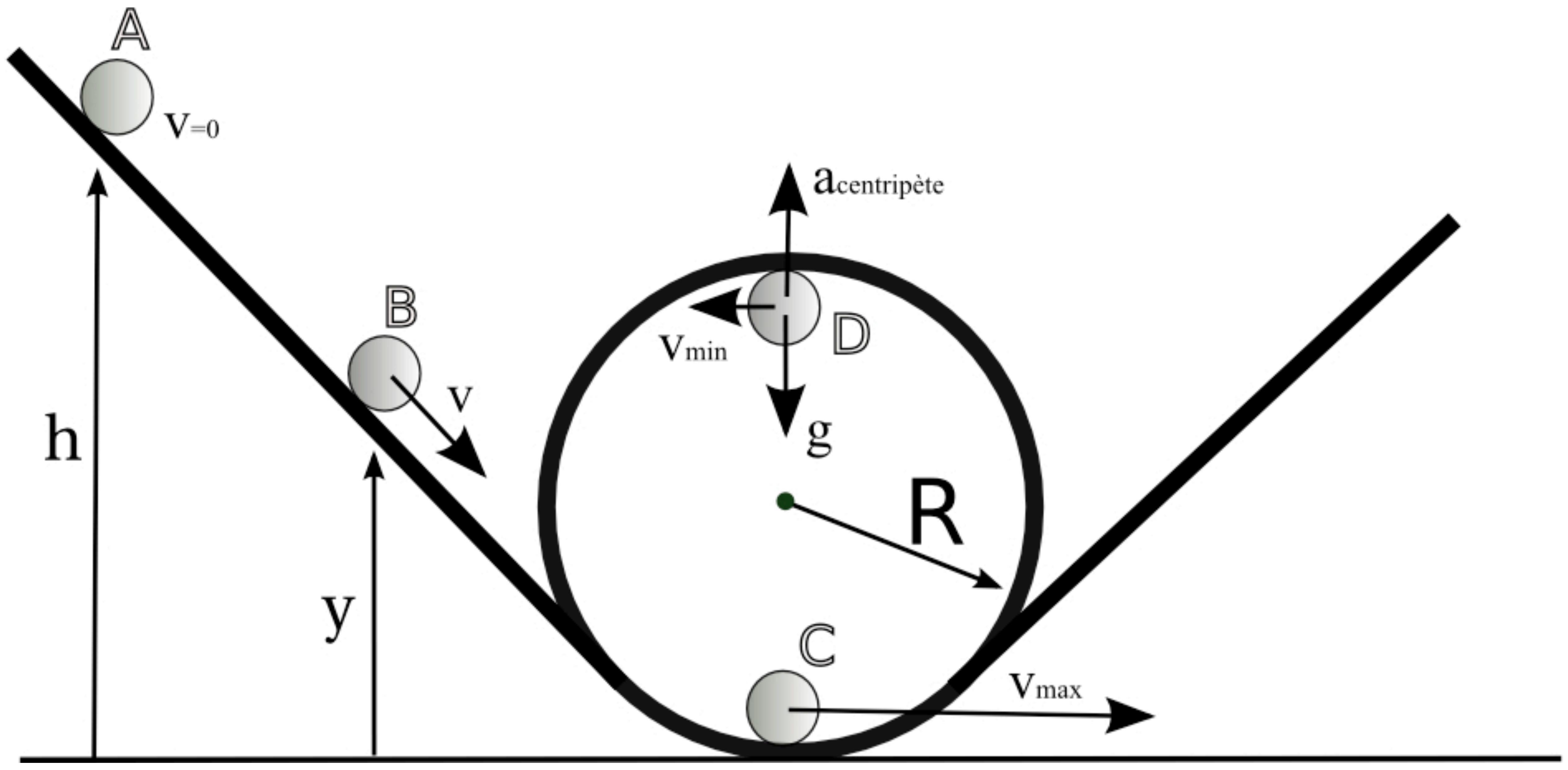
Centripetal acceleration which creates circular motion comes from the normal force and gravity, so:

$$\frac{mV^2}{R} = N + mg$$

Ball moves on the inside of the loop:

$$N > 0$$

REVISITING THE MARBLE IN THE LOOP



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$$N > 0 \implies N = \frac{mV^2}{R} - mg > 0 \implies \boxed{v^2 > gR}$$

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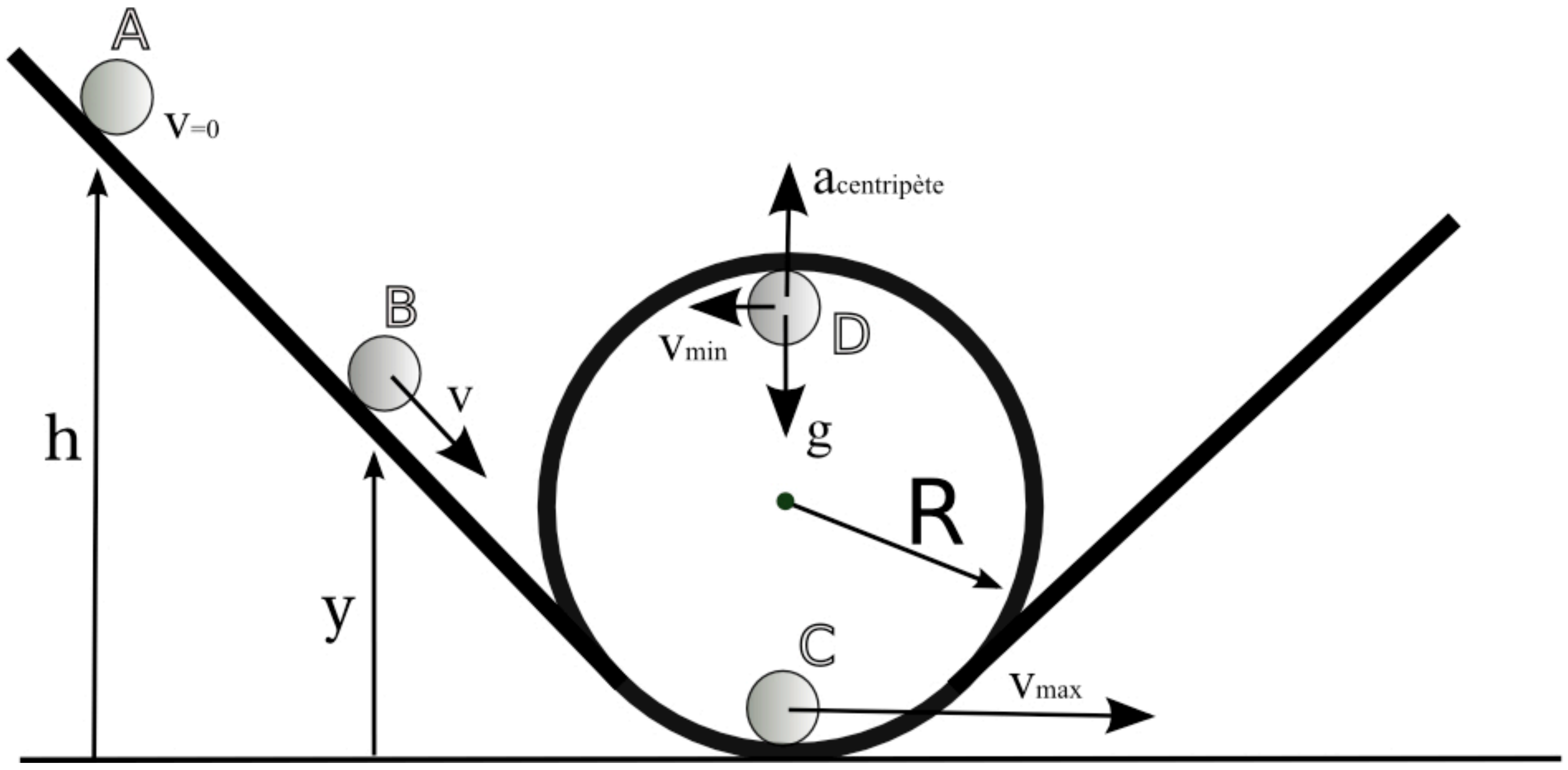

