PHYS-101 WEEK 14

EPFL

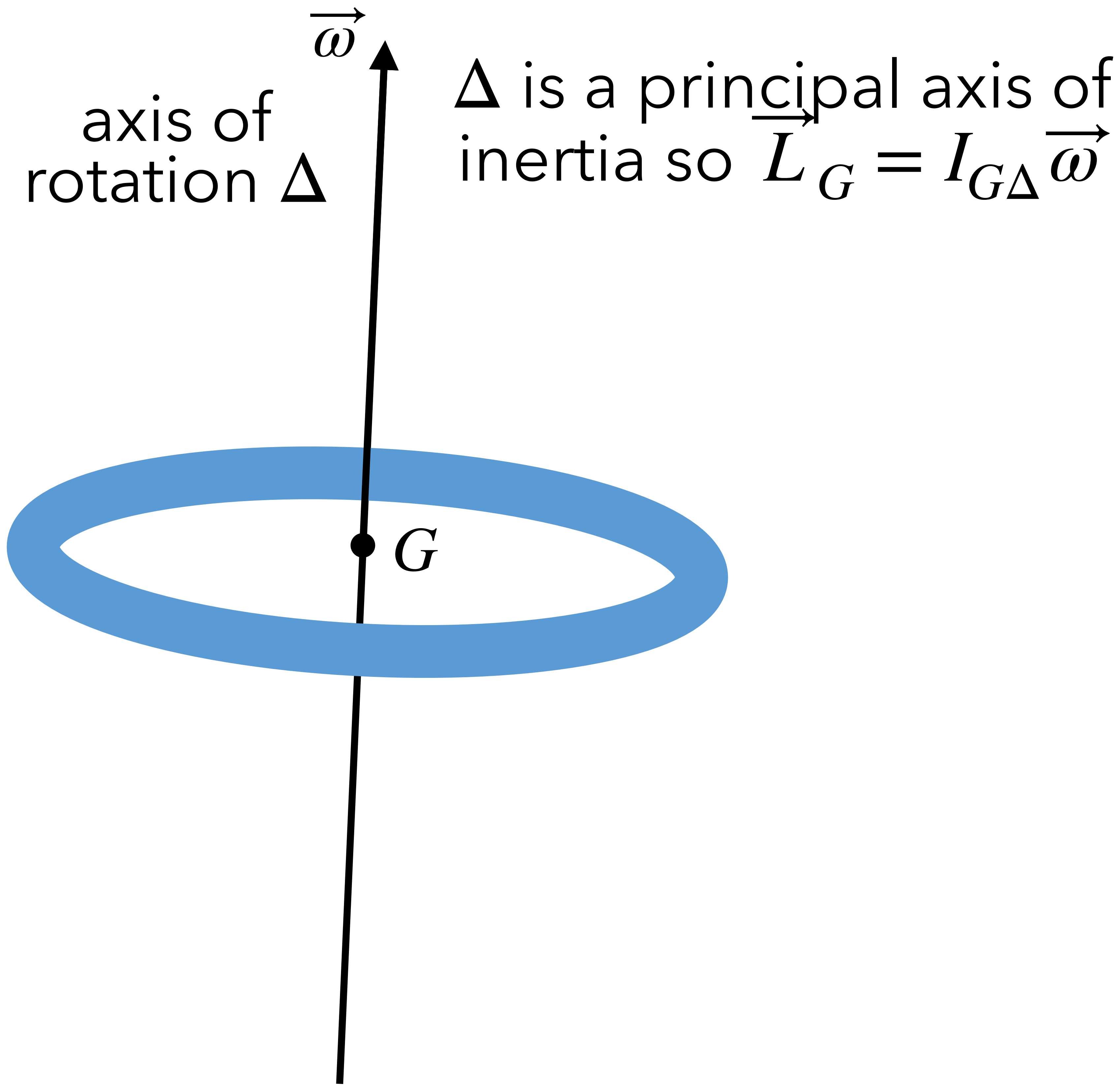
Physique générale : mécanique (classe inversée en anglais)

Prof. Emma Tolley, 8 December 2025

APPLICATIONS OF RIGID BODY MECHANICS

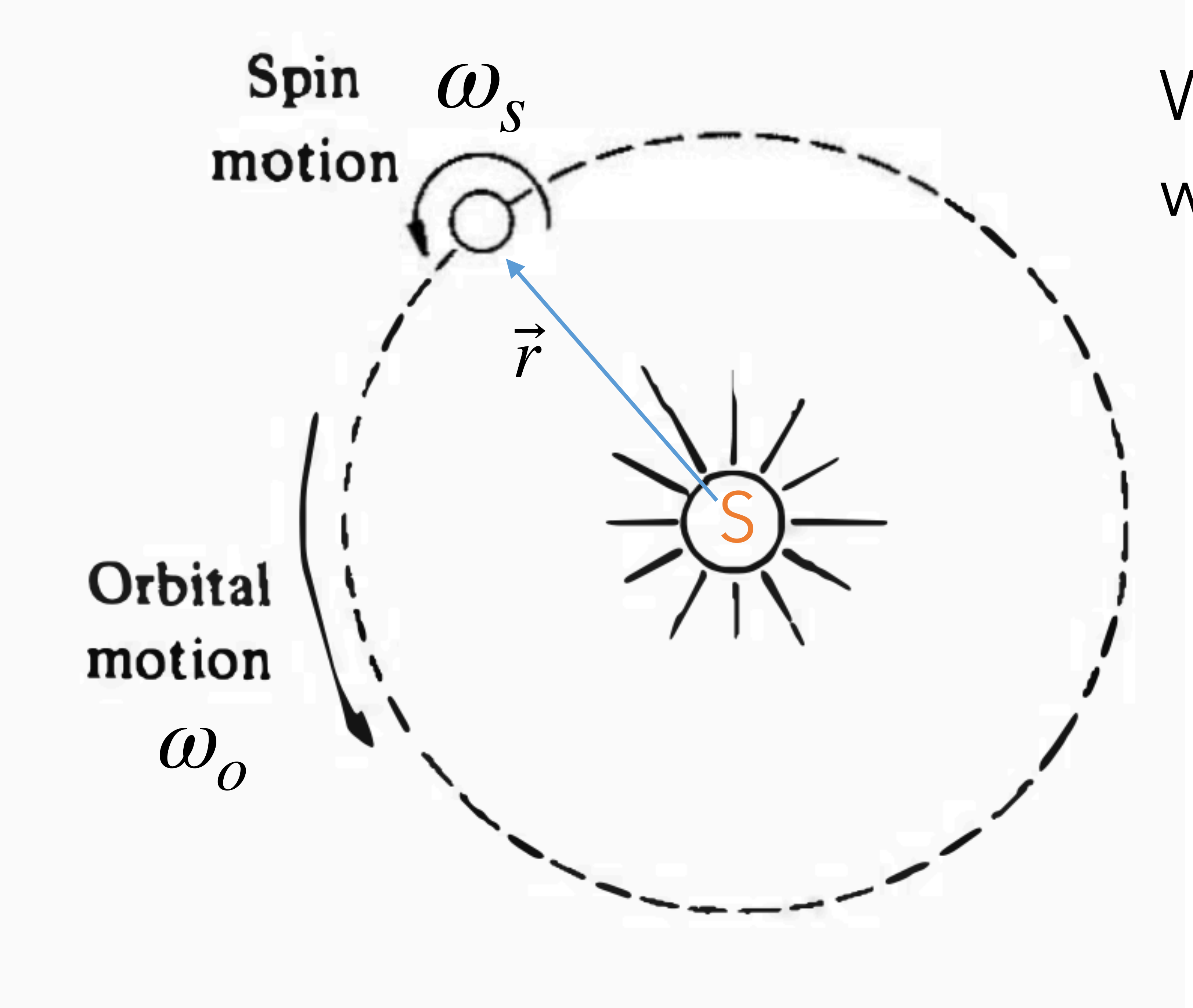
Theory of angular momentum for a point A on the object which is: fixed, the CoM, or moving collinear to the CoM

$$\sum \overrightarrow{M_A} = \frac{d\overrightarrow{L_A}}{dt}$$



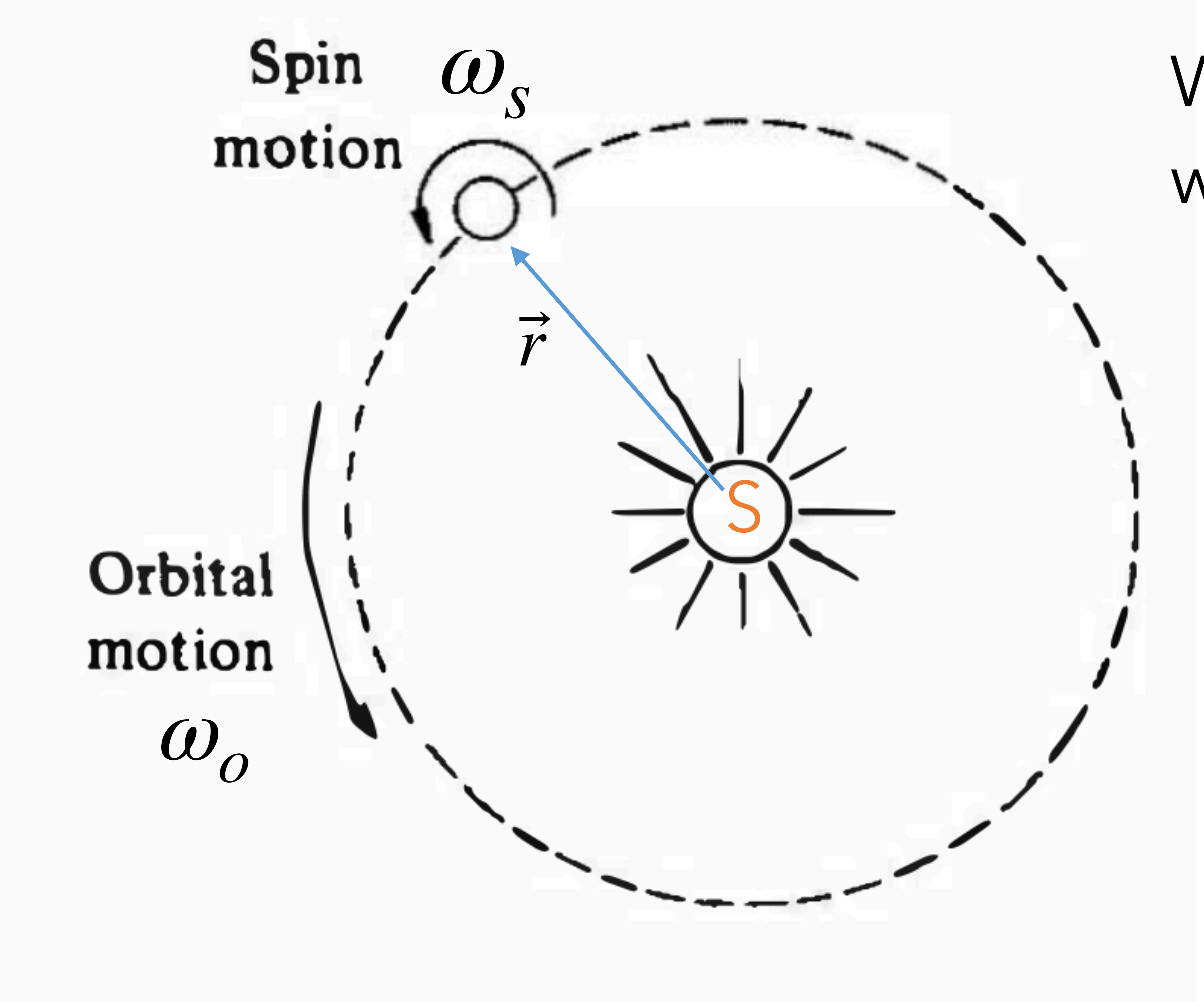
THE ANAGYRE/RATTLEBACK/CELT

APPLICATIONS OF RIGID BODY MECHANICS



What's the total angular momentum of the Earth with mass M with respect to the Sun?

APPLICATIONS OF RIGID BODY MECHANICS

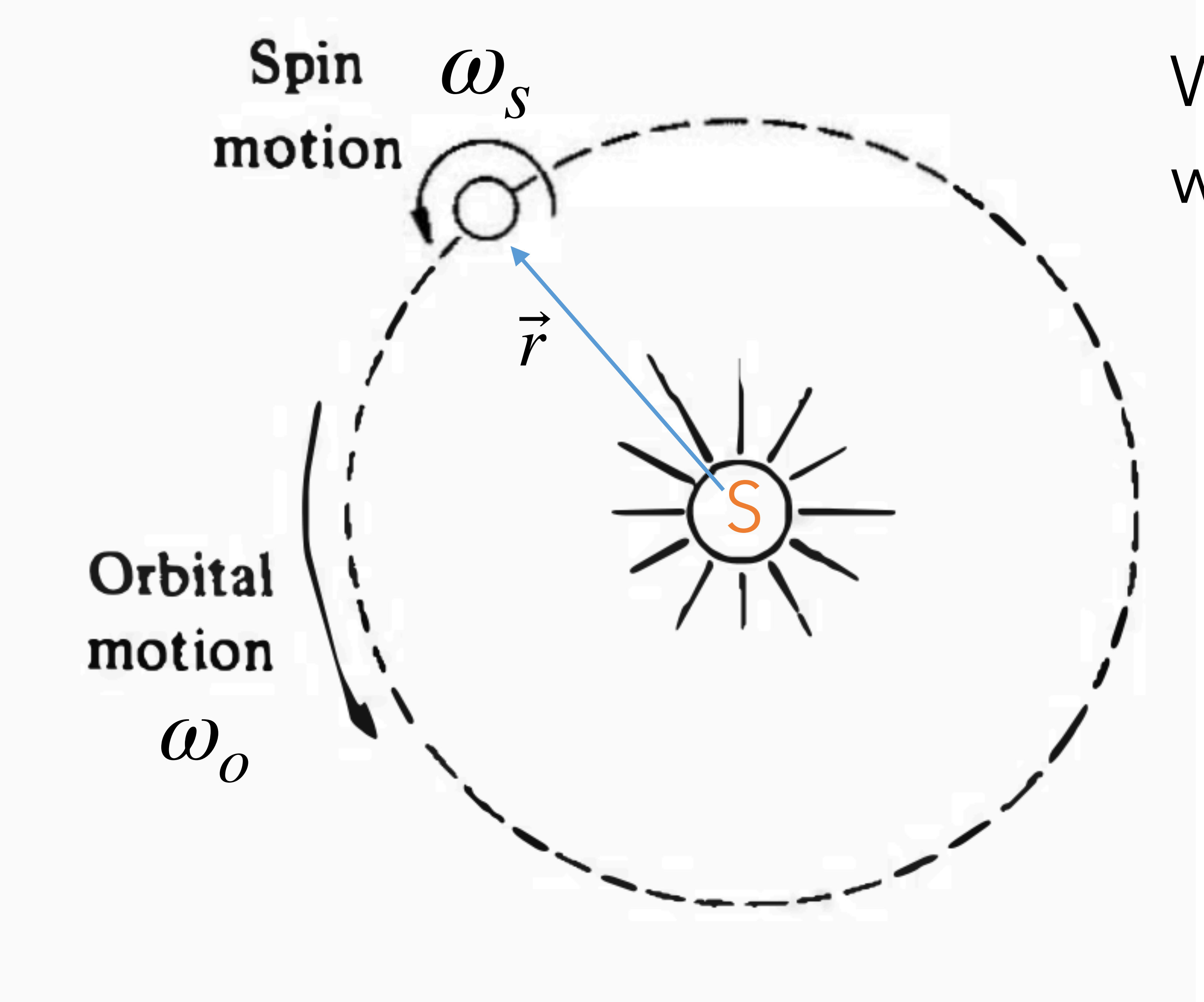


What's the total angular momentum of the Earth with mass M with respect to the Sun?

