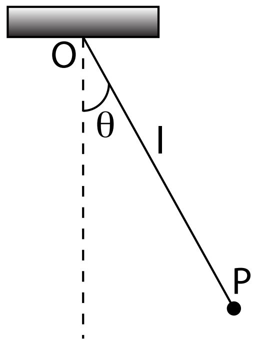
Exercises

Exercise 1

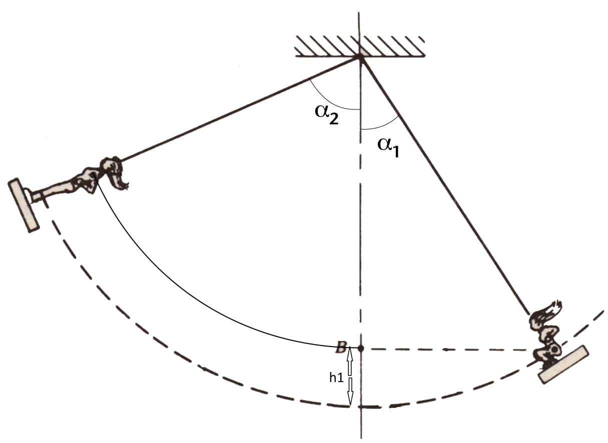


A pendulum consists of a mass m suspended from point P by a string of negligible mass and length l. The string is oriented relative to the vertical by an angle θ. The motion is frictionless.

1. Establish the equation of motion using the angular momentum theorem. Find this equation using the conservation of mechanical energy.
2. Considering oscillations of amplitude θ_0, what is the maximum tension in the wire ?

Exercise 2

A little girl is using a swing. She starts from a crouched position, with no initial speed, while the rope forms an angle α_1 with the vertical.

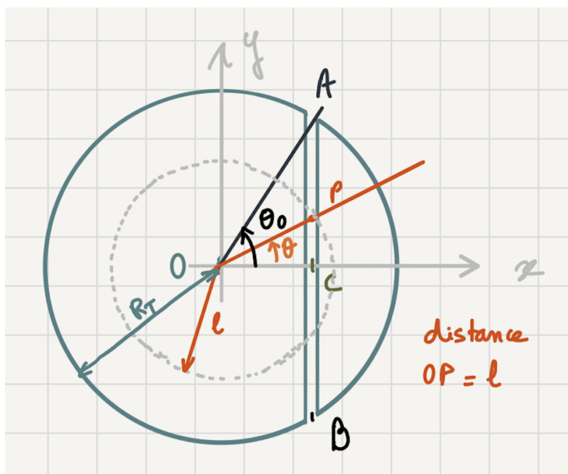


The moment the rope passes through the vertical position, the little girl straightens up, which raises her center of gravity by h_1 and returns it to its starting height.
What is the angle α_2 formed by the rope and the vertical line when the swing swings around to the other side ?
We will ignore friction.

Exercise 3

It is assumed that a straight tunnel can be dug between two arbitrary points on Earth.

1. Express $g(r)$ the acceleration due to gravity at a distance r from the center of the Earth as a function of g_0, the acceleration due to gravity at the Earth's surface, r and R_T. We will assume uniform density.
2. A tunnel is dug between two diametrically opposite points*. An object of mass m is dropped at the entrance to the tunnel on one side. Determine the equation of motion for mass m. From this, deduce the type of motion, the transit time, and the speed as it passes through the center of the Earth.
3. We now dig a tunnel between two arbitrary points A and B. We place ourselves in the coordinate frame indicated in the figure.



We are looking for the differential equation of motion of a mass m dropped at point A in the tunnel. Show that the resultant of the forces on point P marked by θ is

$$\sum \vec{F} = -mg(l)\sin\theta \vec{e}_y$$

4. Deduce the differential equation of the motion of the mass. It will be expressed using the variable y, the coordinate of P on (Oy).
5. Show that the time taken to cross the tunnel is the same regardless of the starting and ending points.

∗. Note : Two points located on exactly opposite sides of the Earth are called antipodes. For example, the antipode of Lausanne lies in the South Pacific Ocean, with the nearest land being around Waitangi, New Zealand !

Exercise 4 The Little Prince
This exercise was given in the 2013 general physics I exam.

In *The Little Prince*, Saint-Exupéry introduces us to a character who lives on an asteroid. He seems to move around on its surface as he would on Earth. We will therefore assume that his "planet" has a mass M_A such that on its surface, g has the same value as on Earth. Its radius R_A will be taken as 2 meters (illustration) and the height h of the little prince as 1 meter.
Furthermore, since its days appear to be of Earth-like duration, we can assume that the asteroid's period of revolution is 24 hours, as for Earth.
The radius of the Earth, R_T, is $6,400$ kilometers.

1. Calculate the density of the asteroid ρ_A based on the density of the Earth ρ_T and the radii R_A and R_T.
2. Numerically evaluate the ratio ρ_A/ρ_T.
3. Calculate g_{head} the value of the acceleration due to gravity at the level of the Little Prince's head based on the data.
4. Numerically evaluate the ratio g_{head}/g.
5. Comment on this result.
6. Calculate the escape velocity v_l of an object from the surface of the asteroid, based on g and the data.
7. Evaluate v_l numerically. Note that the world records for the 100 m correspond to an average speed of 10 m/s.
8. The little prince stands at a latitude of 45° on his planet and builds a Foucault pendulum : a simple pendulum with a string of length l. The mass at the end of the string is close enough to the ground that we can take g as the acceleration due to gravity. He observes the rotation of the plane of oscillation of the pendulum due to the rotation of his planet. Compared to the rotation of the plane of the pendulum on Earth, on the Little Prince's planet, is it identical, faster, or slower ? Justify your answer.
9. While jogging in the morning, the little prince wanted to sprint and clumsily sent himself into orbit around his planet. We assume that he is in a circular orbit at h_o=1 m above the ground.
   (a) Calculate his mechanical energy and kinetic energy.
   (b) How fast did he sprint ?