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$$\Delta E_c = Ep^G(h) - Ep^G(z = 2R)$$

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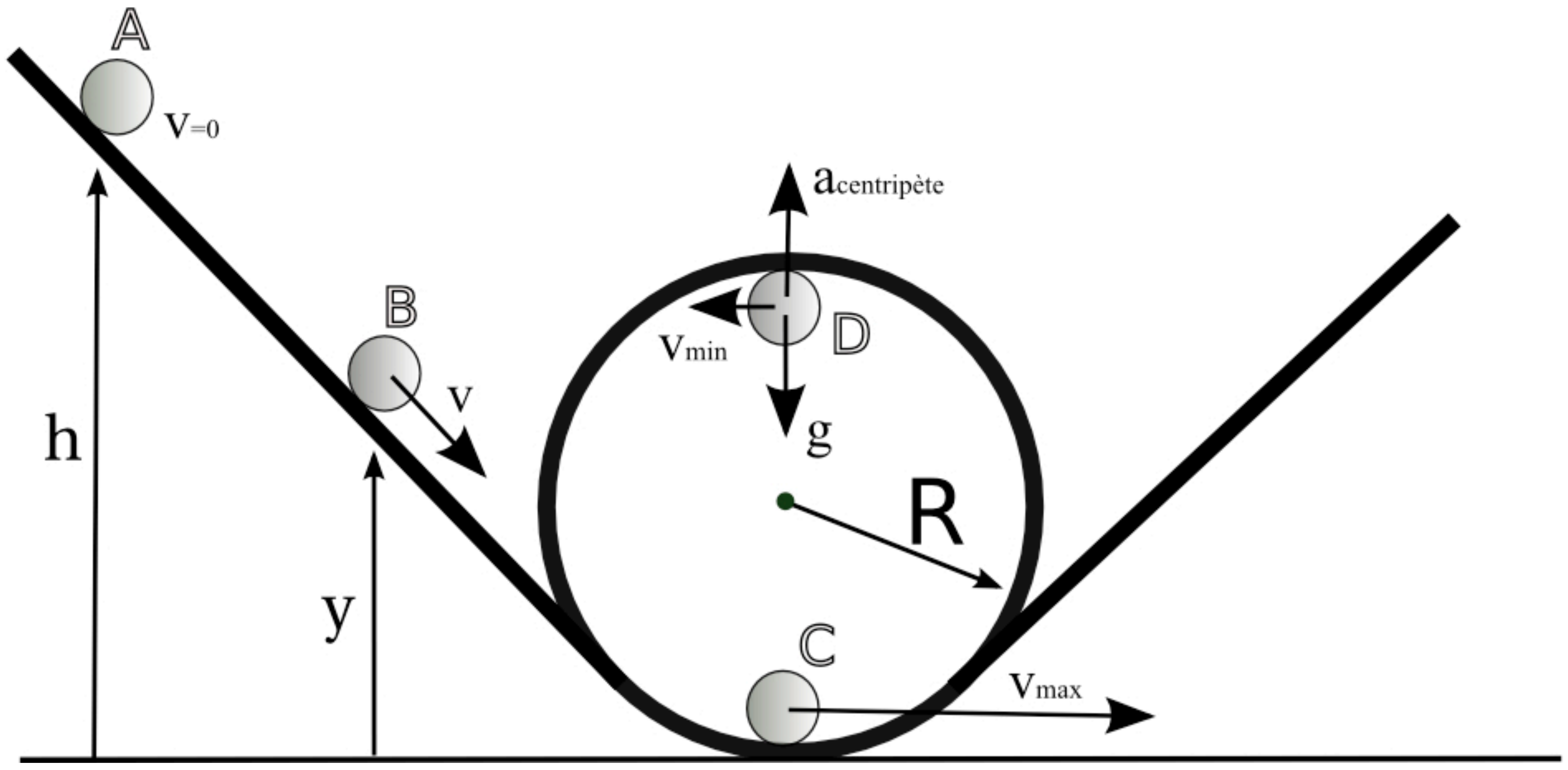