$$\vec{L}_{\text{total}} = \vec{L}_s + \vec{L}_o$$

Sum angular momentum around the center of mass $\vec{L}_s$ with the orbital angular momentum $\vec{L}_o$

APPLICATIONS OF RIGID BODY MECHANICS



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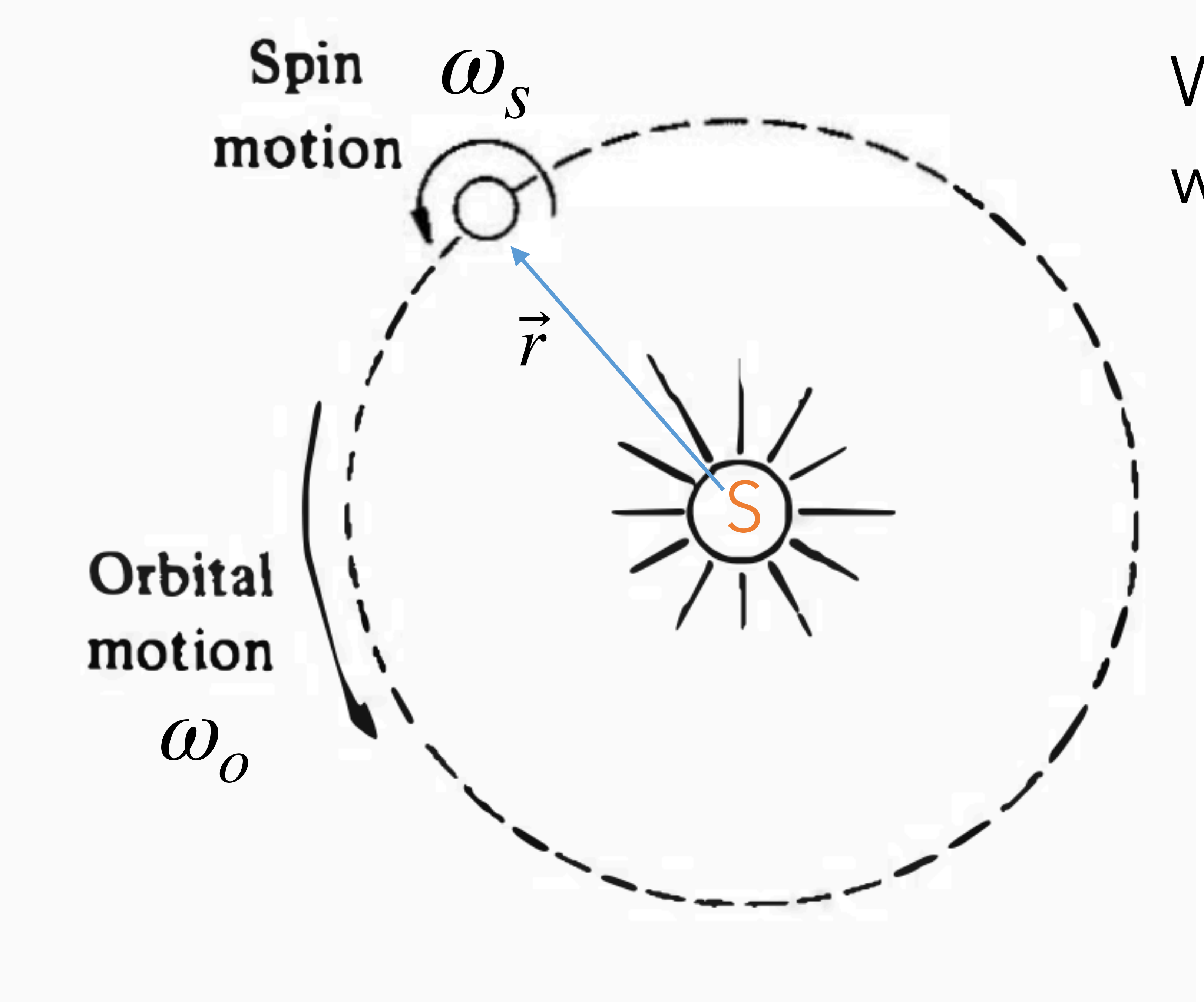
APPLICATIONS OF RIGID BODY MECHANICS



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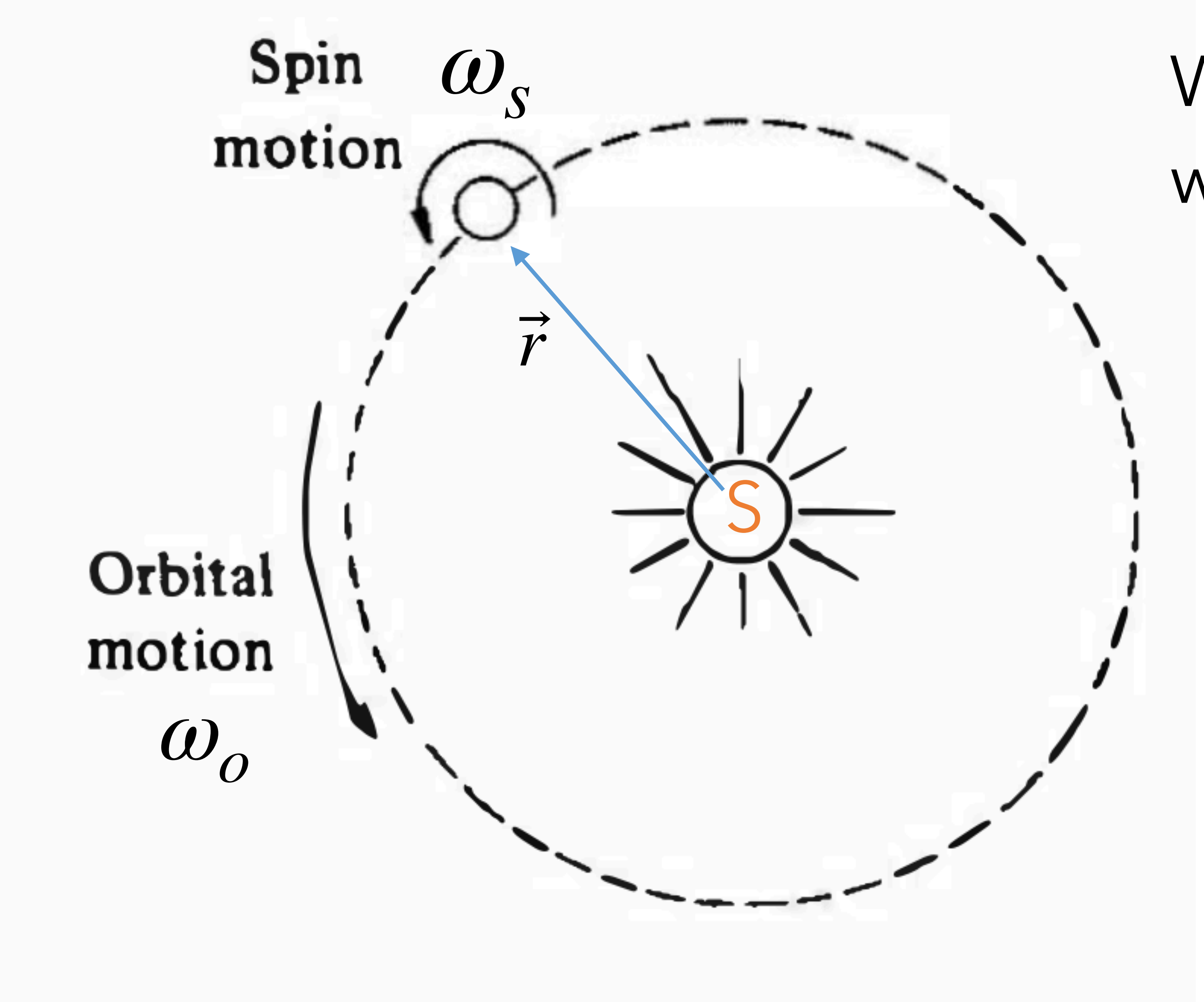
Circular motion implies: $\vec{v} = \vec{\omega}_o \wedge \vec{r}$ (Eq. 1.30 in Course Notes)

Use $\vec{r} = r\vec{e}_\rho, \quad \vec{\omega}_o = \omega_o \vec{e}_z$

APPLICATIONS OF RIGID BODY MECHANICS



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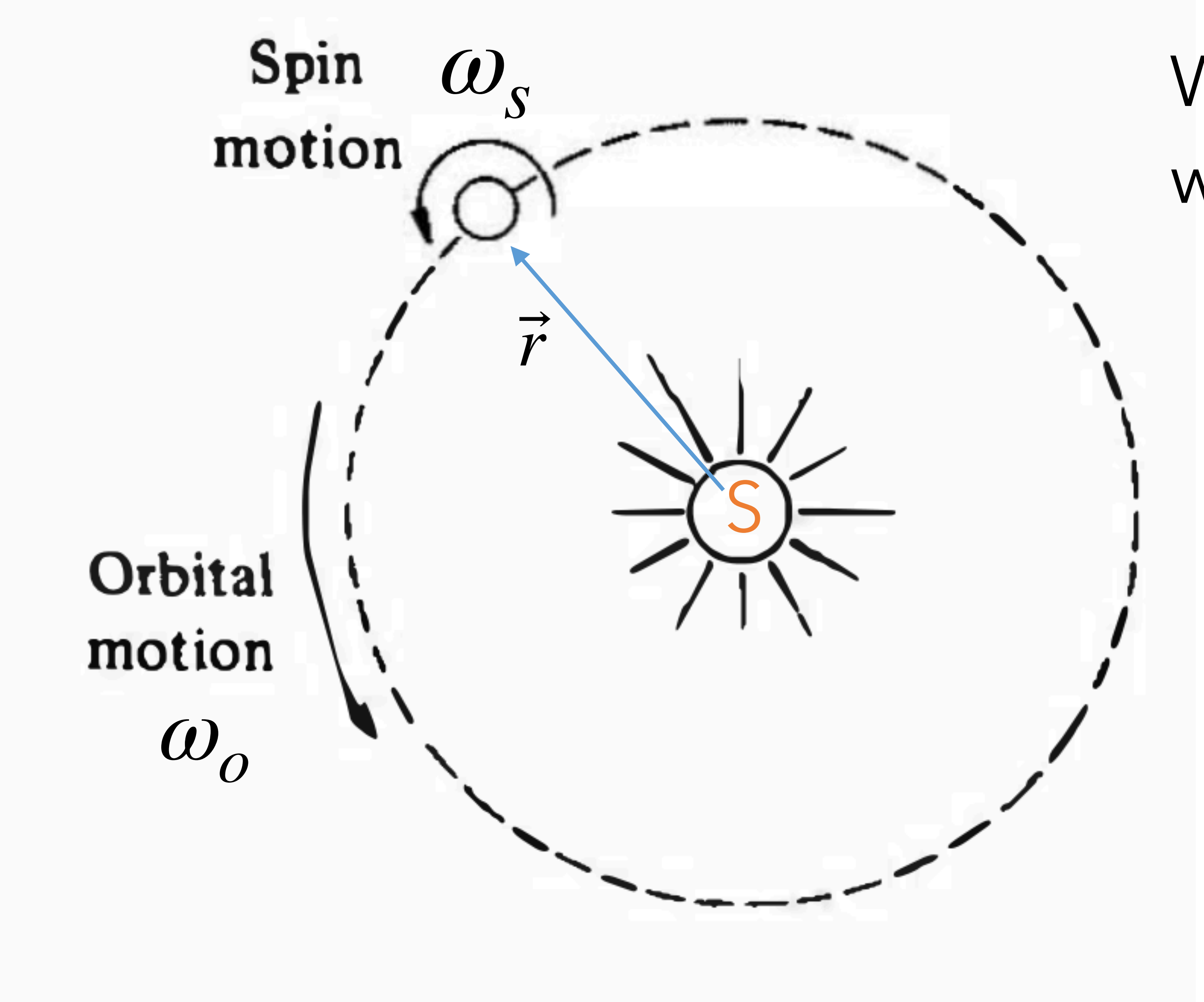
$$\vec{r} \wedge M\vec{v} = (r\vec{e}_\rho) \wedge M(\omega_o\vec{e}_z \wedge r\vec{e}_\rho) = Mr^2\vec{\omega}_o e_z = Mr^2\vec{\omega}_o$$

APPLICATIONS OF RIGID BODY MECHANICS



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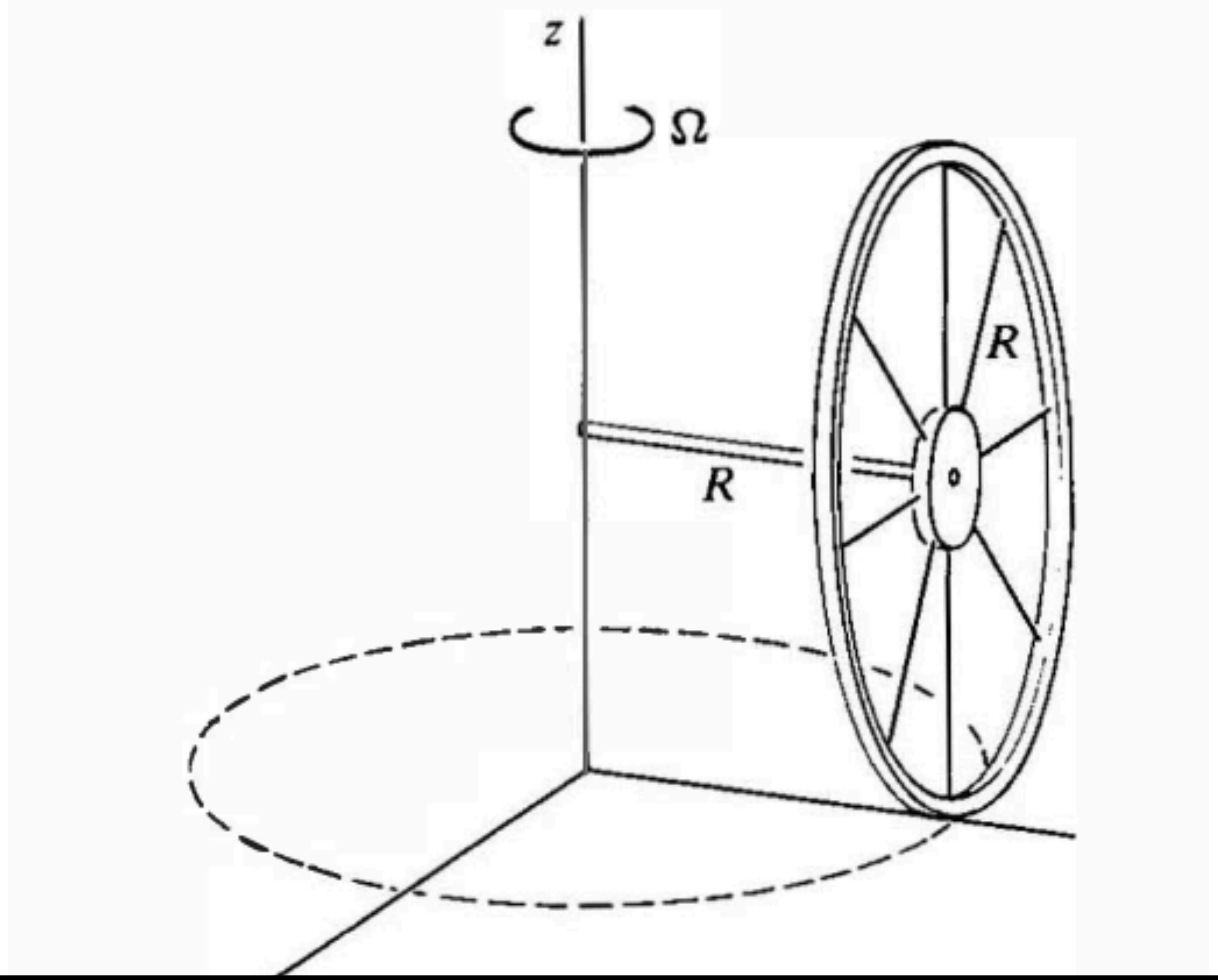
If $\omega_o = \omega_s$, then $\vec{L}_{\text{total}} = I_0\vec{\omega}_s + Mr^2\vec{\omega}_s = (I_0 + Mr^2)\vec{\omega}_s$

EXERCISE: WHEEL



A thin hoop of mass M and radius R rolls without slipping about the z axis. It is supported by an axle of length R through its center, as shown. The hoop circles around the z axis with angular speed $\mathbf{\Omega}$.

