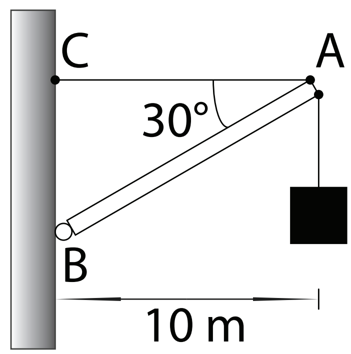
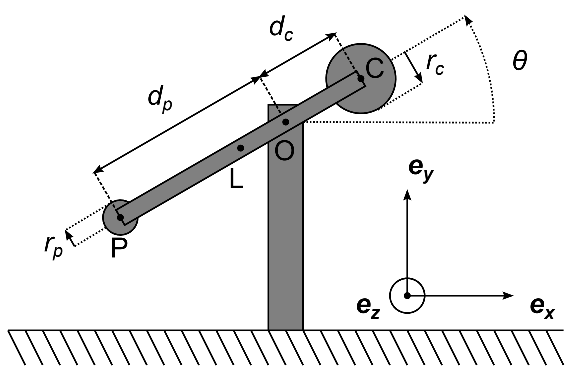
Exercises

Exercise 1 *Tense atmosphere in the bistro*
A bistro sign is hung as shown in the opposite diagram. AB is a beam connected to the wall by a pivot at B. AC is a cable that holds the beam and the sign which is also secured by a cable. The cables are negligible in mass and the mass of the beam and the sign are 15 kg and 300 kg respectively.



Find the forces $\vec{F}_B$ and $\vec{F}_C$ acting on B and C respectively.

Exercise 2 *To be catapulted into the problem*
We propose to study the dynamics of the catapult shown in the figure below.



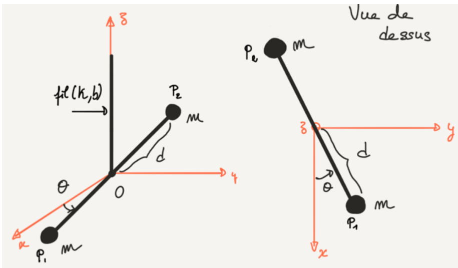
The catapult consists of a lever similar to a thin homogeneous rod of mass m_l attached to a support at point O. The projectile is a solid ball of mass m_p and radius r_p attached to the end P of the lever at a distance d_p from the axis of rotation. A solid ball with mass m_c and radius r_c placed at the other end C at a distance d_c from the axis of rotation serves as a counterweight to operate the catapult. The angle θ is defined as the angle between the horizontal $\vec{e}_x$ and the vector $\overrightarrow{OC}$. We assume that a mechanism ejects the projectile when the angle θ_e reaches the desired value.

1. Place the forces acting on the catapult on the figure.
2. Calculate the moments of inertia relative to the axis of rotation for the projectile (I_p), the counterweight (I_c), and the lever (I_l). From this, deduce the overall moment of inertia I_O of the (projectile+counterweight+lever) system.
3. What condition must be met between m_c, d_p, m_p, d_c, and m_l for the catapult to work ?

4. Give the differential equation on $\theta(t)$ that describes the motion of the catapult using the overall moment of inertia I_O of the system.
5. Give the velocity of the projectile as a function of the launch angle, knowing that the initial angle $\theta(t=0) = \theta_0$ and that the initial angular velocity is zero.

Exercise 3 *Balance ton Cavendish (Exam 2019)*
The Cavendish balance is an instrument used to experimentally determine the gravitational constant G. It consists of two material points P_1 and P_2 of equal mass m connected by a massless rod to a wire, forming a torsion pendulum. Two large spheres of mass M, S_A and S_B, can be placed in such a way as to deflect the pendulum in one direction or the other by the effect of gravity.

Part 1 : Study of the torsion pendulum
The masses P_1 and P_2 are connected by a massless rod of length $2d$, and constrained to rotate around O in the horizontal plane (O, x, y).

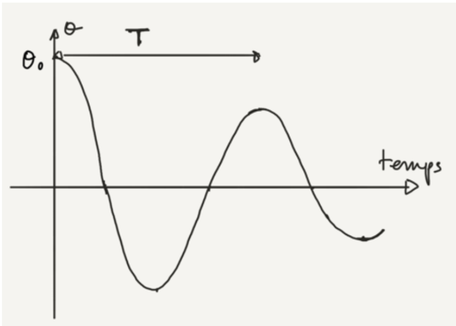


a) Calculate the moment of inertia I_O of the torsion pendulum relative to the axis (Oz).

The wire is characterized by two constants, κ and b, defined as follows :

- the wire exerts an elastic moment depending on the angle of deflection θ, given by $\overrightarrow{M_O^{el}} = -\kappa\theta\vec{e}_z$
- and the internal friction of the wire exerts the torque $\overrightarrow{M_O^{f}} = -b\dot{\theta}\vec{e}_z$

The pendulum is moved from its equilibrium position by an angle θ_0 and released without imparting any angular velocity. The angle of deviation is measured as a function of time, and decreasing oscillations with a pseudo-period T are observed (see schematic).



b) Establish the differential equation of motion on the variable θ.

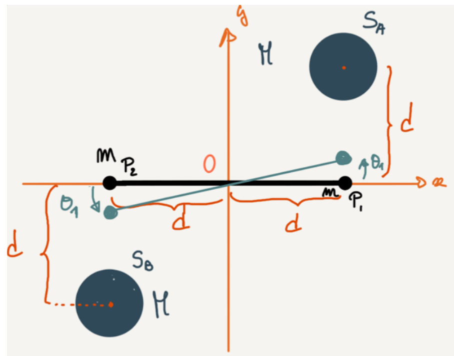
c) What is the natural frequency of the torsion pendulum ?

d) Give the general form of the solution to the differential equation without calculating the integration constants. Explain the pseudo-period and the damping factor based on the data in the problem.

e) Assume very weak damping ($b \approx 0$) and measure T. Determine κ as a function of T, m and d.

Part 2 : Influence of the gravitational force of the two large spheres on the two point masses

The two large spheres (S_A, S_B) of mass M are brought opposite the point masses (P_1, P_2) at a distance d from the axis (Ox), and the pendulum is allowed to balance with the angle of deviation θ_1. We assume that the angle θ_1 is very small ($\theta \ll 1$).



a) Express (vectorially) the moment $\overrightarrow{M_{O,1}^{tot}}$, relative to O on the pendulum, linked to the gravitational force of S_A on P_1 and of S_B on P_2.

b) Express (vectorially) the moment $\overrightarrow{M_{O,2}^{tot}}$ related to the gravitational force of S_A on P_2 and of S_B on P_1.

c) Show that for an order-of-magnitude calculation, we can neglect $\|\overrightarrow{M_{O,2}^{tot}}\|$ compared to $\|\overrightarrow{M_{O,1}^{tot}}\|$.

d) Express the angle θ_1 at equilibrium as a function of G, M, m, d, and κ.

e) Deduce the expression for G as a function of M, m, d, T and θ_1, quantities that are known or easily measurable.

f) Supplementary question (not part of the exam) : Use the data from the experiment to estimate the order of magnitude of G. The period T is 8 minutes, $M = 1.5$ kg, $m = 15$ g, $d = 5$ cm, and we measure θ_1 using the deviation of the laser beam, which is 20 cm over the 13.5 m of the lecture hall.