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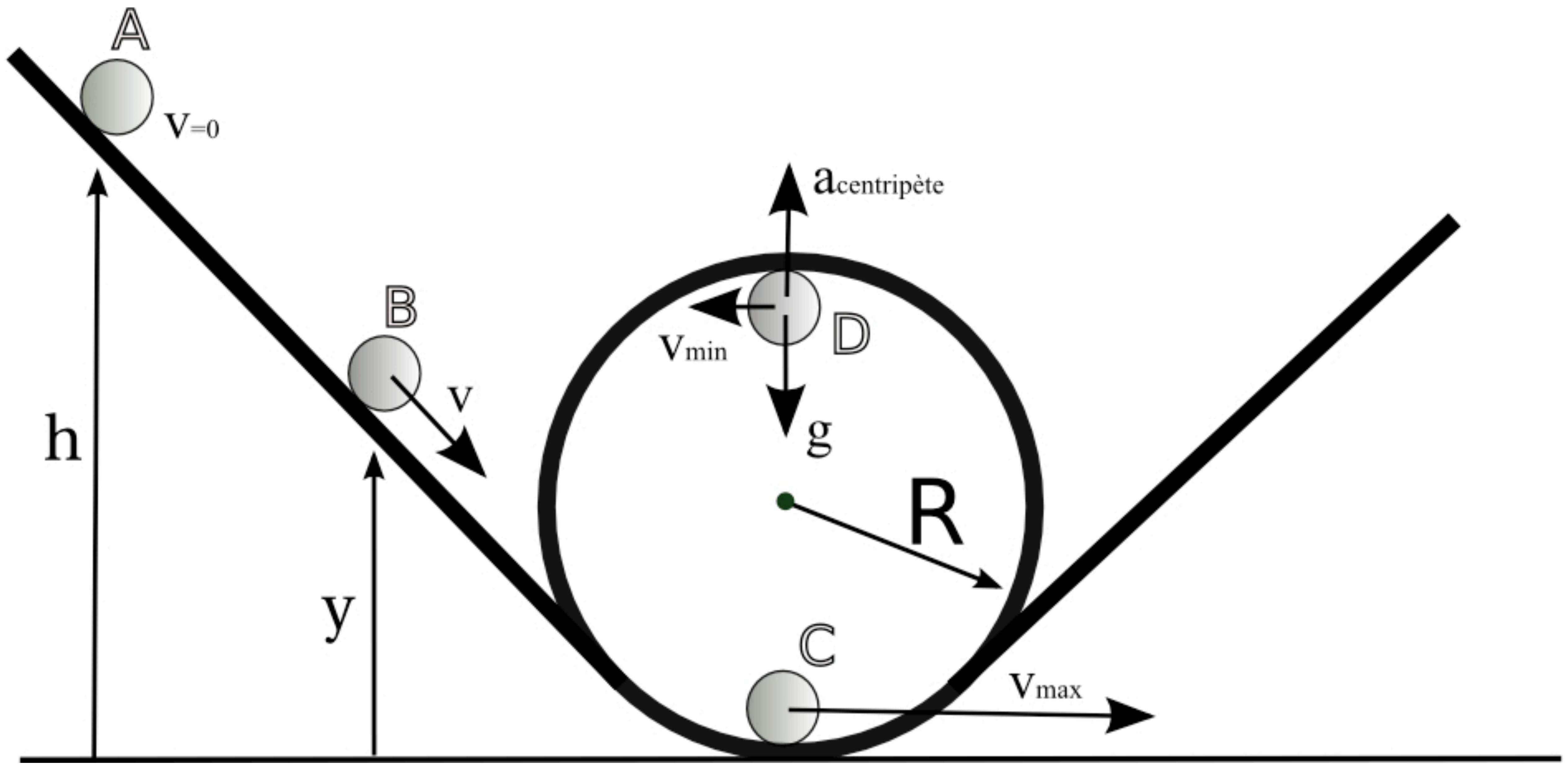
$$\Delta E_c = Ep^G(h) - Ep^G(z = 2R)$$

$$\frac{1}{2}mv^2 = mg(h - 2R) \implies \boxed{v^2 = 2g(h - 2R)}$$

$$2g(h - 2R) > gR \implies \boxed{h > 2.5R}$$

REVISITING THE MARBLE IN THE LOOP



This ignores rotation!!

Centripetal acceleration which creates circular motion comes from the normal force and gravity, so: $$\frac{mV^2}{R} = N + mg$$

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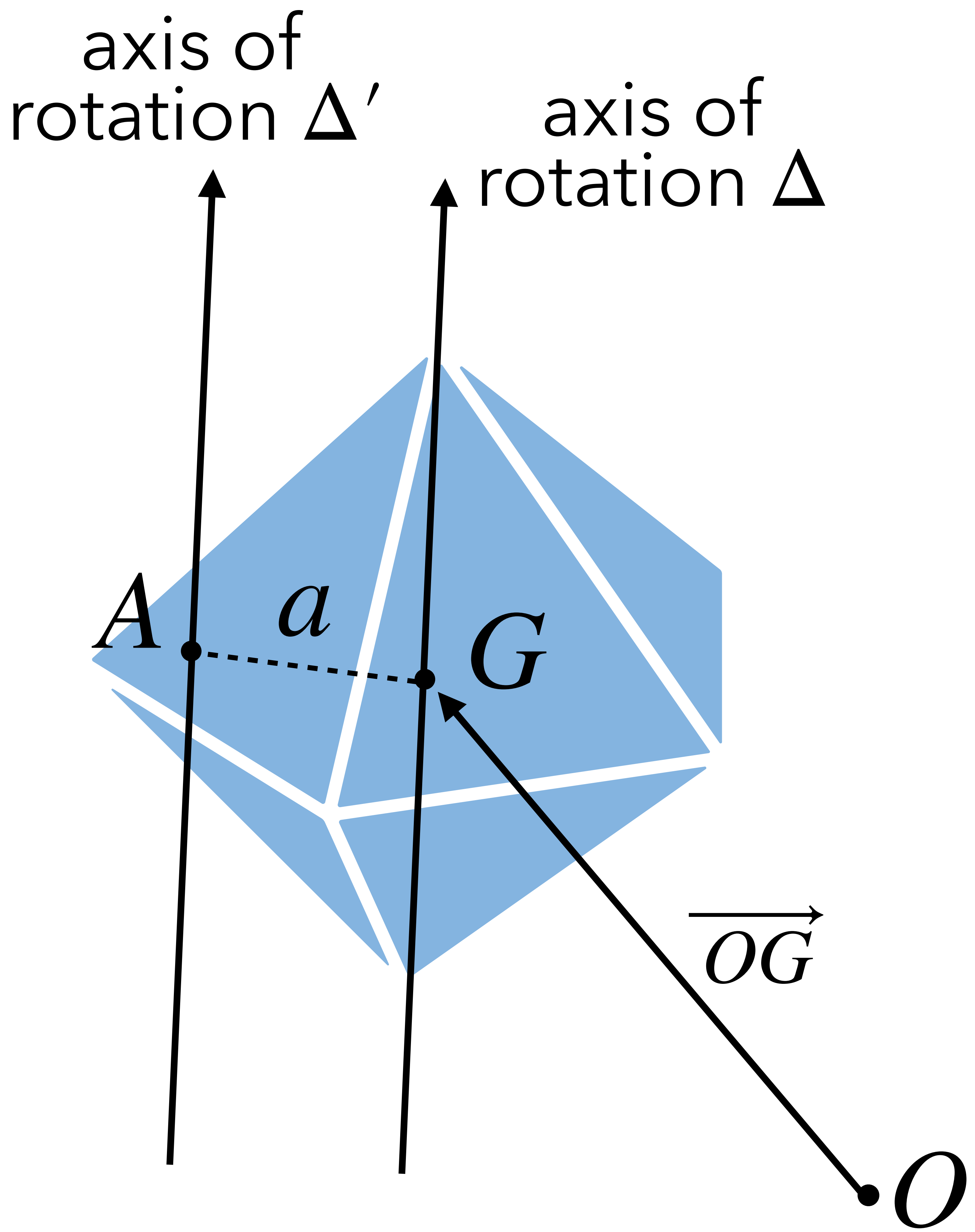
MAIN CONCEPTS, CONTINUED