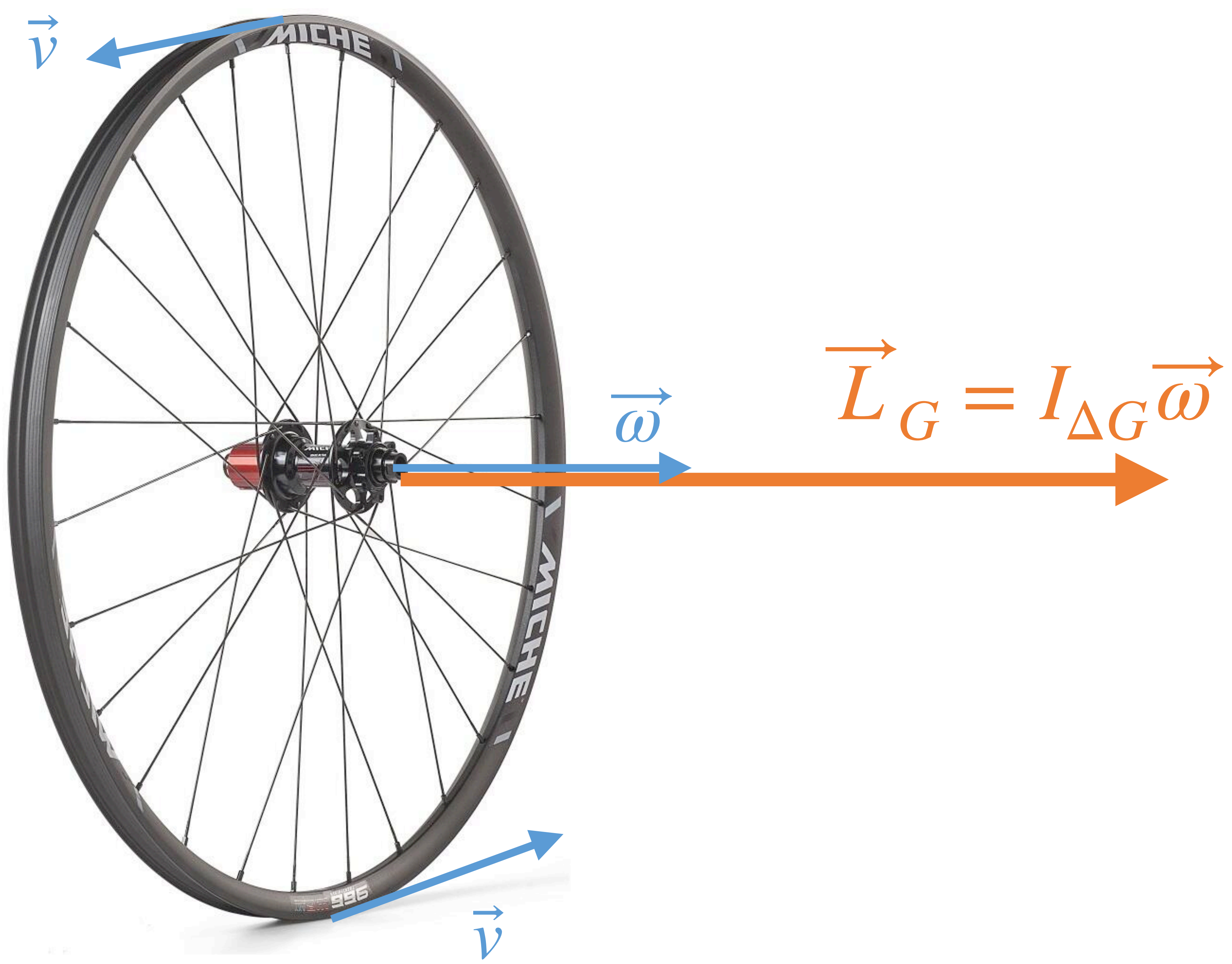
A) What is the total angular velocity ω of the hoop?

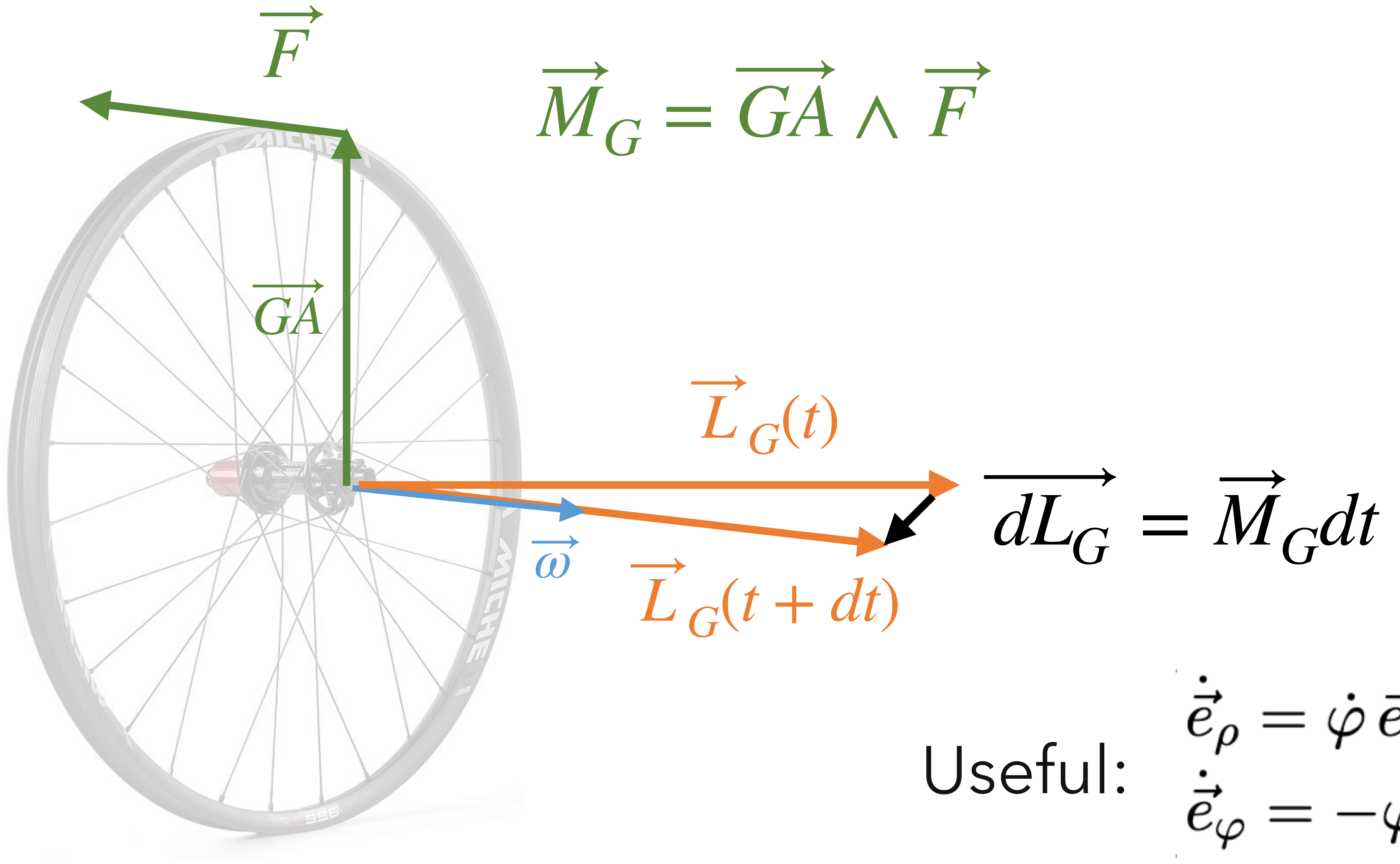
B) What is the angular momentum L of the hoop? Is L parallel to ω? (Note: the moment of inertia of a hoop for an axis along its diameter is $\frac{1}{2}MR^2$)

GYROSCOPES

$\vec{v}$
$\vec{\omega}$
$\overrightarrow{L}_G = I_{\Delta G}\overrightarrow{\omega}$
$\vec{v}$
$\vec{v}$
$\overrightarrow{L}_G(t)$
$\overrightarrow{dL_G} = \overrightarrow{M}_G dt$
$\overrightarrow{\omega}$
$\overrightarrow{L}_G(t + dt)$
$\vec{v}$

GYROSCOPES

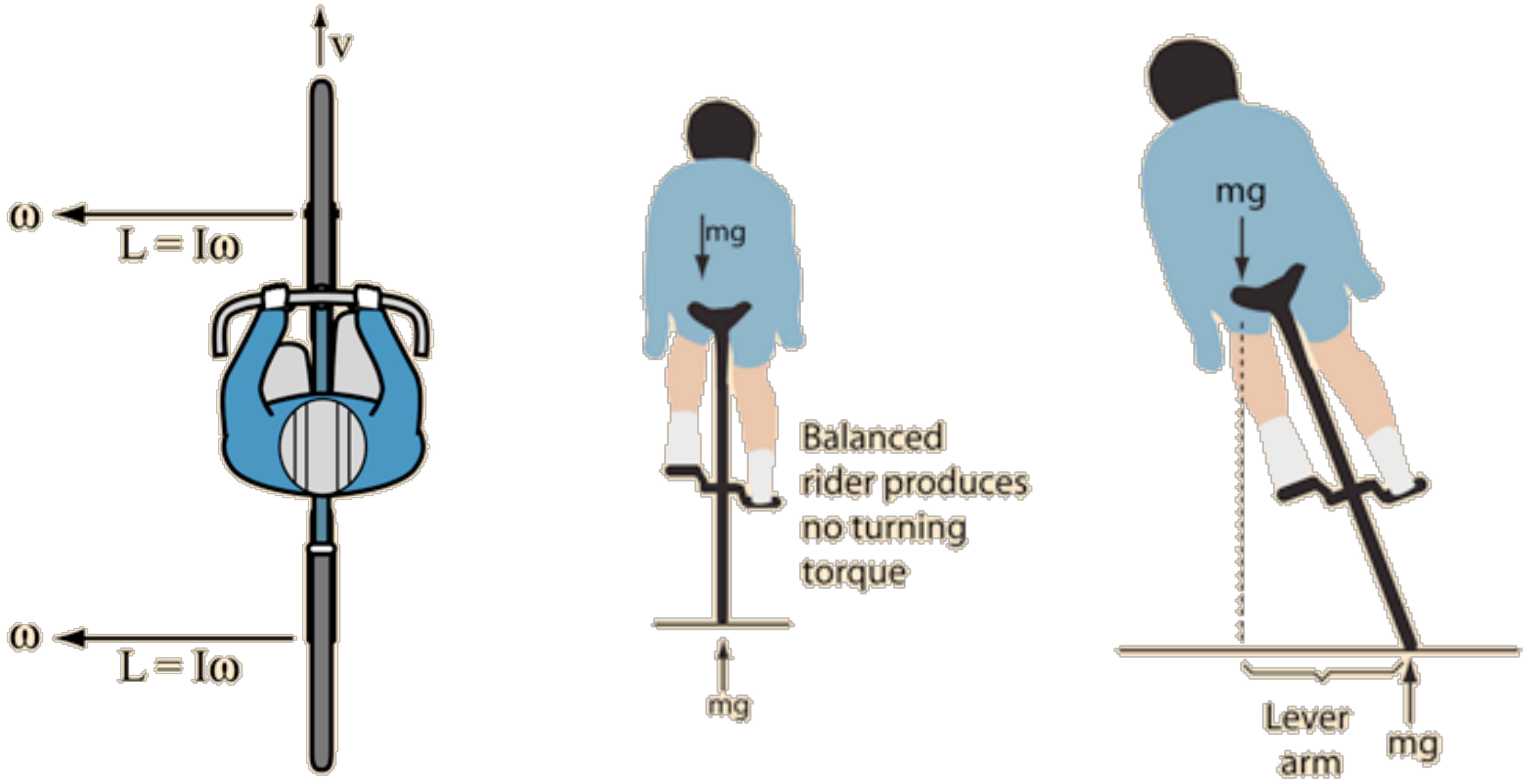




Useful:

$$\dot{\vec{e}}_\rho = \dot{\varphi}\,\vec{e}_\varphi$$
$$\dot{\vec{e}}_\varphi = -\dot{\varphi}\,\vec{e}_\rho$$

