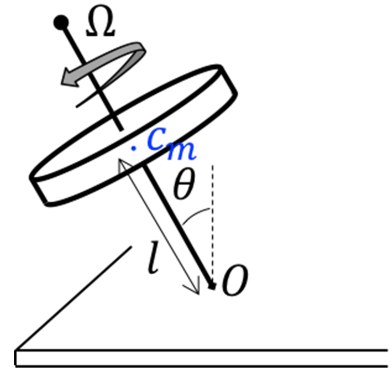
Exercises

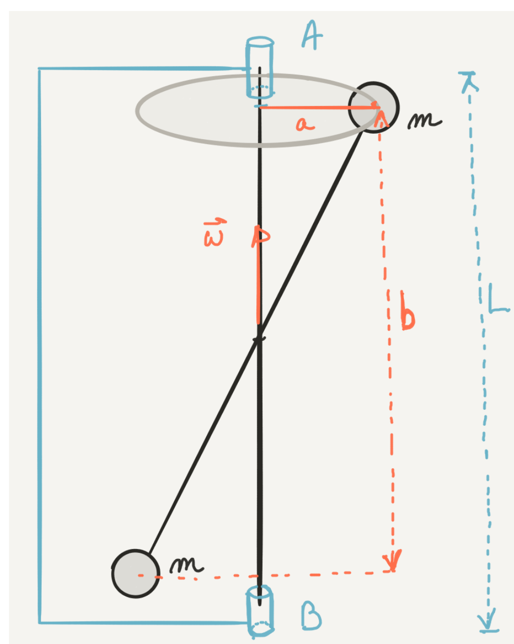
Exercise 1 *Reaction of the ground on a spinning top*



A top of mass m rotates on the ground at a high angular velocity Ω. Its point of contact with the ground O is fixed. The axis of the spinning top undergoes a precession motion at an angular velocity $\omega_p = \frac{mgl}{I\Omega}$, where l is the distance between O and the center of mass c_m of the spinning top, and I is its moment of inertia about its axis. The spinning top rotates in the direction shown in the diagram (the points of the spinning top facing you move from top to bottom / from right to left).

1. What forces are acting on the spinning top ?
2. Draw the angular momentum of the spinning top on the diagram.
3. Show in which direction the precession movement occurs.
4. Apply Newton's laws to the center of mass and deduce the components of the reaction force from the ground.

Exercise 2 *Hold steady at the helm and hold steady in the wind*
The vertical bar in the figure below, supported at its ends by frictionless ball bearings, can rotate freely around its axis. Two equal masses, held in place as shown in the figure by rigid rods of negligible mass, are arranged symmetrically about the center of the bar.

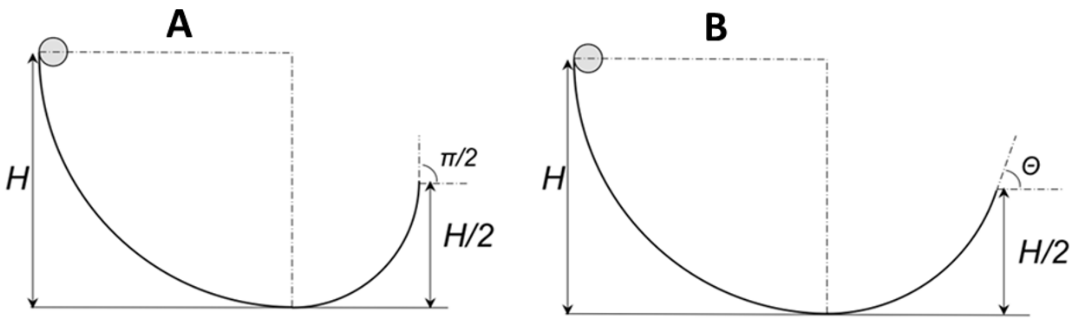


Find :

1. the angular momentum of the system relative to the center of gravity when the system rotates with angular velocity ω ;
2. the forces exerted on the ball bearings.

Exercise 3 *Cylinder on a springboard*

A homogeneous solid cylinder with radius r and mass m is dropped from the top of a springboard with zero initial velocity. The top of the springboard is at height H and the launch point is at height $H/2$. Consider the two types of springboard (A and B) shown above, and the following three scenarios :



Case 1 : The cylinder slides without rolling along ramp A and is ejected vertically.
Case 2 : The cylinder rolls without slipping along ramp A and is ejected vertically.
Case 3 : The cylinder rolls without slipping along ramp B and is ejected at an angle θ to the horizontal.

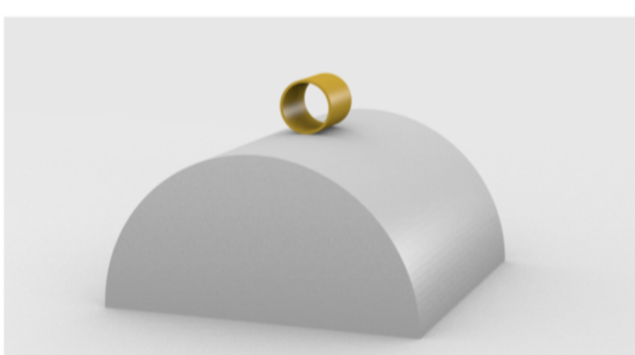
Let h_1, h_2, and h_3 denote the maximum heights reached by the cylinder after its ejection from the springboard for these three cases, respectively.

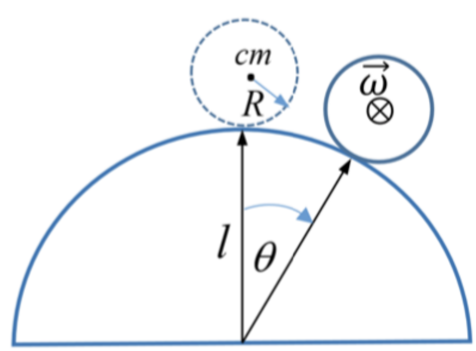
1) Rank qualitatively (without calculation) in descending order, or by equality if necessary, the values (h_1, h_2, h_3, and H). Explain the reasons for this ranking.

2) Calculate h_1, h_2, and h_3 quantitatively based on the data in the problem.

Exercise 4 *Hollow cylinder that rolls and then takes off (2018/2019 exam)*

A hollow cylinder of mass m, length L, radius R, and negligible thickness rests on a support shaped like a half-cylinder with radius l, as shown in the diagram below. The two axes of symmetry of the cylinders are parallel. The hollow cylinder is initially stationary at the top of the support ($\theta = 0$), then it begins to roll without slipping along the support. The position of the hollow cylinder is marked by the angle θ, as shown in the figure below. Air friction is neglected. Let g be the acceleration due to gravity.





1. Show that the moment of inertia I_{cm} of the hollow cylinder for rotation about its axis of symmetry is $I_{cm} = mR^2$.

2. Indicate the forces acting on the hollow cylinder. Care should be taken to specify their point of application. Draw these forces on the diagram on the right, for the position $\theta > 0$.
3. What is the trajectory of the hollow cylinder after leaving the support ?
4. Calculate the critical angle of lift-off θ_c.
5. Determine the differential equation of motion of the hollow cylinder in terms of θ, for $\theta < \theta_c$ (while it rolls without slipping on the support). Express this equation in terms of R, l, and g.
6. If the hollow cylinder slid without friction (no rotation), would the critical angle of lift-off θ_c be greater or smaller ? Explain your reasoning without using calculations.