Angular momentum if the object is rotating along a *principal axis of inertia*

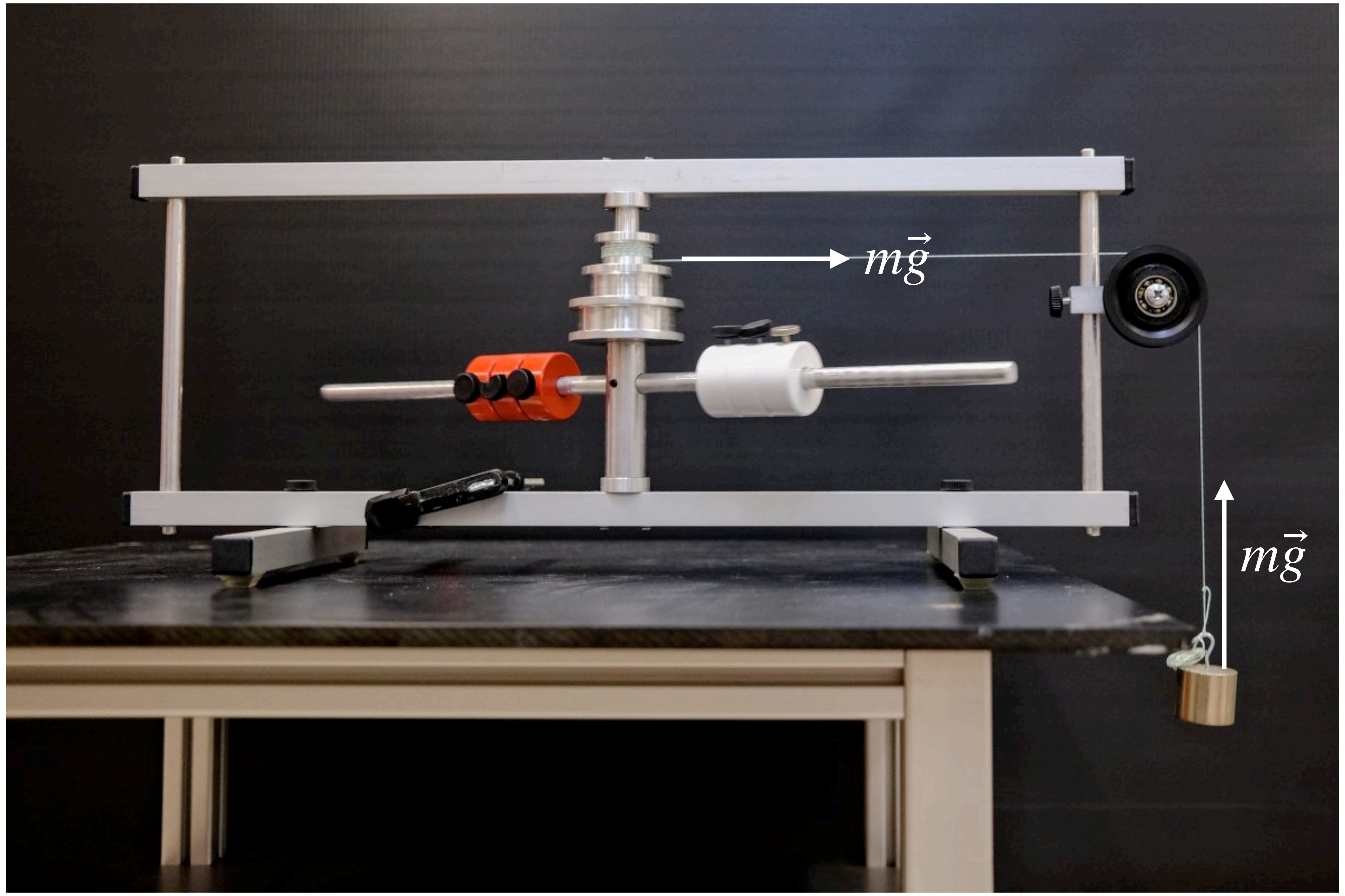
$$\overrightarrow{L_A} = I_{\Delta'}\overrightarrow{\omega} \qquad \overrightarrow{L_G} = I_{\Delta}\overrightarrow{\omega}$$

Theory of angular momentum for a point A on the object which is: fixed, the CoM, or moving collinear to the CoM

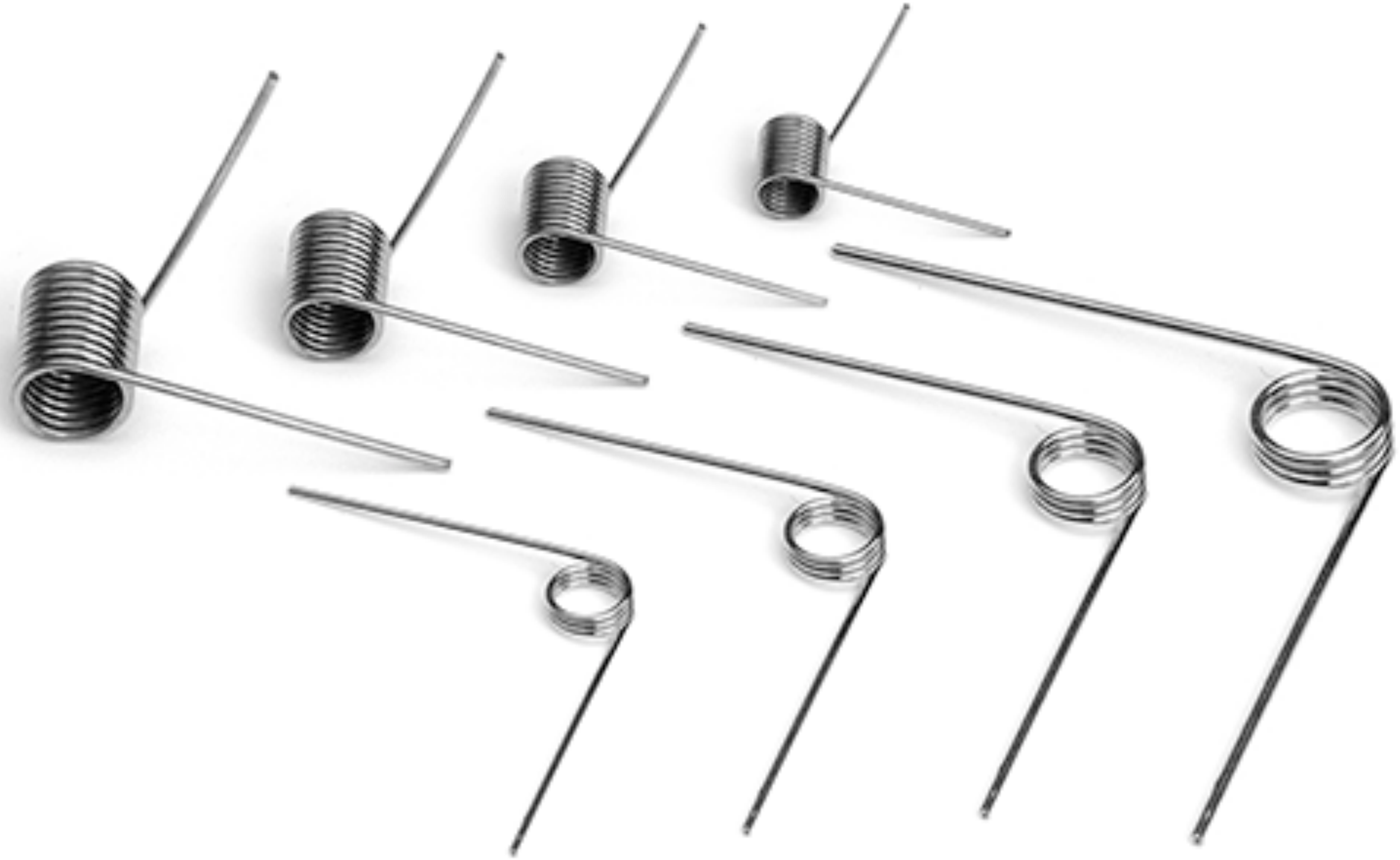
$$\sum \overrightarrow{M_A} = \frac{d\overrightarrow{L_A}}{dt}$$