BICYCLE AS A GYROSCOPE



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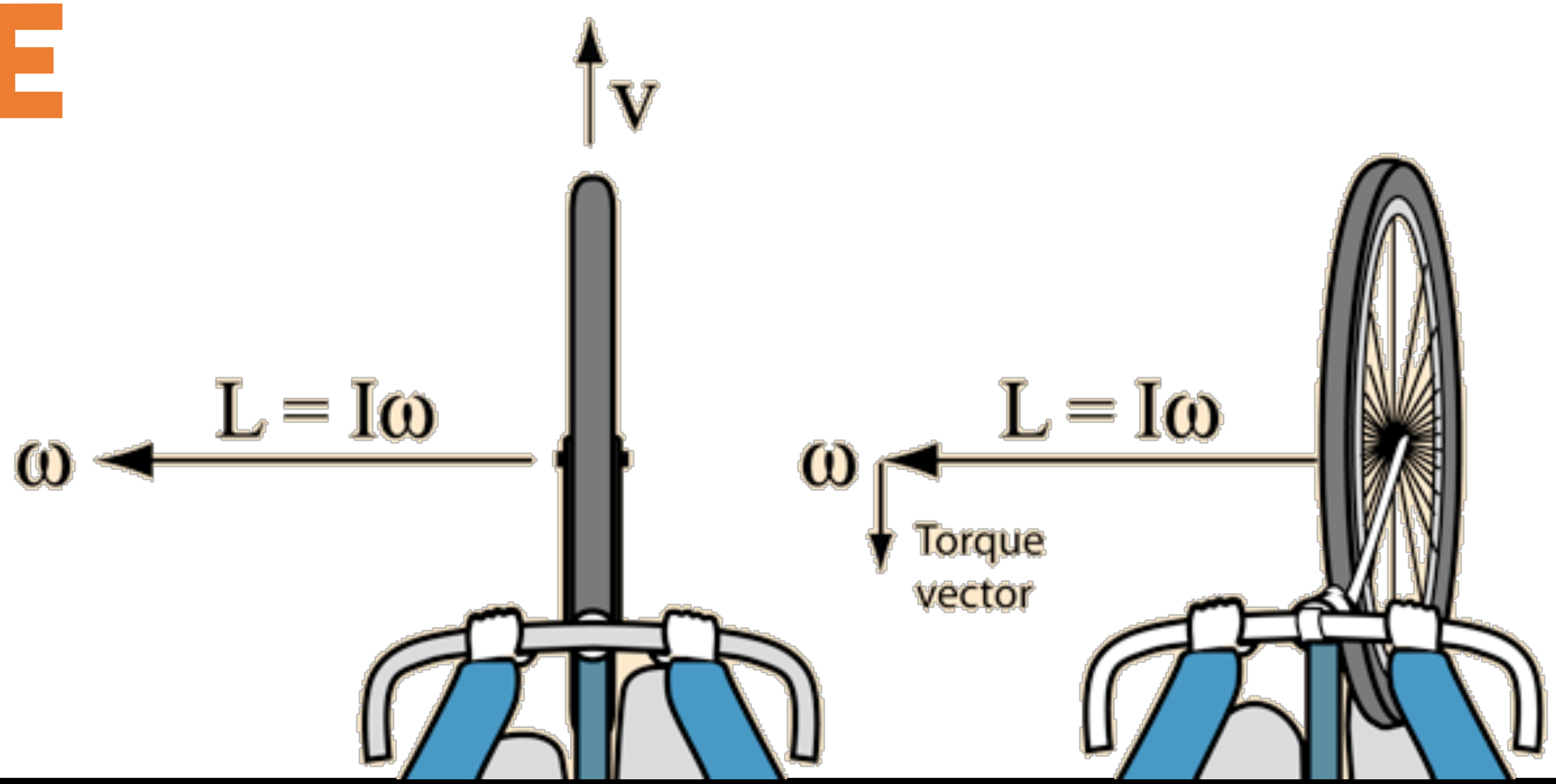
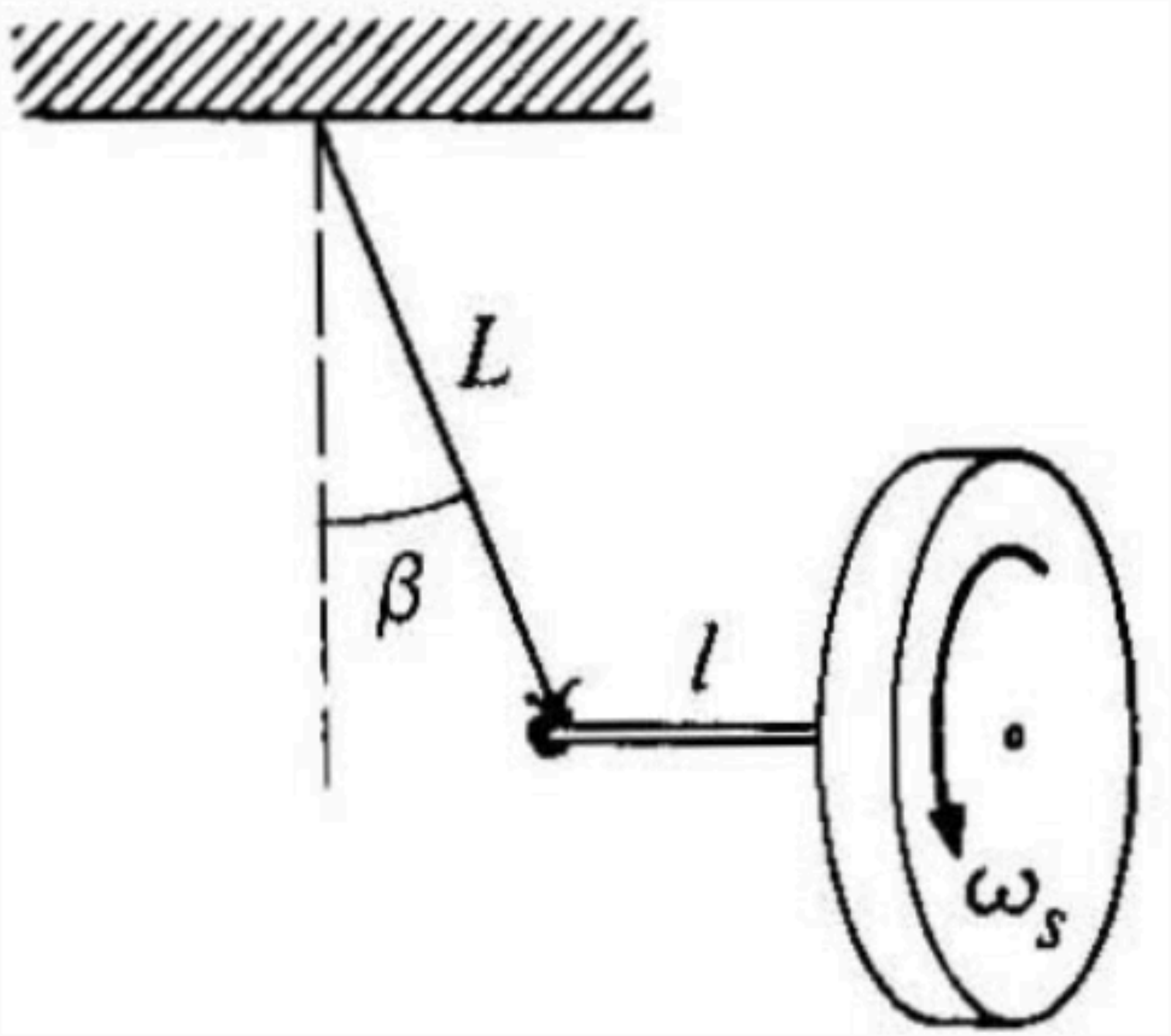


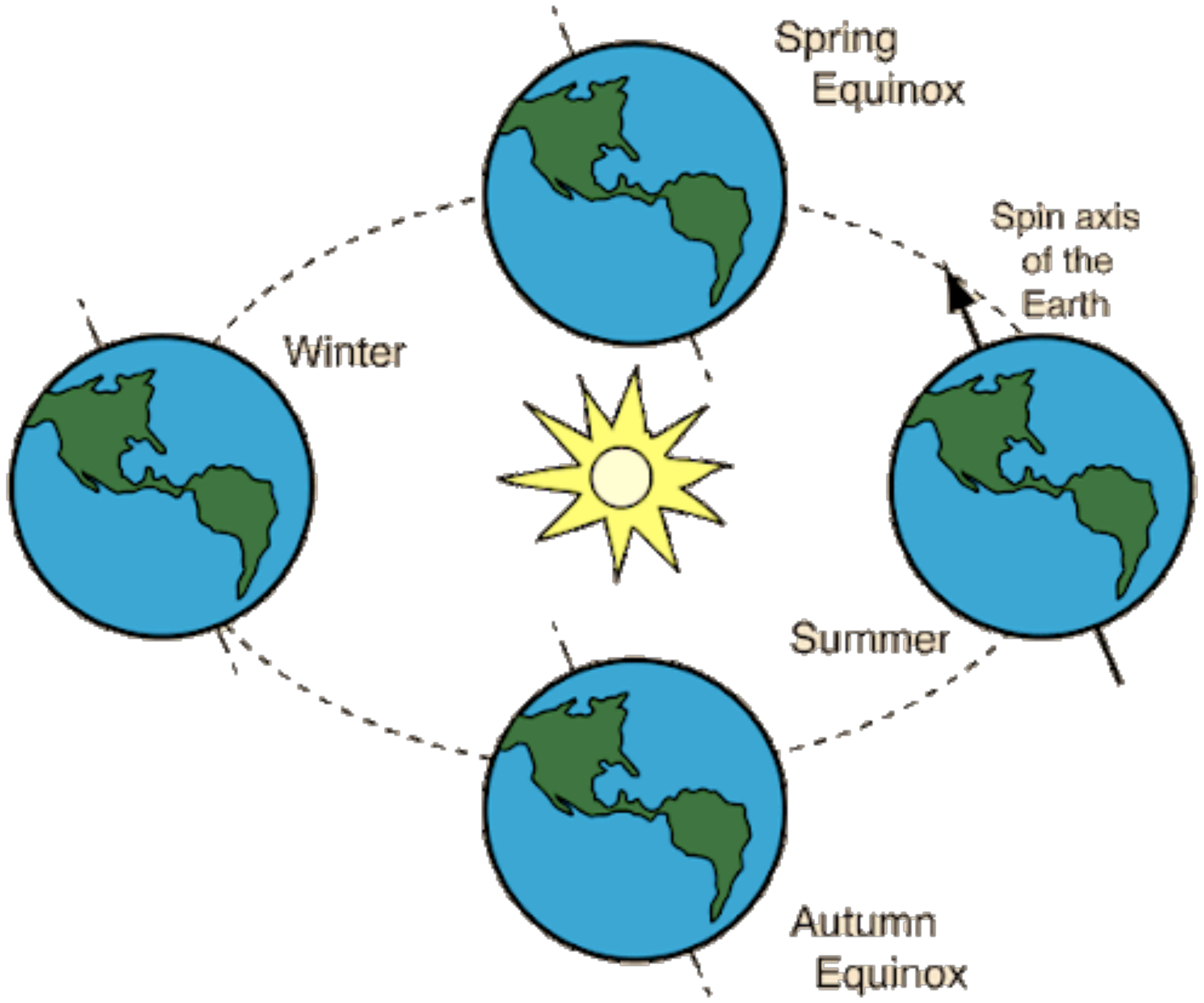
EXERCISE: GYROSCOPE ON A WIRE



A gyroscope wheel is at one end of an axle of length l. The other end of the axle is suspended from a string of length L. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its center of mass I_G, Its spin angular velocity is ω_s. Neglect the mass of the shaft and of the string.

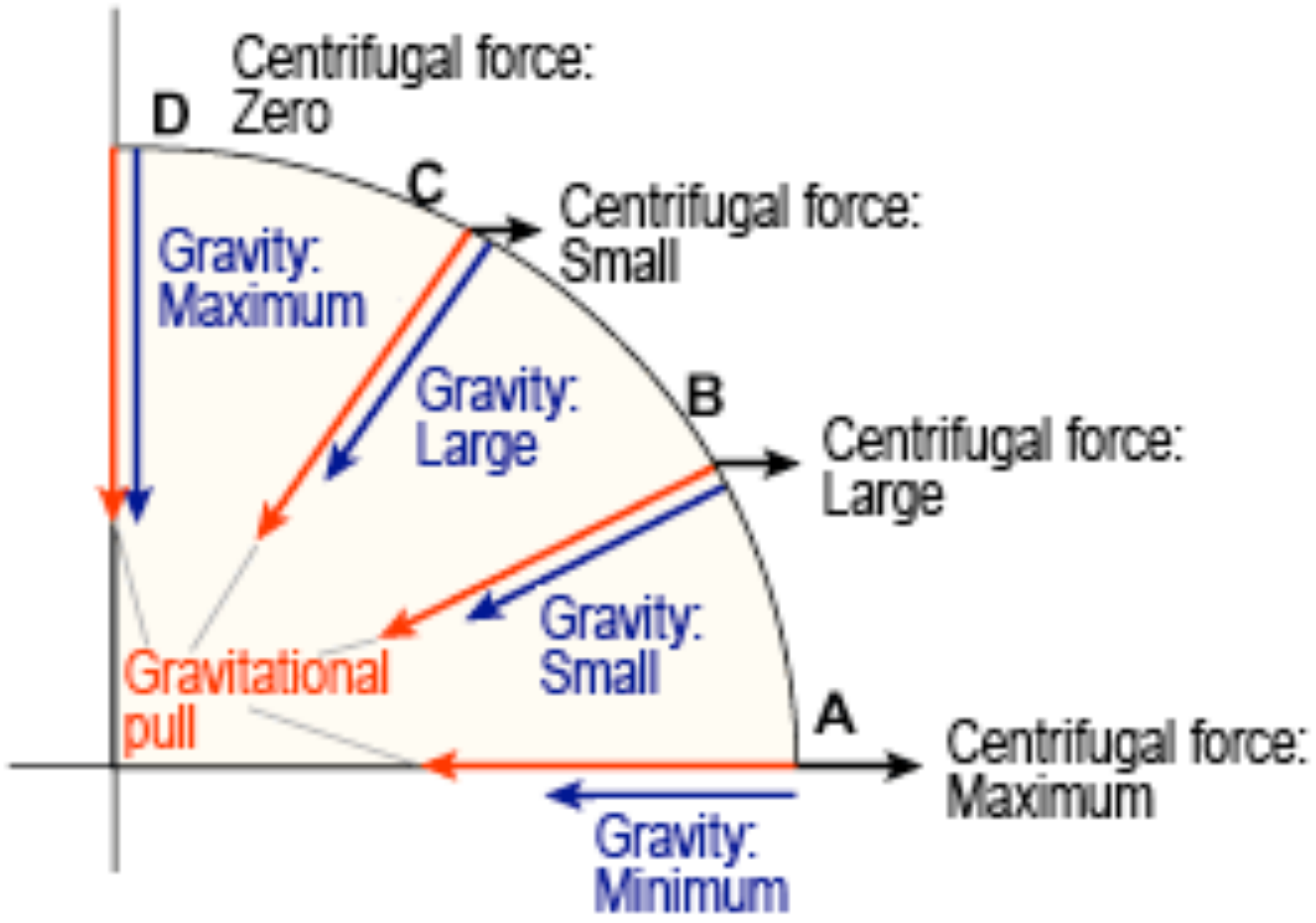
Find the angle β that the string makes with the vertical. Assume that β is so small that approximations like $\sin\beta \approx \beta$ are justified, and that the precession angular velocity is much smaller than ω_s