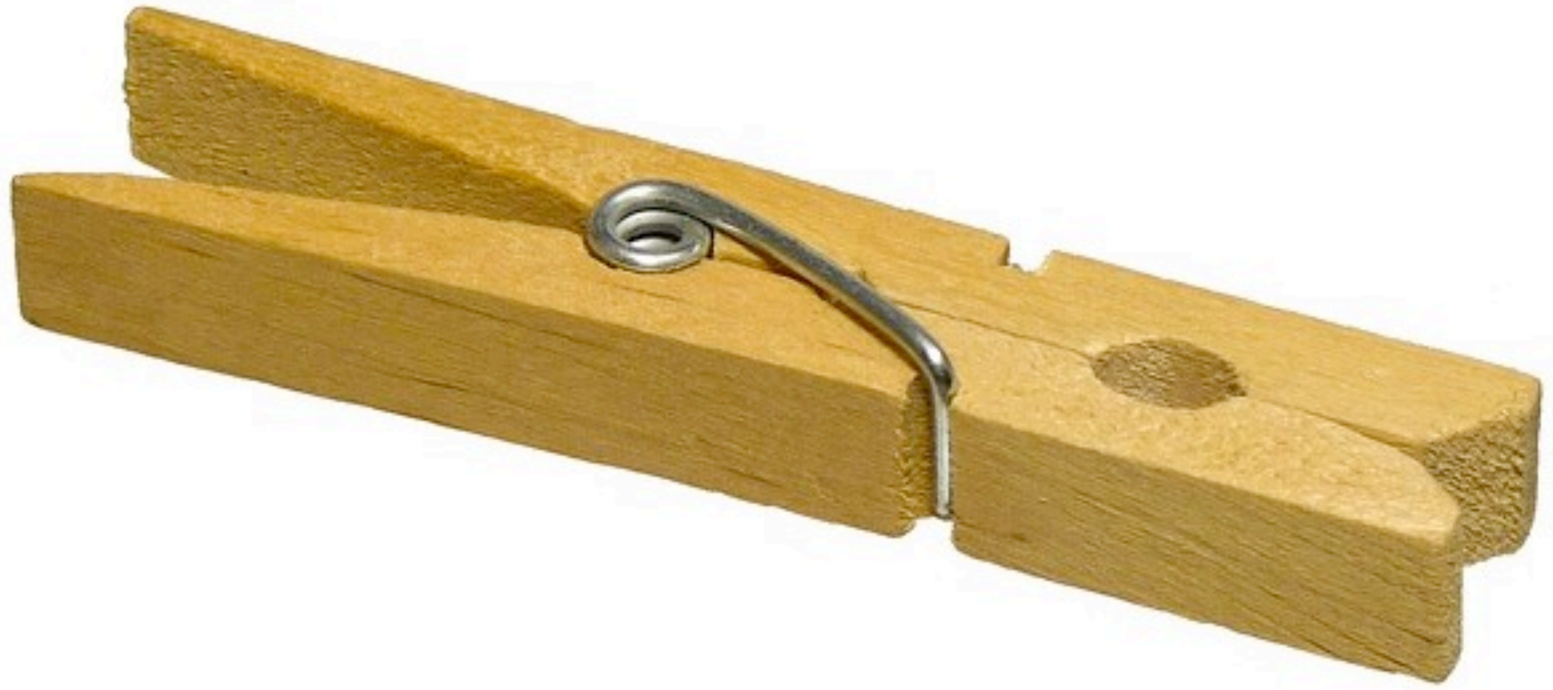


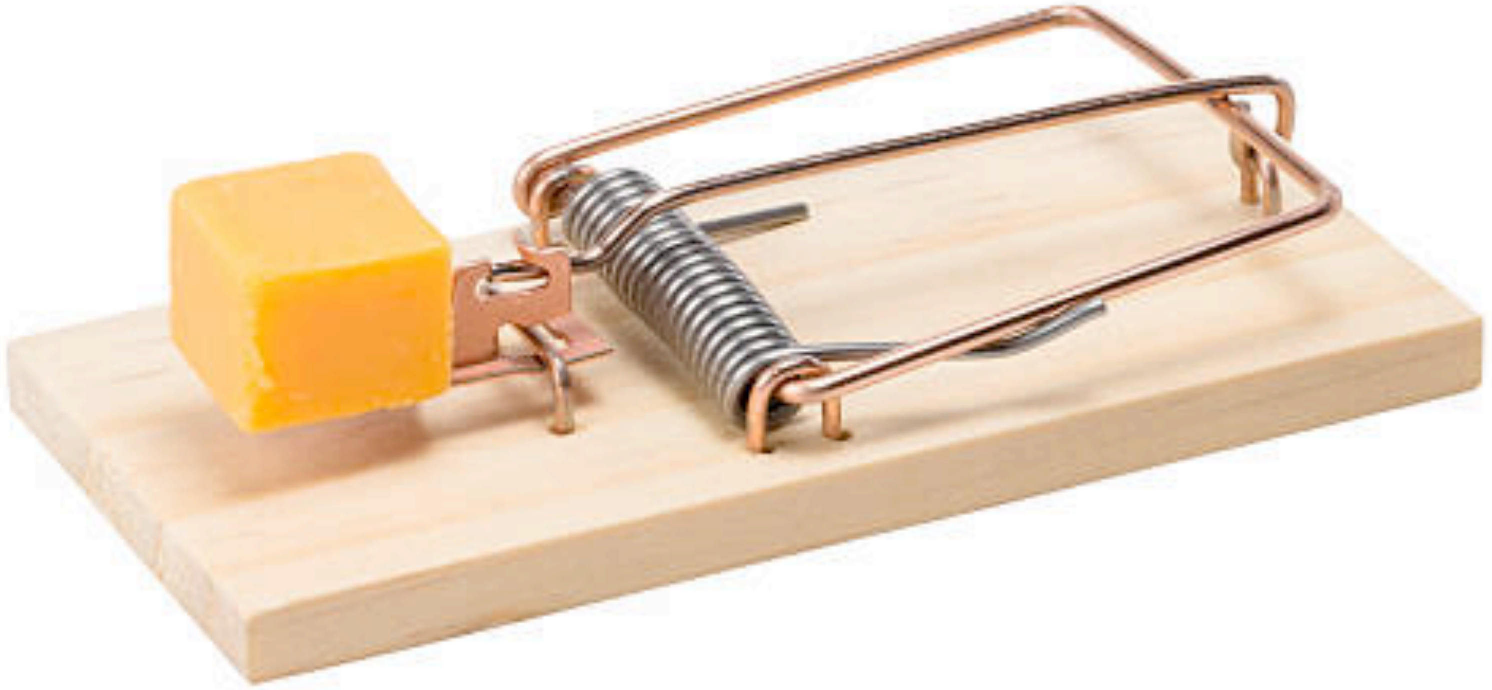
UNROLLING SPOOL



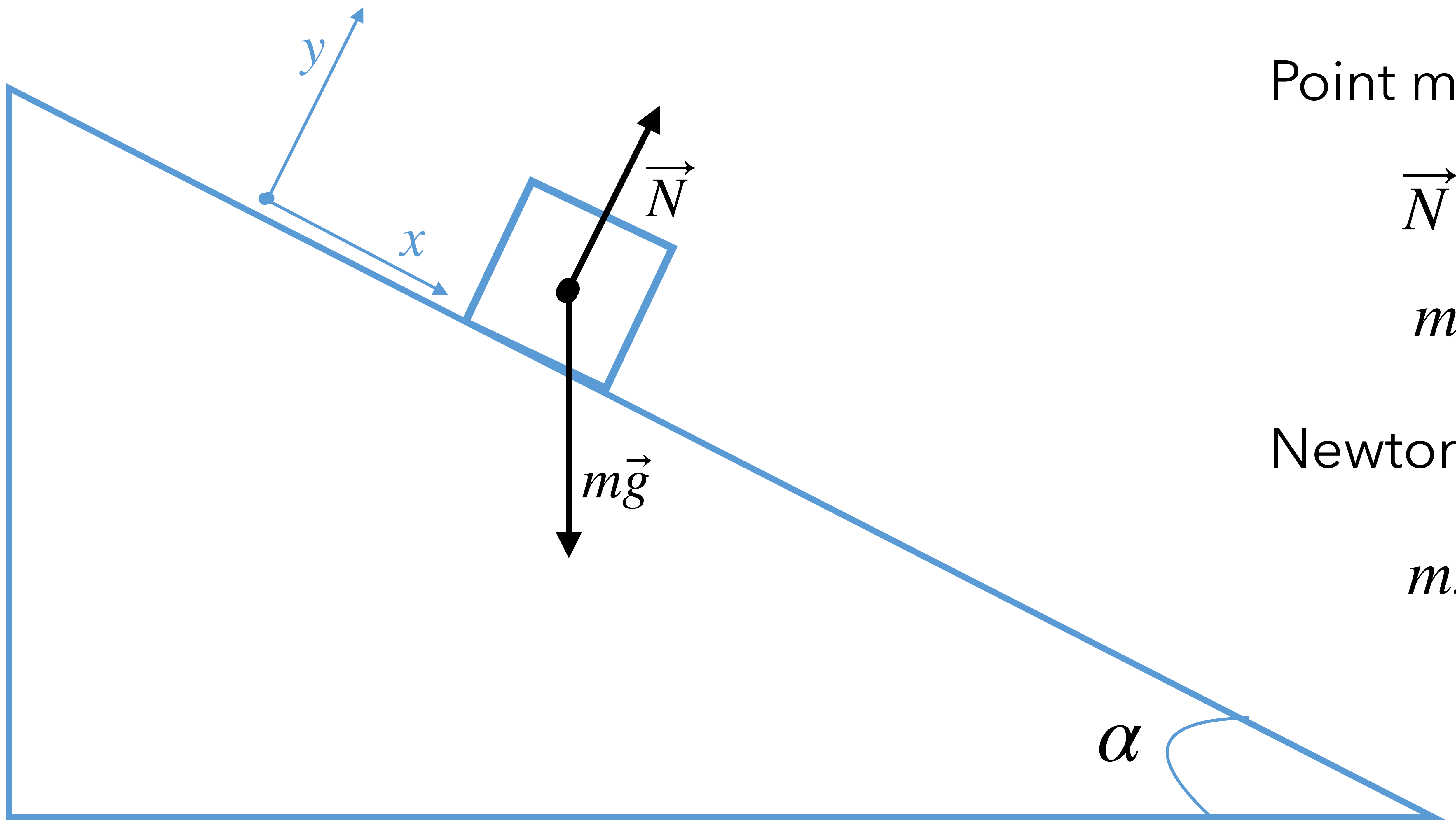
TORSION PENDULUM







MOTION ALONG AN INCLINED PLANE



Point mass m slides without friction:

$$\vec{N} = N\vec{e}_y, \quad N \geq 0 \quad \text{s.t.}\ \dot{y} = 0$$