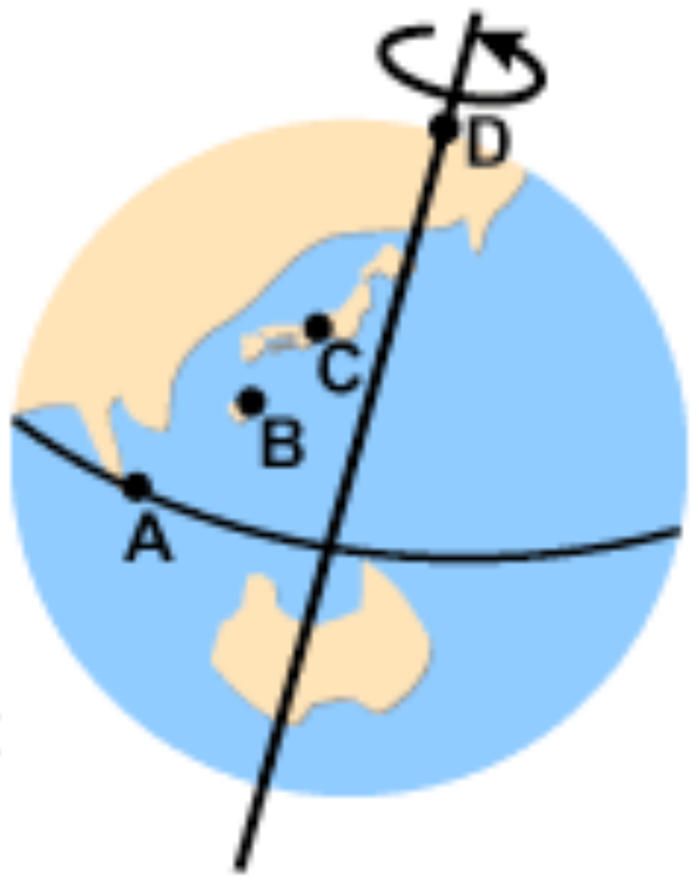
ANGULAR MOMENTUM OF THE EARTH

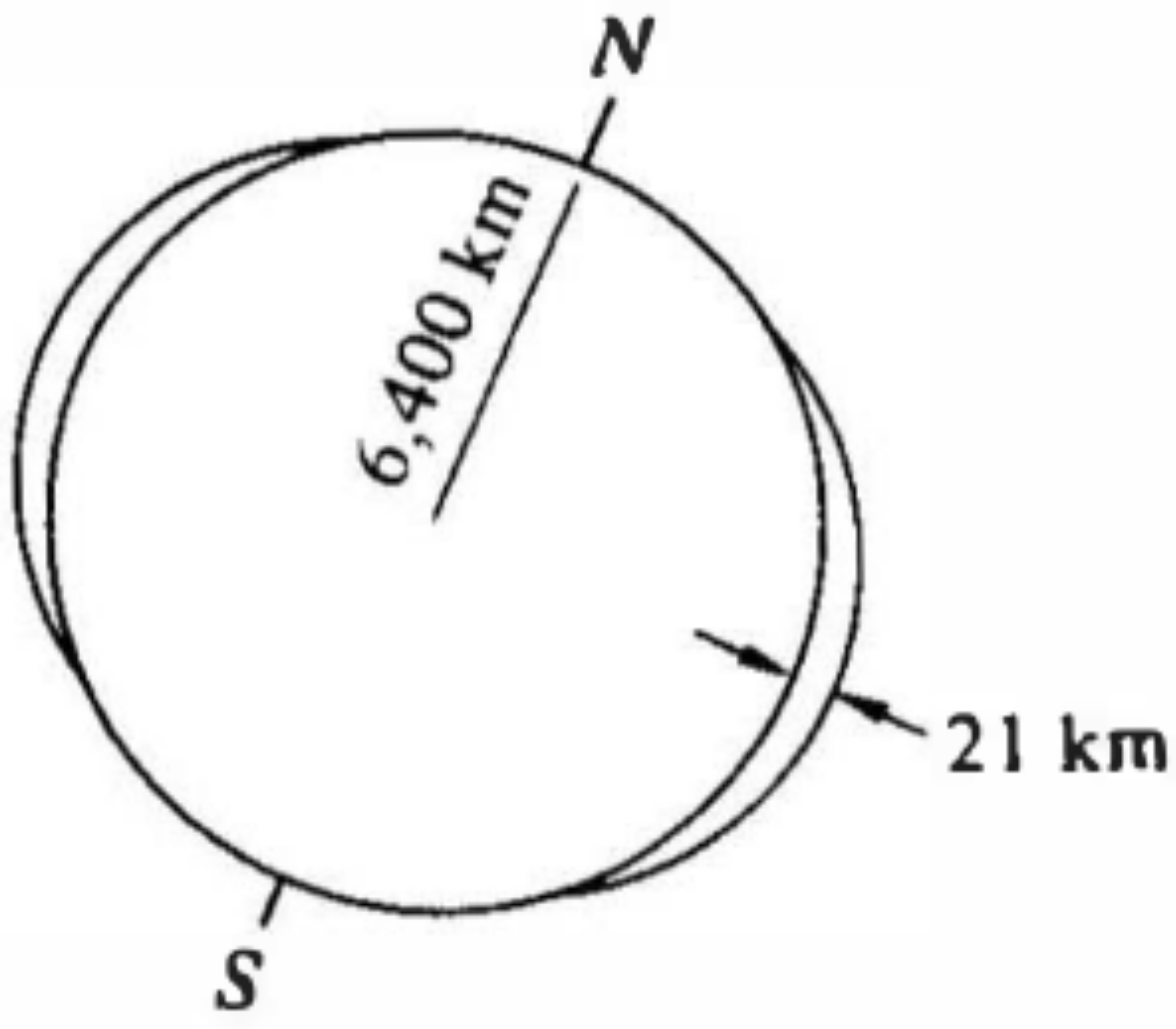



Matt Molloy

ANGULAR MOMENTUM OF THE EARTH







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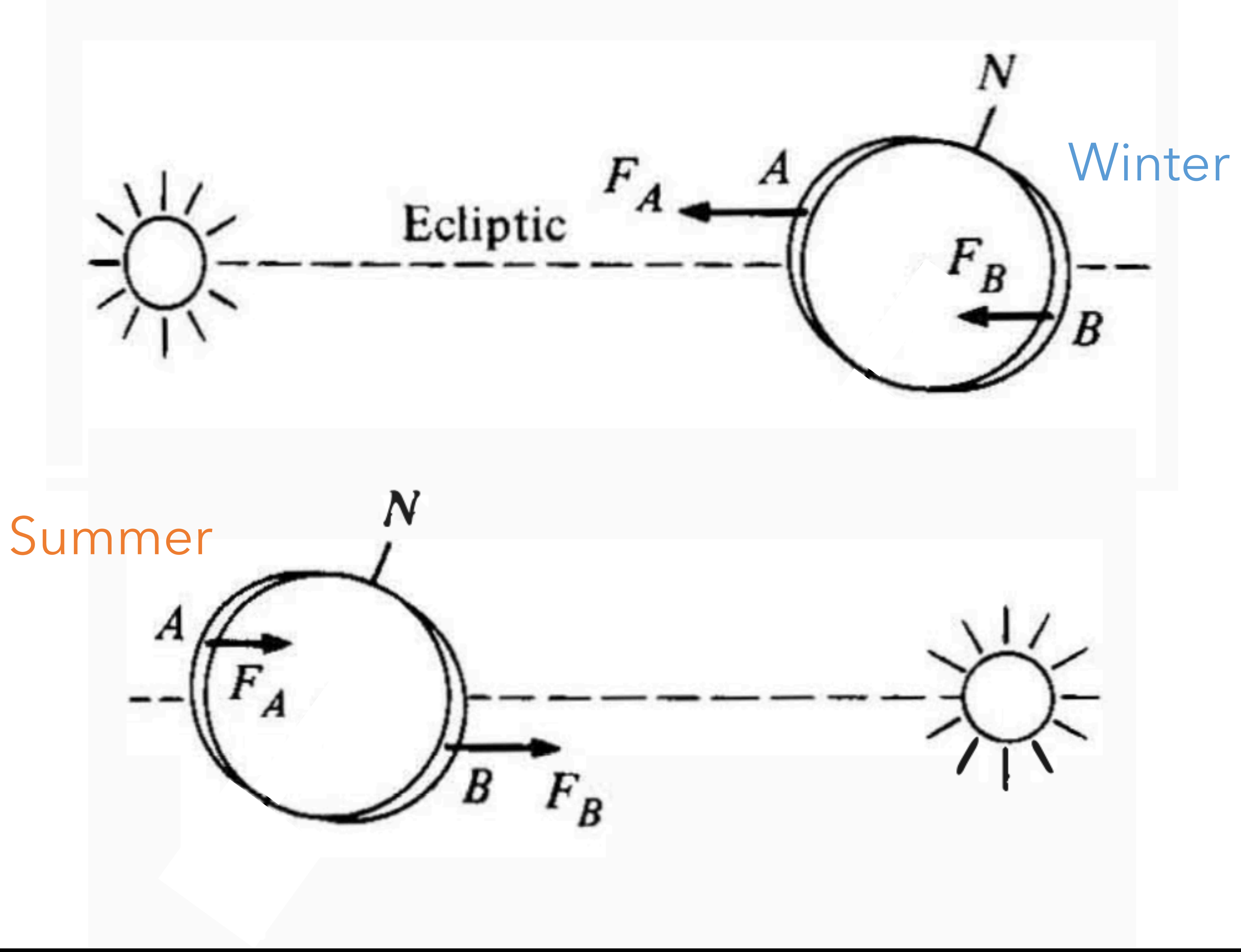

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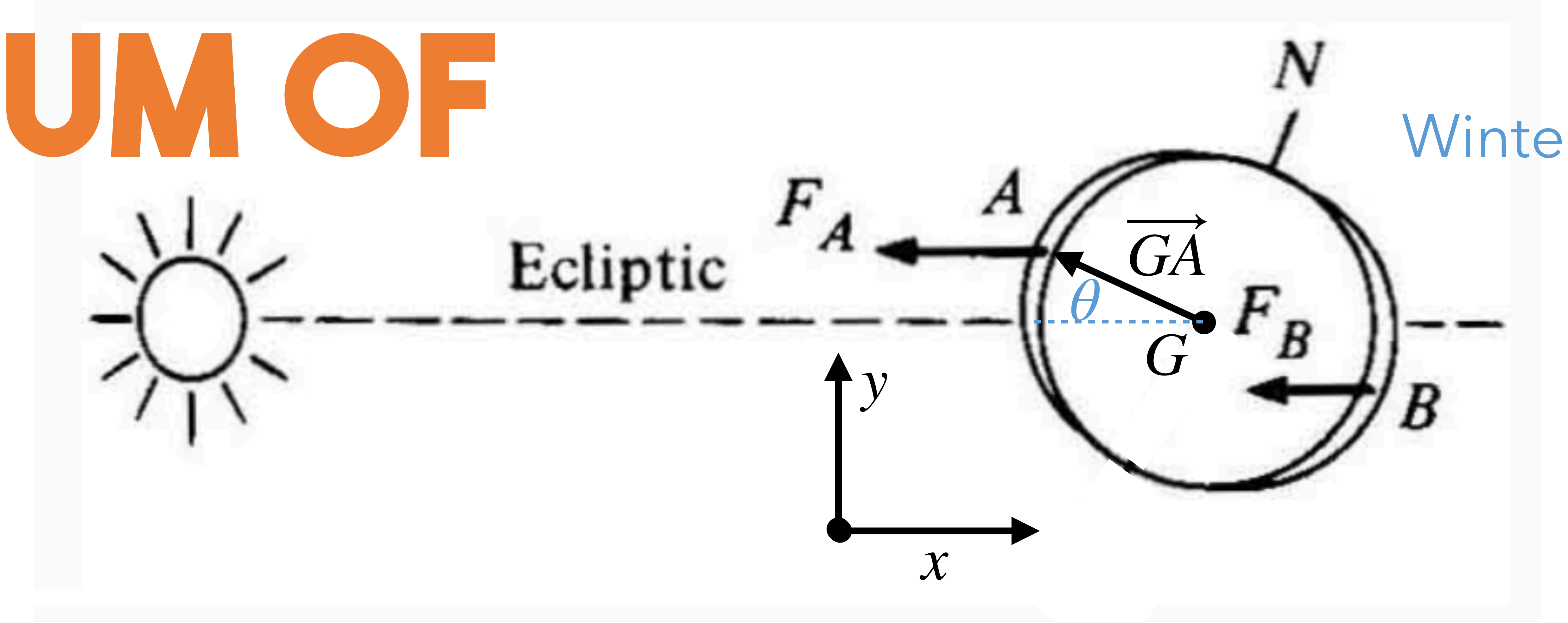

Forces

$$\overrightarrow{F}_A = -F_A\vec{e}_x$$

$$\overrightarrow{F}_B = -F_B\vec{e}_x$$

$$F_A > F_B$$

Lever arm

$$\overrightarrow{GA} = -\overrightarrow{GB} = R(-\cos\theta\vec{e}_x + \sin\theta\vec{e}_y)$$

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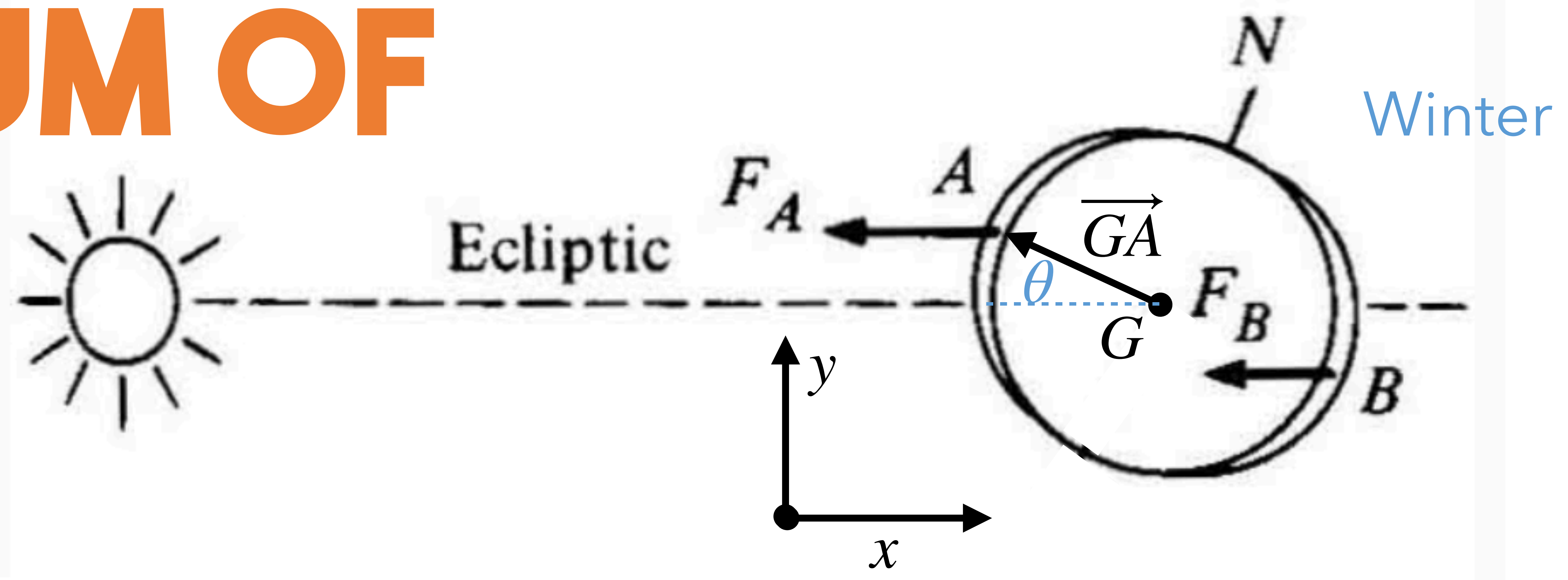

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Torque

$$\vec{M}_G = \overrightarrow{GA} \wedge \vec{F}_A + \overrightarrow{GB} \wedge \vec{F}_B$$

$$= \overrightarrow{GA} \wedge \vec{F}_A - \overrightarrow{GA} \wedge \vec{F}_B$$

$$= \overrightarrow{GA} \wedge (\vec{F}_A - \vec{F}_B)$$

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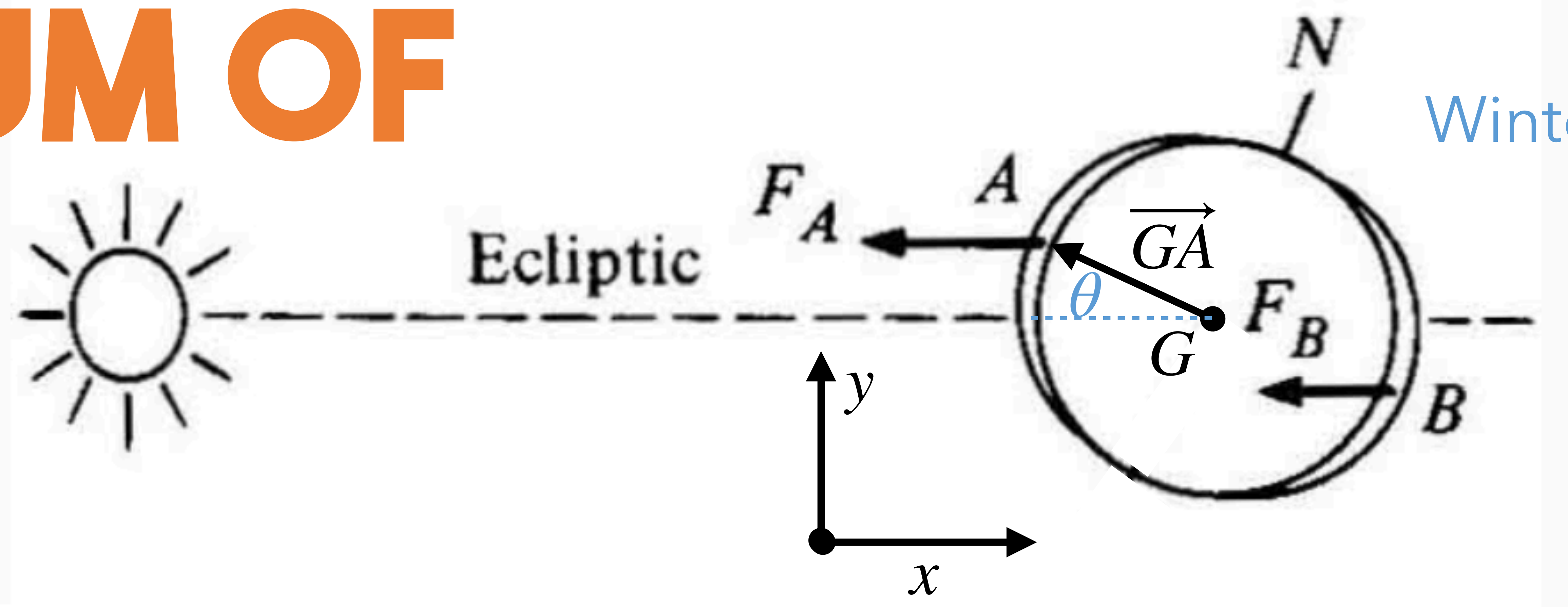

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$$= \overrightarrow{GA} \wedge \overrightarrow{F}_A - \overrightarrow{GA} \wedge \overrightarrow{F}_B$$

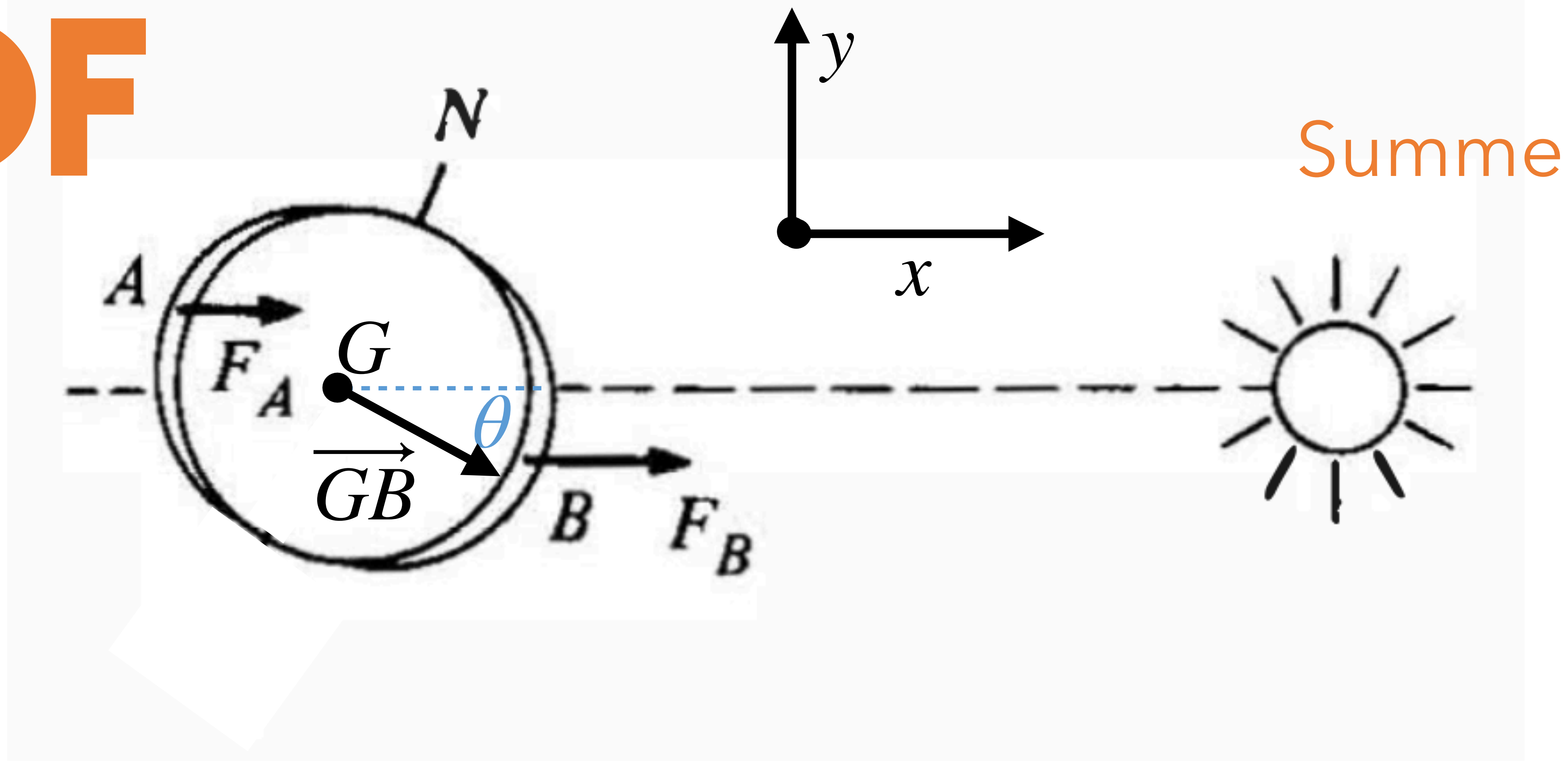
$$= \overrightarrow{GA} \wedge (\overrightarrow{F}_A - \overrightarrow{F}_B) = \overrightarrow{GA} \wedge \vec{e}_x(-F_A + F_B)$$

$$= R\ (F_B - F_A)\ \sin\theta\ \vec{e}_y \wedge \vec{e}_x$$

$$\overrightarrow{M}_G = R\ (F_A - F_B)\ \sin\theta\ \vec{e}_z$$

Points in positive z direction

ANGULAR MOMENTUM OF THE EARTH



Summer

Forces

$$\vec{F}_A = F_A \vec{e}_x$$

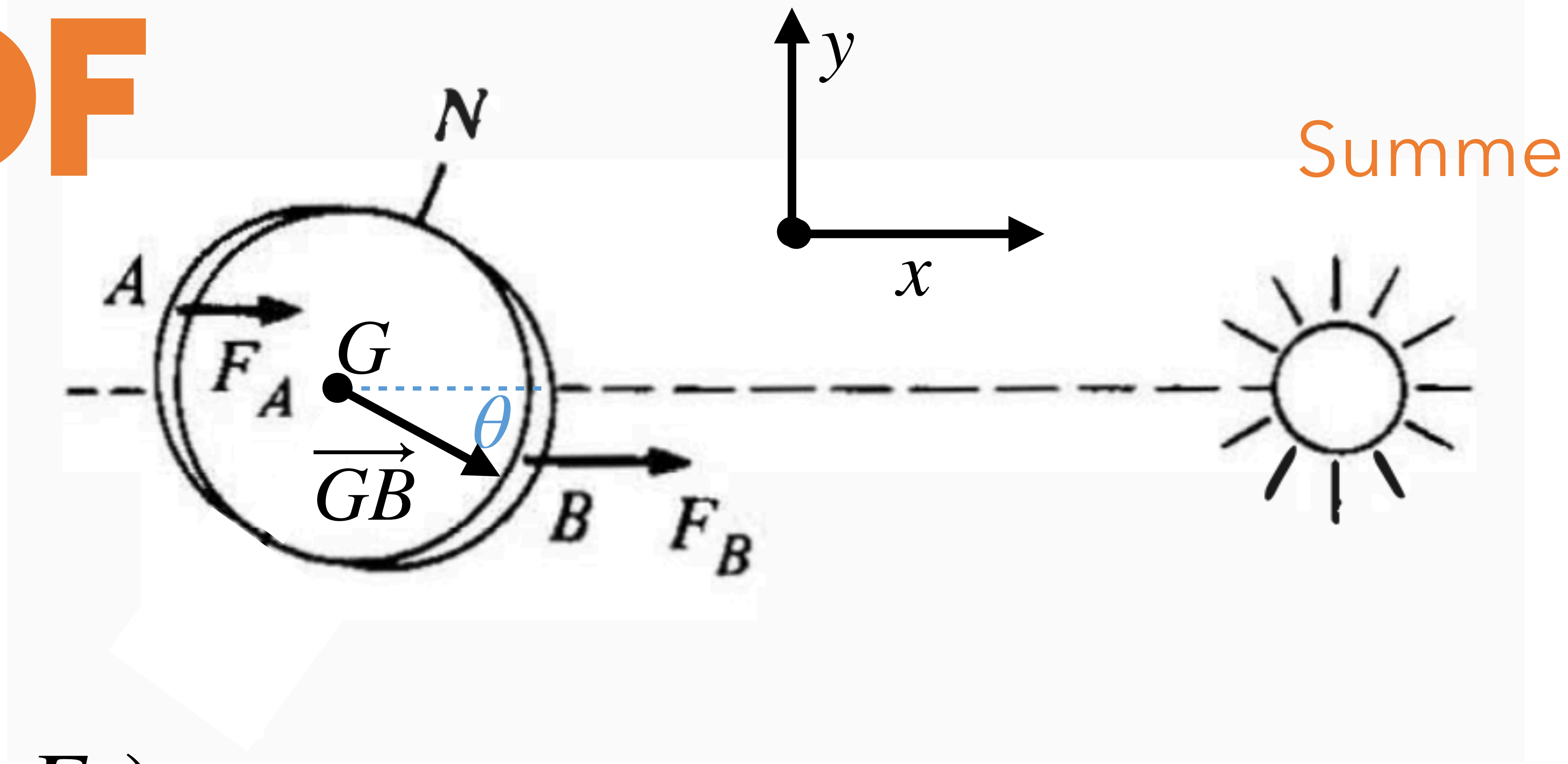
$$\vec{F}_B = F_B \vec{e}_x$$

$$F_B > F_A$$

Lever arm

$$\overrightarrow{GB} = -\overrightarrow{GA} = R(\cos\theta \vec{e}_x - \sin\theta \vec{e}_y)$$

ANGULAR MOMENTUM OF THE EARTH



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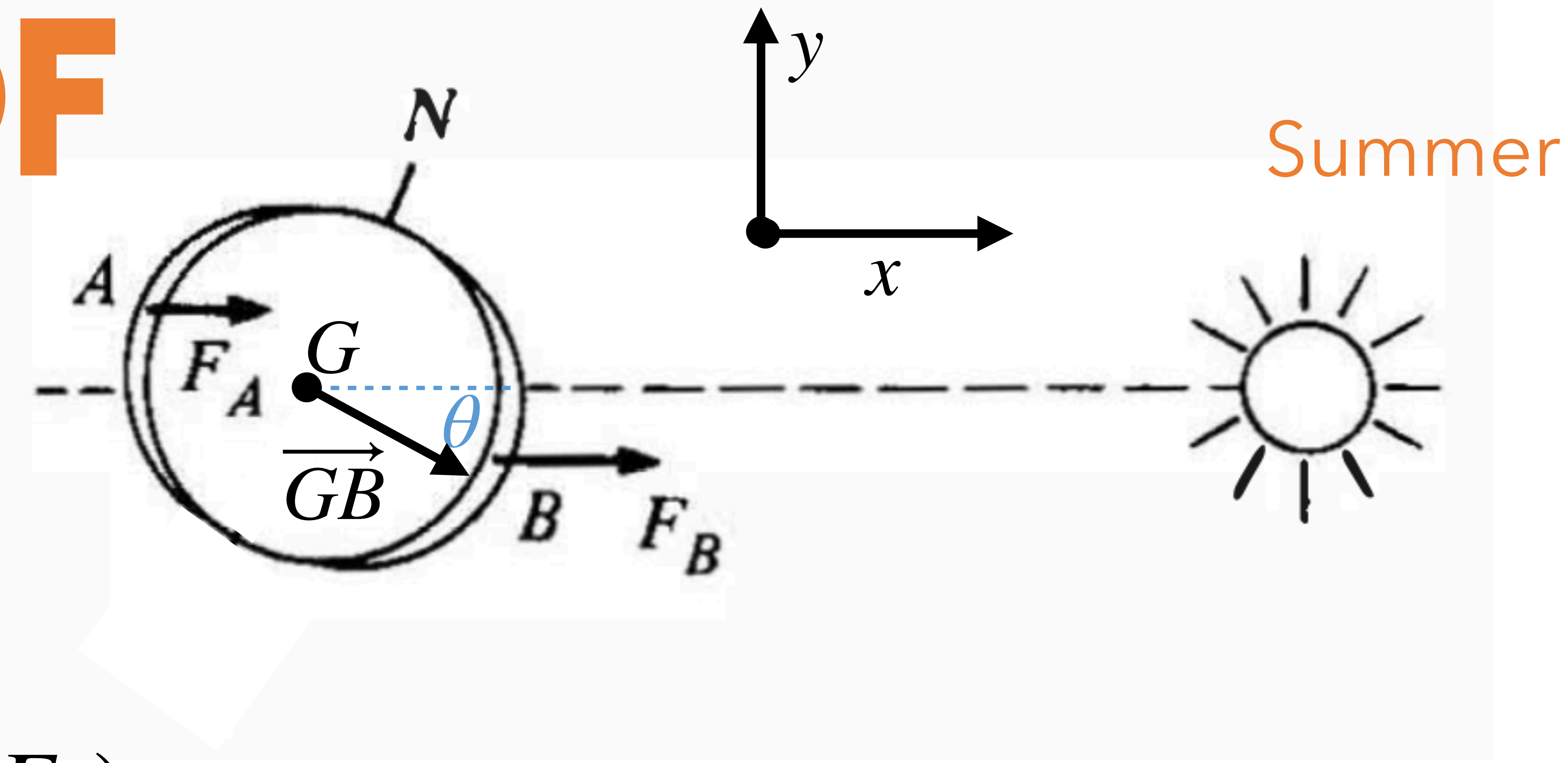