$$m\vec{g} = mg\sin\alpha\vec{e}_x - mg\cos\alpha\vec{e}_y$$

Newton's 2nd Law on along x-axis:

$$m\ddot{x} = mg\sin\alpha \implies \boxed{\ddot{x} = g\sin\alpha}$$

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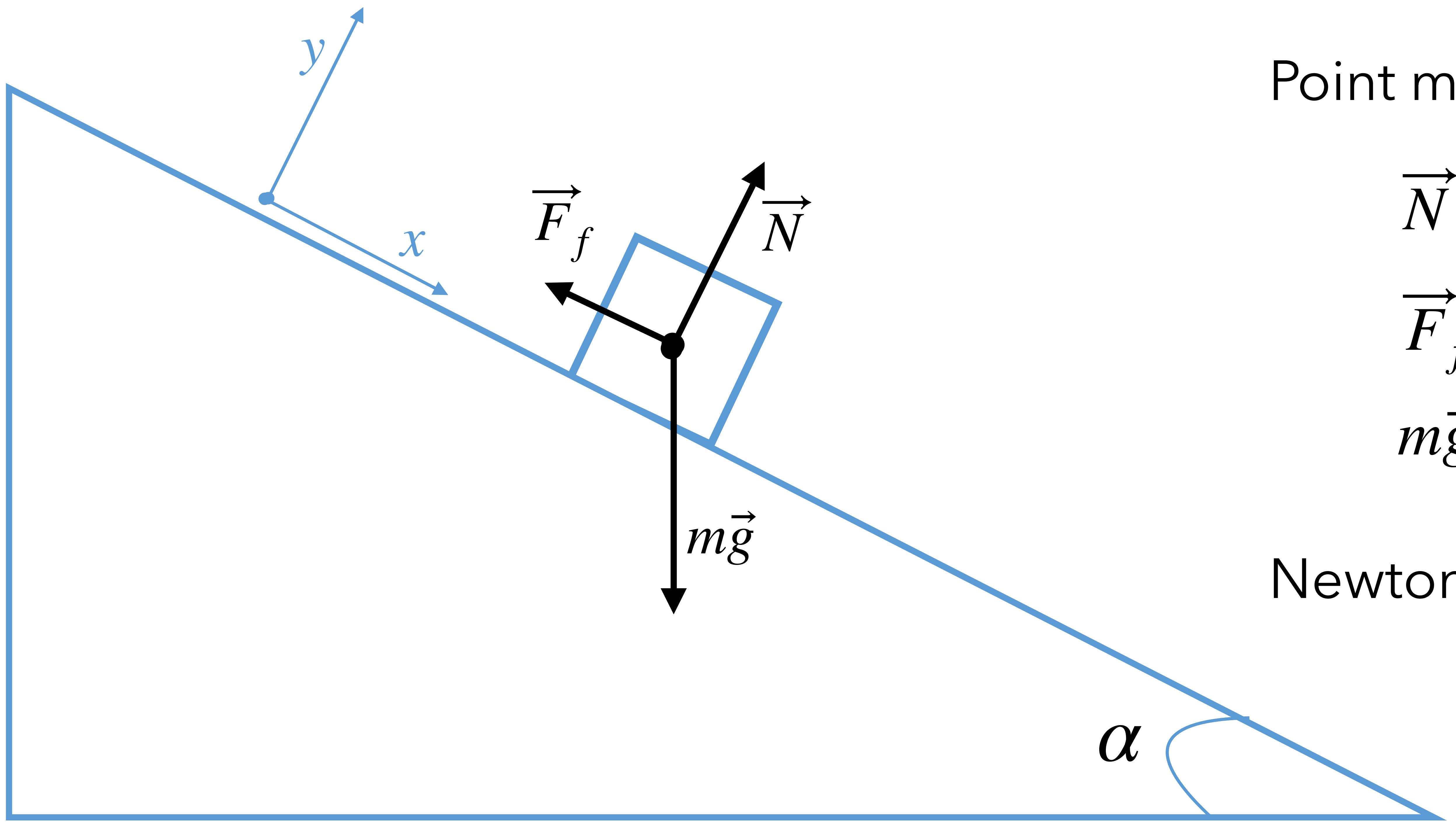