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$$= \overrightarrow{GB} \wedge (\vec{F}_B - \vec{F}_A) = \overrightarrow{GB} \wedge \vec{e}_x (F_B - F_A)$$

$$= R\ (F_B - F_A)(-\sin\theta)\ \vec{e}_y \wedge \vec{e}_x$$

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Points in positive z direction

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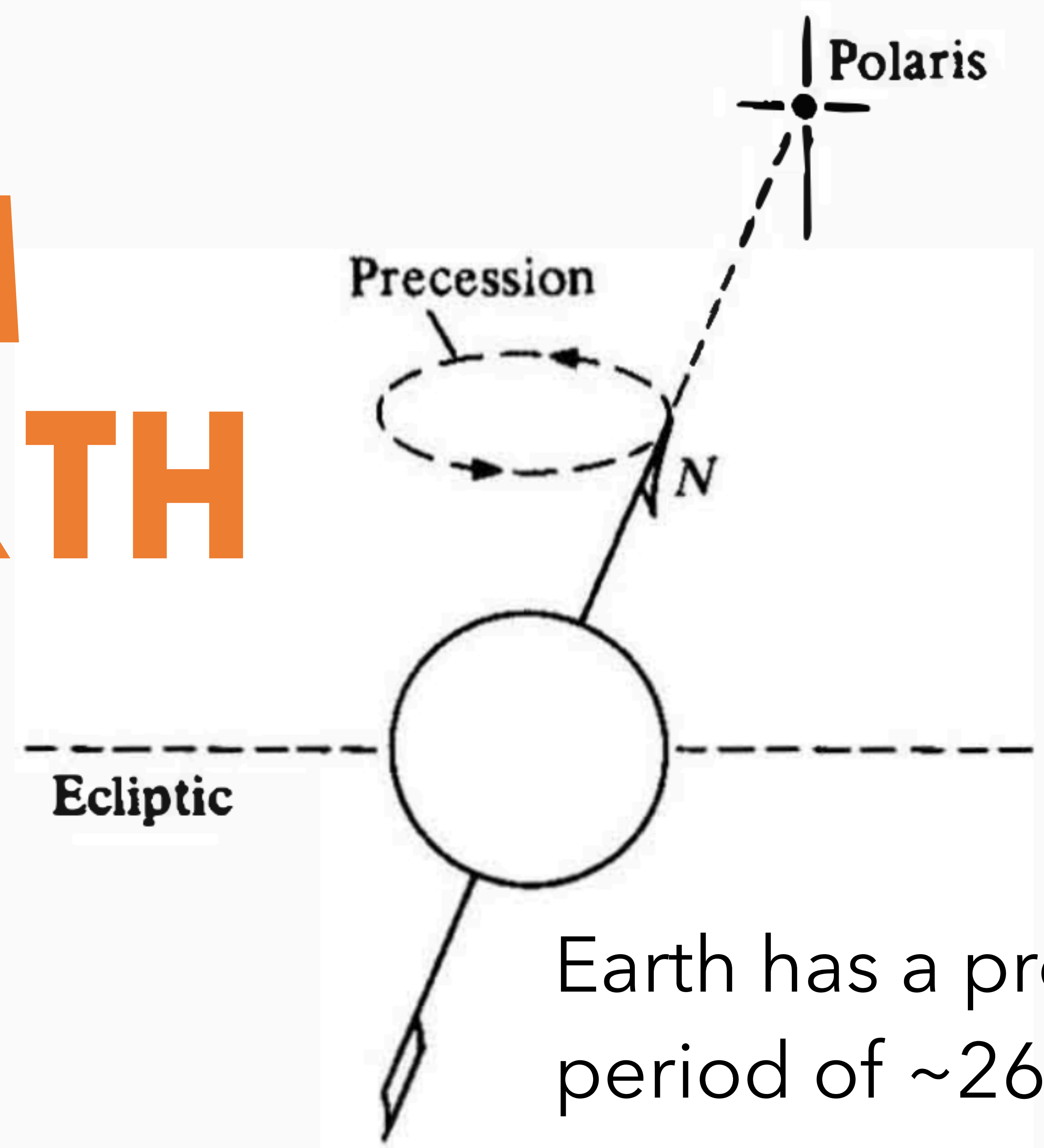
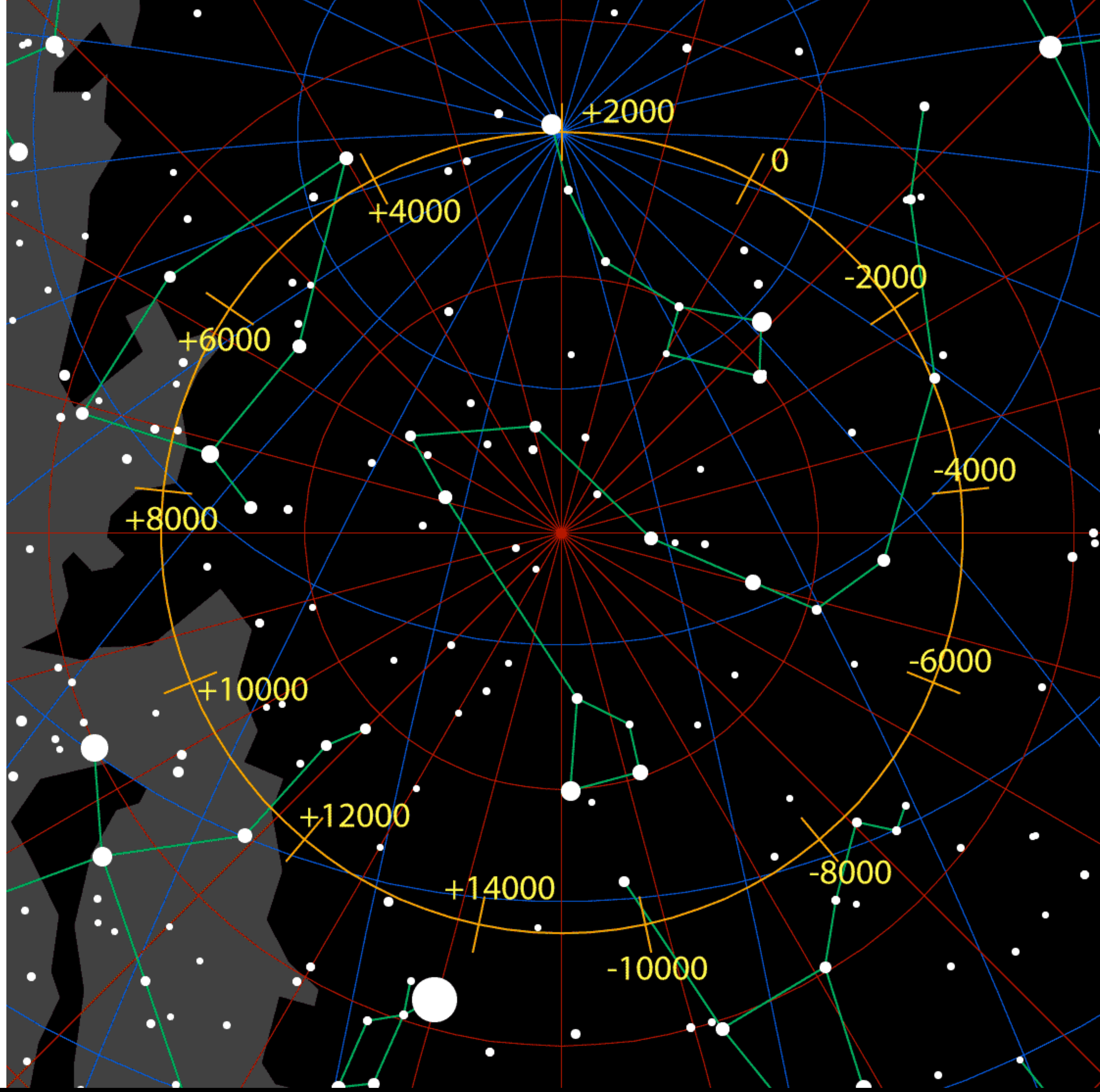
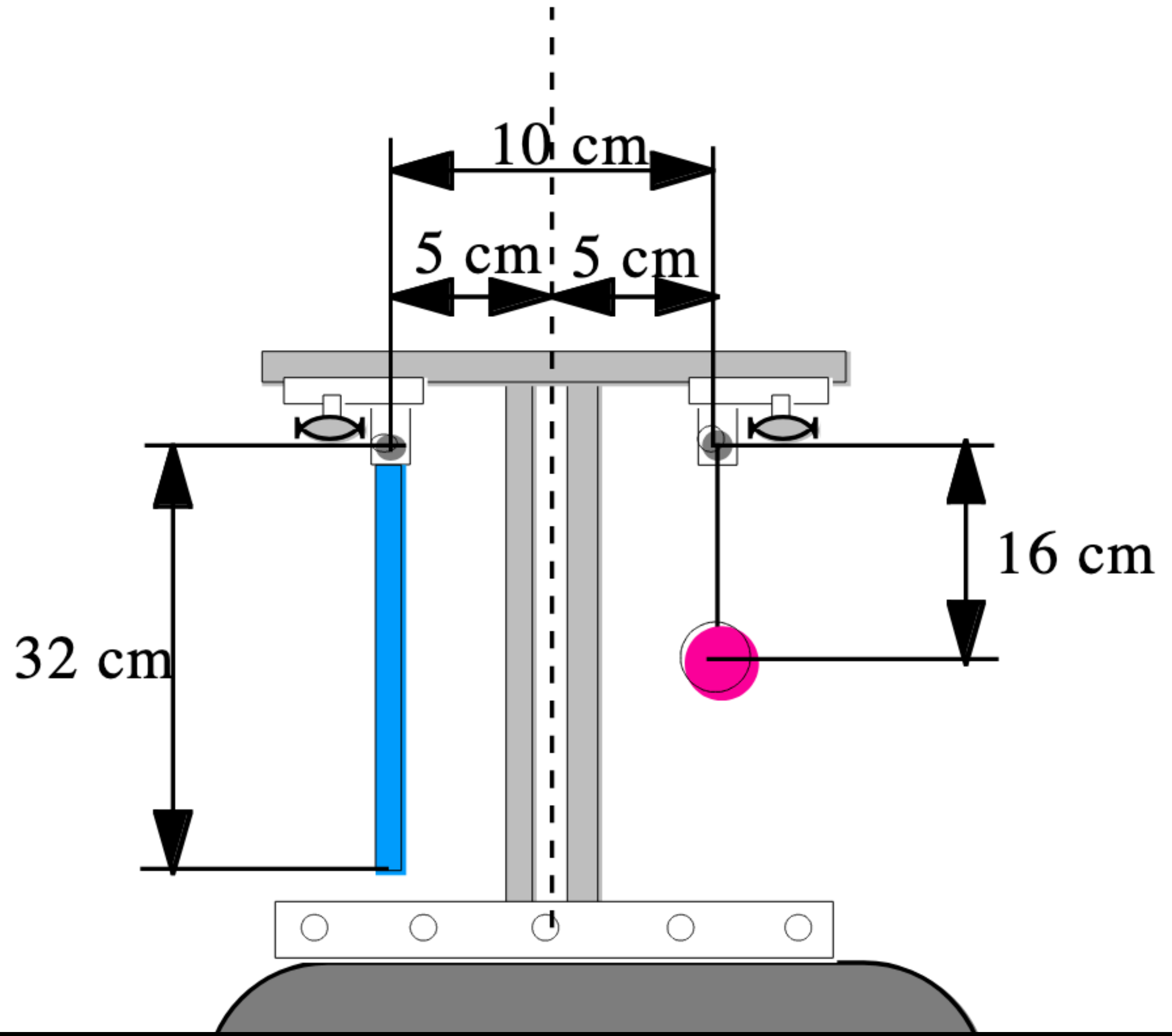