Point mass m with static friction:

$$\vec{N} = N\vec{e}_y, \;\; N \geq 0 \;\; \text{s.t. } \dot{y} = 0$$

$$\vec{F}_f = -F_f\vec{e}_x, \;\; F_f \geq 0 \;\; \text{s.t. } v = 0$$

$$m\vec{g} = mg\sin\alpha\vec{e}_x - mg\cos\alpha\vec{e}_y$$

Newton's 2nd Law on along x-axis:

$$m\ddot{x} = mg\sin\alpha - F_f \implies \boxed{\ddot{x} = 0, \; F_f = mg\sin\alpha}$$

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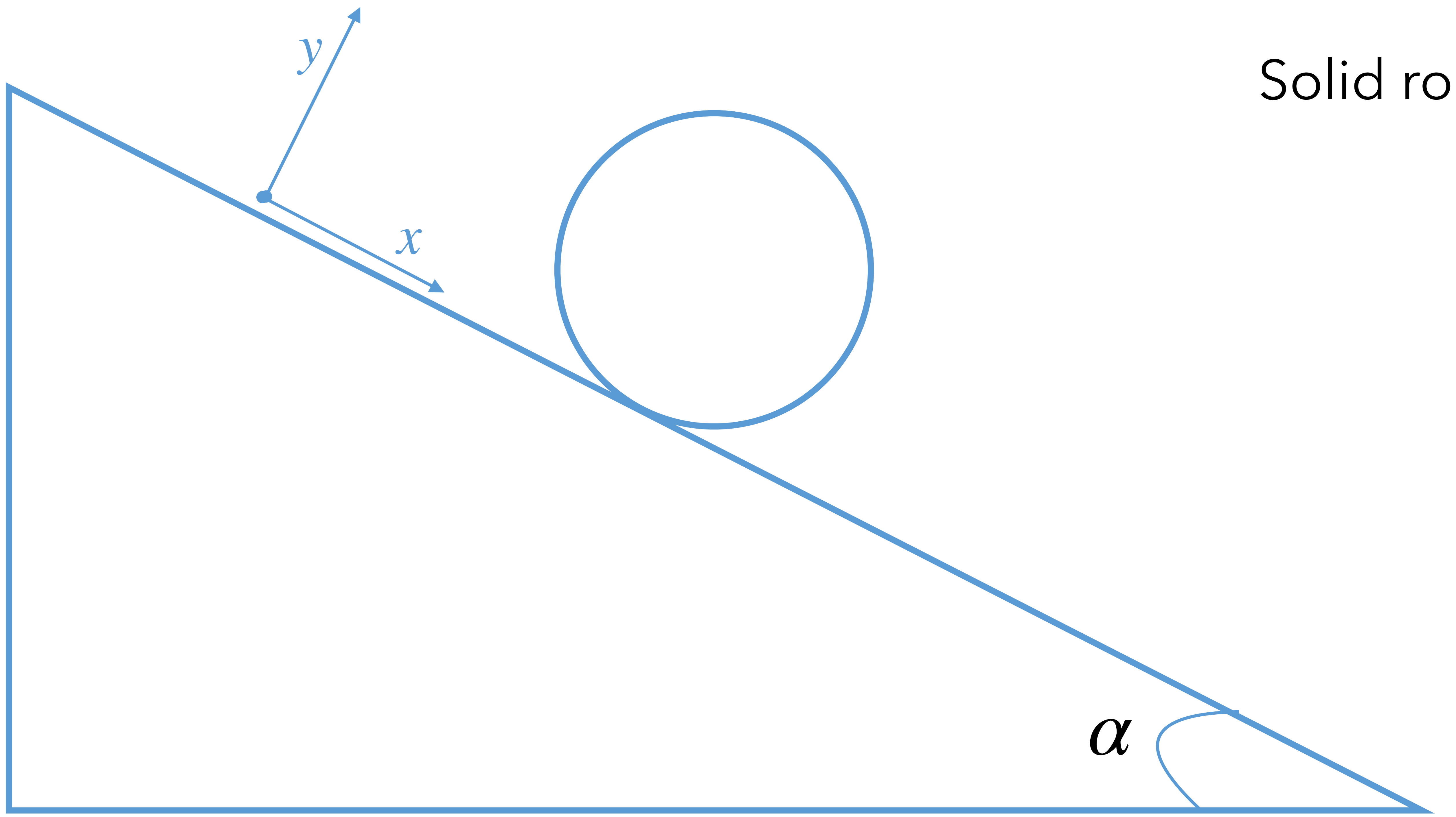
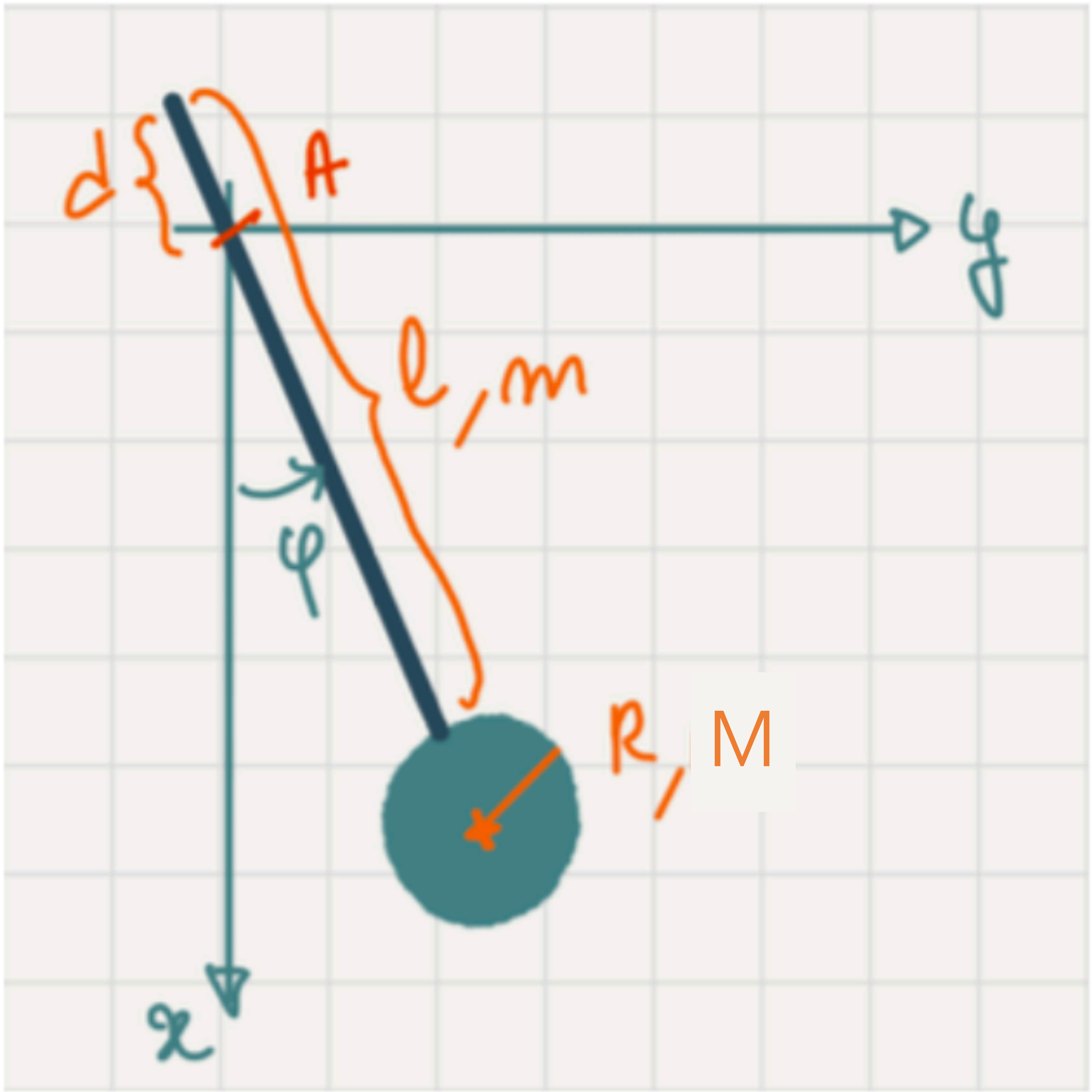


Solid rolling object?

EXERCISE: SIMPLE PENDULUM



A simple pendulum consists of a bar of mass m and length l attached to a pivot, with a disk of radius R and mass M attached to the end of the bar. The pivot point is a distance d from the edge of the bar. At t = 0, it is displaced by an angle ϕ_0 from its equilibrium position and released from rest.

Establish the differential equation of motion and solve it in the case of small oscillations.

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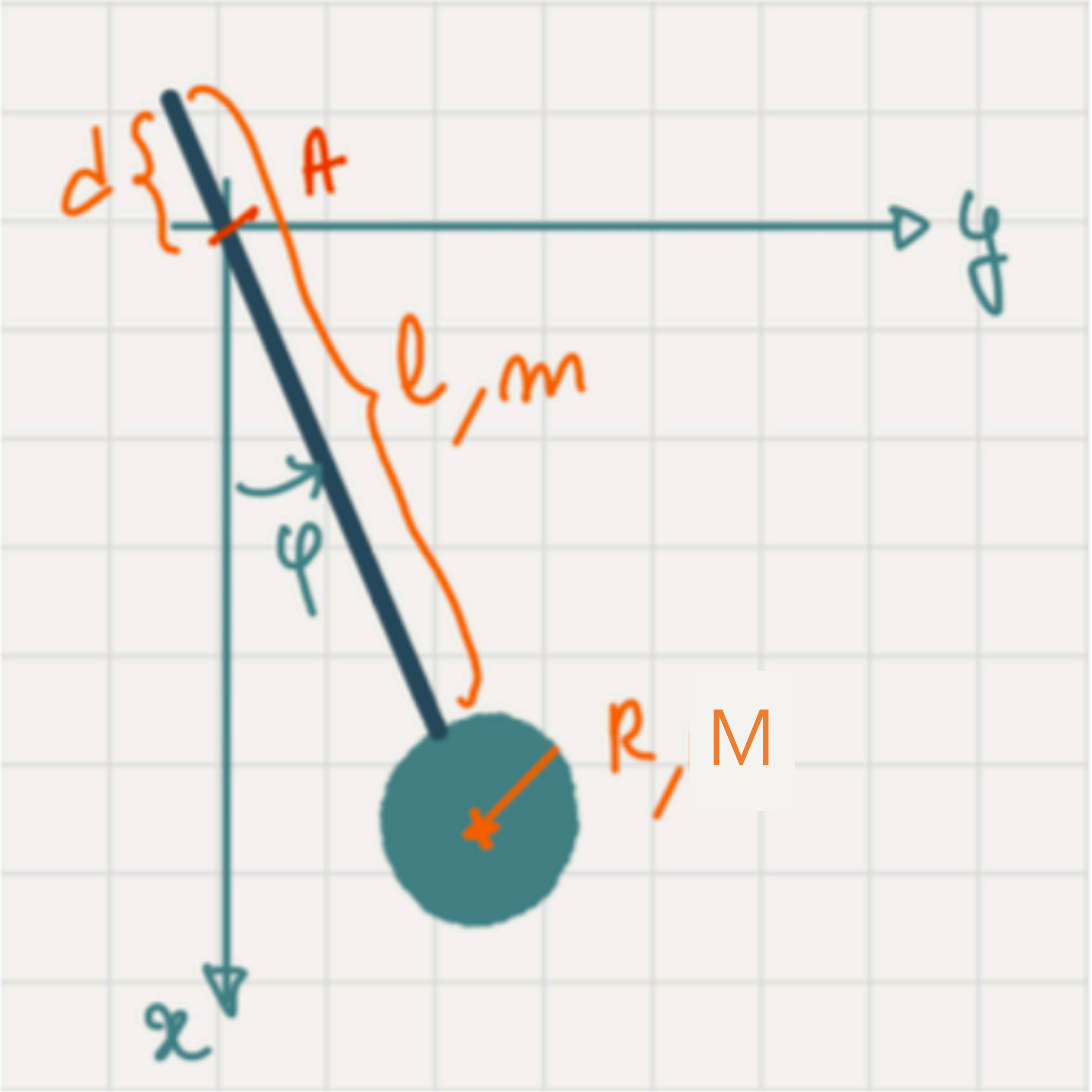

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Establish the differential equation of motion and solve it in the case of small oscillations.

Step 1: find the **CoM**

Step 2: calculate the **moment of inertia**

Step 3: find an expression for the **torque** acting on the system

Step 4: use the **theory of angular momentum**

Sanity check: see that we recover the normal equation of motion for a pendulum if d=0, m=0, R=0