Earth has a precession period of ~26,00 years



SPINNING PHYSICAL PENDULUM



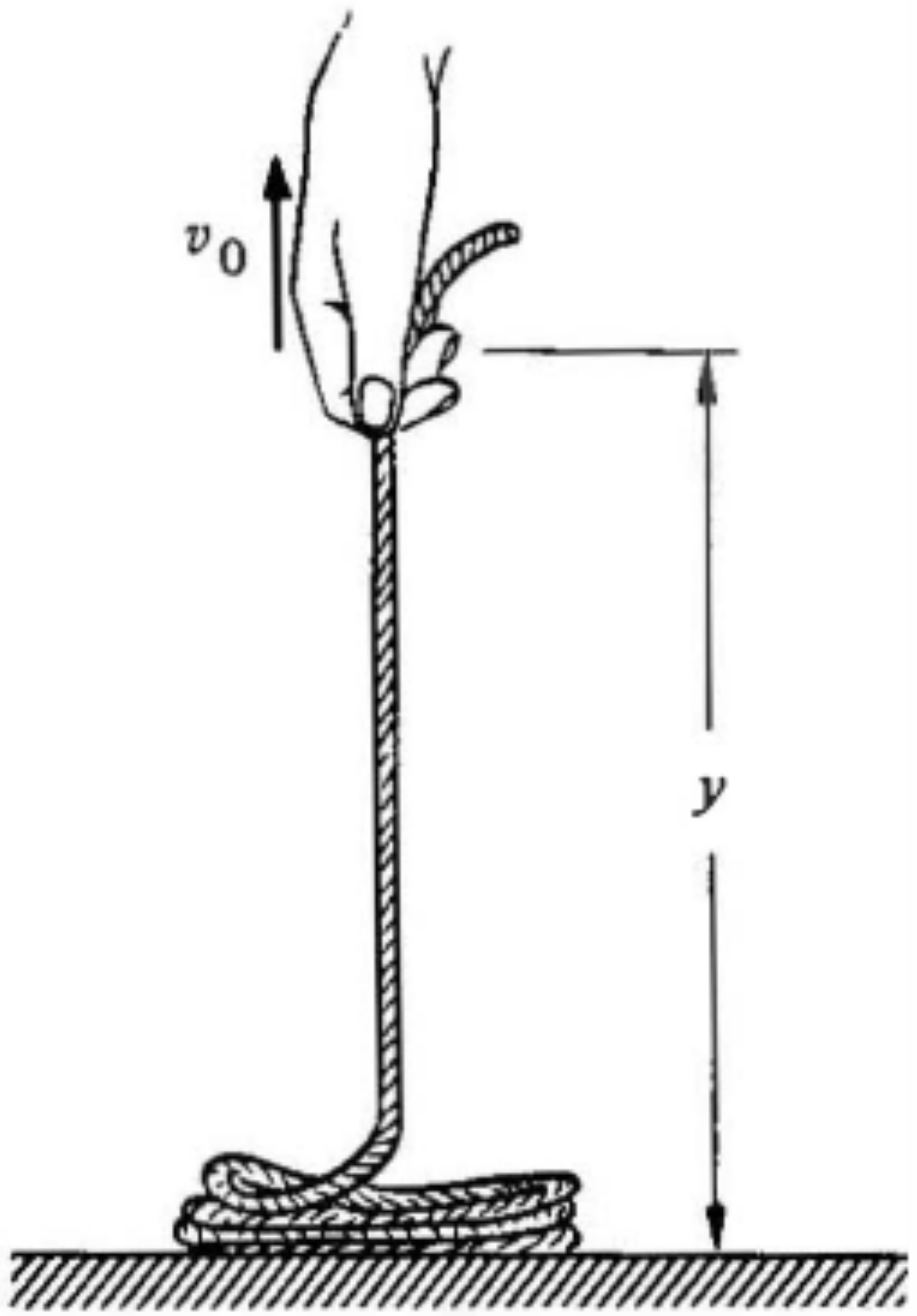
Rotation = 75 t/min
Poids de la bille : 65.5 gr
Poids de la barre : 65.5 gr

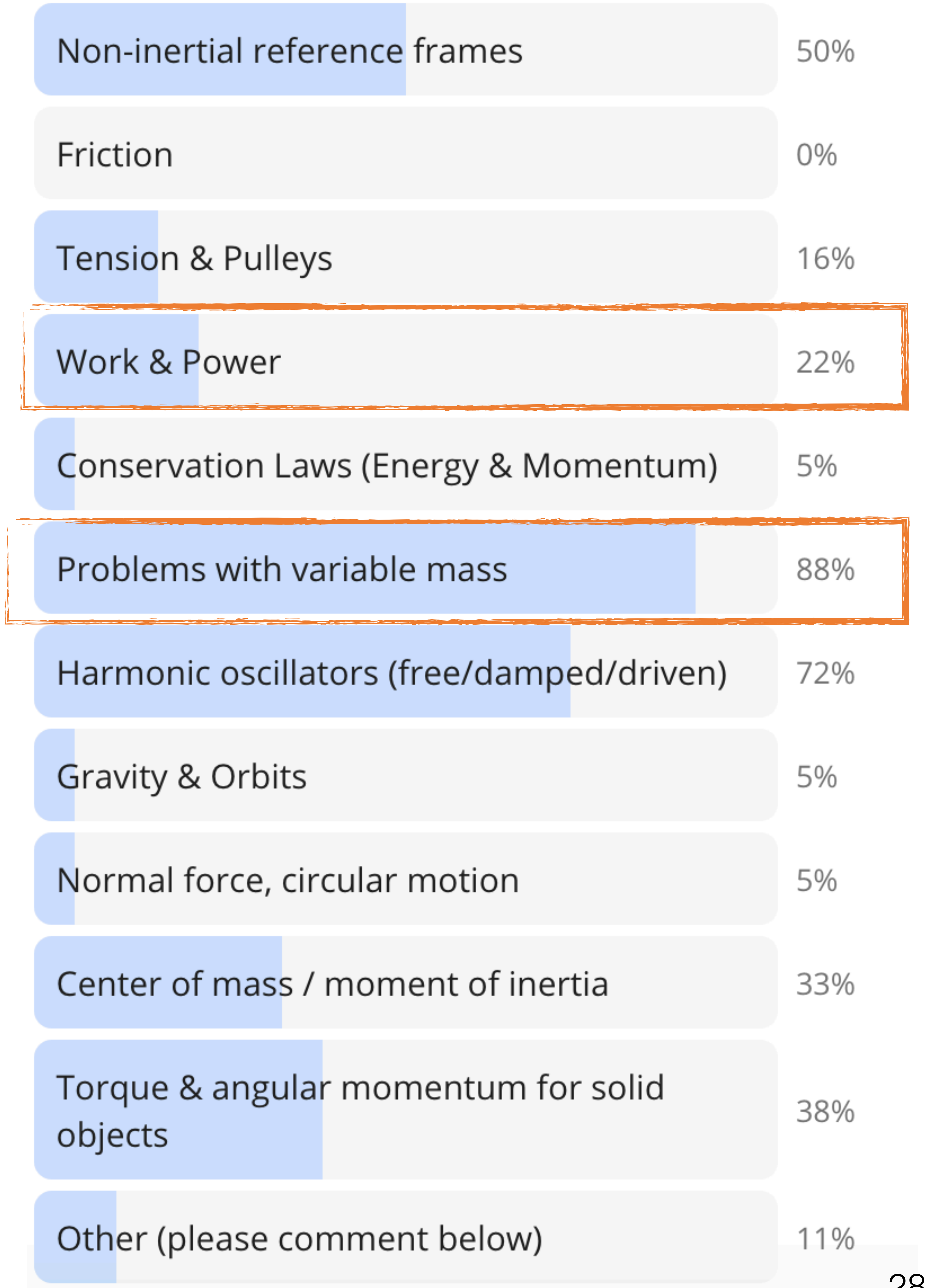
EXERCISES FOR WEEK 15

A uniform rope with linear mass density λ is coiled on a smooth horizontal table. One end is pulled straight up with constant speed V_o.

A) Find the force exerted on the end of the rope as a function of height y.

B) Compare the power delivered to the rope with the rate of change of the rope's total mechanical energy





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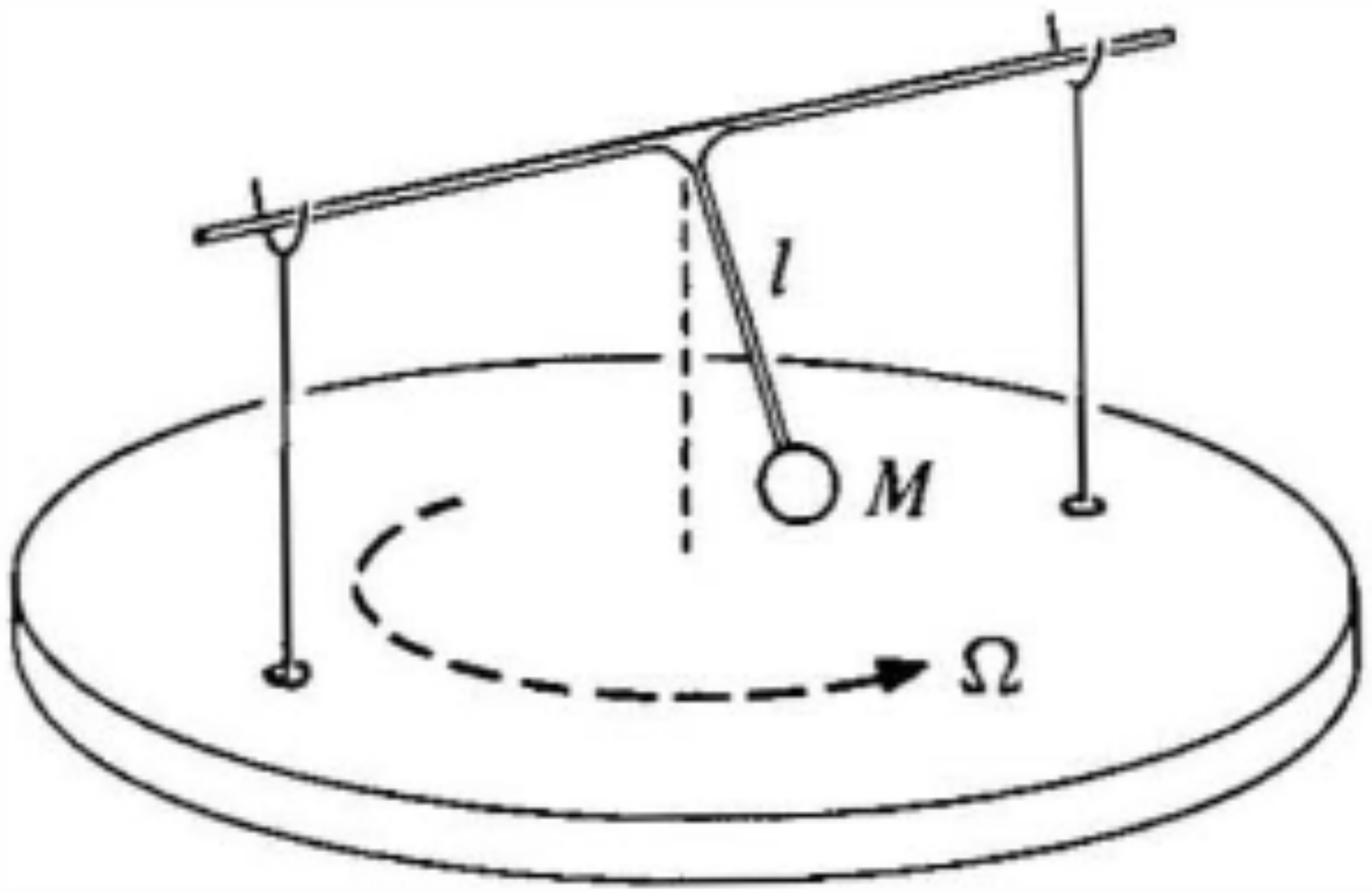
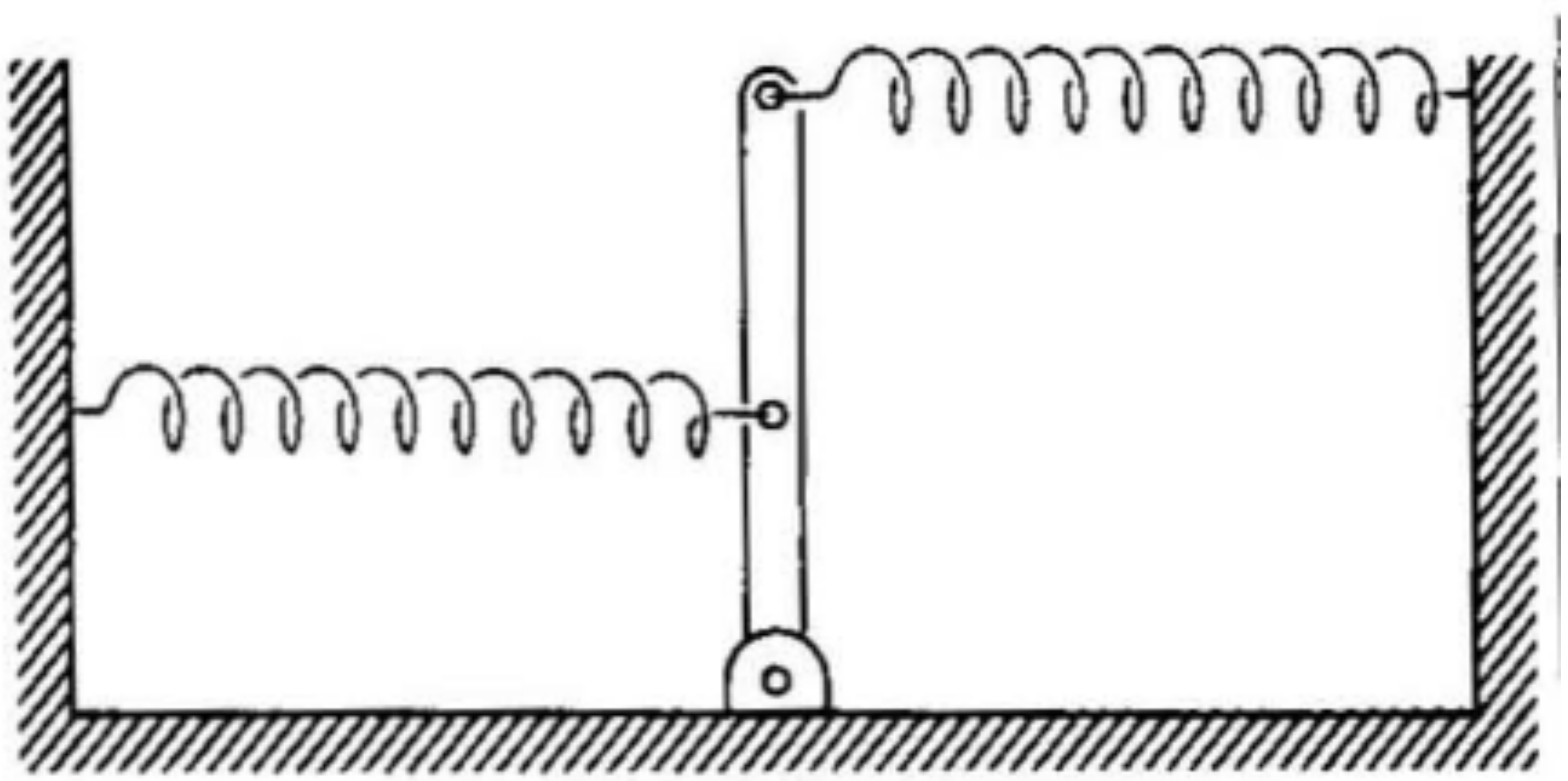
A pendulum is rigidly fixed to an axle held by two supports so that it can swing only in a plane perpendicular to the axle. The pendulum consists of a mass M attached to a massless rod of length l. The supports are mounted on a platform which rotates with constant angular velocity Ω. Find the pendulum's angular frequency assuming that the amplitude is small.



Non-inertial reference frames	50%
Friction	0%
Tension & Pulleys	16%
Work & Power	22%
Conservation Laws (Energy & Momentum)	5%
Problems with variable mass	88%
Harmonic oscillators (free/damped/driven)	72%
Gravity & Orbits	5%
Normal force, circular motion	5%
Center of mass / moment of inertia	33%
Torque & angular momentum for solid objects	38%
Other (please comment below)	11%

EXERCISES FOR WEEK 15

A rod of length l and mass m, pivoted at one end, is held by a spring at its midpoint and a spring at its far end, both pulling in opposite directions. The springs have spring constant k, and at equilibrium their pull is perpendicular to the rod. Find the frequency of small oscillations about the equilibrium position.



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