PHYS-101 WEEK 11

EPFL

Physique générale : mécanique (classe inversée en anglais)

Prof. Emma Tolley, 17 November 2025

EXAMEN BLANC / PRACTICE EXAM

Practice exam instead of a group exercise for week 11. Corrections will be posted on Friday.

Grading: 50 points total, 12 points QCM / 38 points for exercises
Duration: 3 hours
~ 7 min / QCM question
~ 50 min, 60 min, 25 min for exercises 2, 3, 4

You can use the formula sheet linked on the Moodle, and one 1-sided hand-written sheet. No calculators, computers, phones, etc

FREQUENCY & ANGULAR FREQUENCY

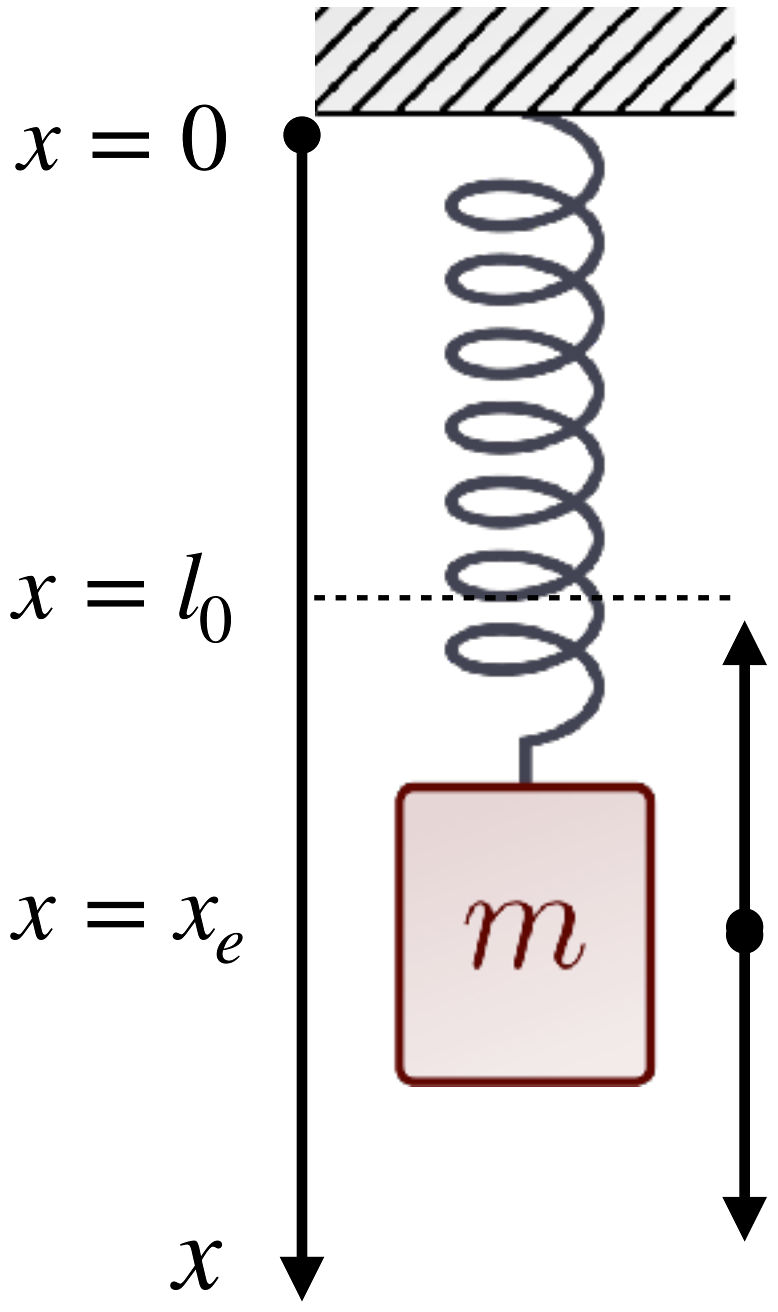
- **Angular frequency**: radians/second
  - Natural angular frequency Ω_0
  - Pseudo angular frequency $\omega = \sqrt{\Omega_0^2 - \gamma^2}$
  - Driving/forcing frequency ω_e
  - Resonance angular frequency $\omega_{\text{res}} = \sqrt{\Omega_0^2 - 2\gamma^2}$
- **Frequency:** revolutions/second $f = \dfrac{\omega}{2\pi}$ $f_0 = \dfrac{\Omega_0}{2\pi}$
  - etc
- **Period**: seconds/revolution $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$

DRIVEN HARMONIC OSCILLATOR

- **Driven/forced harmonic oscillator** equation of motion
  - $\ddot{x} + 2\gamma\dot{x} + \Omega_0^2 x = f_0 \cos(\omega_e t)$
- Solution: $x(t) = x_1(t) + x_2(t)$
  - $x_2(t)$ is the solution to $\ddot{x} + 2\gamma\dot{x} + \Omega_0^2 x = 0$ (damped oscillator)
  - $x_1(t) = A(\omega_e)\cos(\omega_e t + \phi(\omega_e))$ (steady-state solution)

$$A(\omega_e) = \frac{f_0}{\sqrt{(\omega_e^2 - \Omega_0^2)^2 + 4\gamma^2\omega_e^2}} \qquad \tan\phi = \frac{2\gamma\omega_e}{\omega_e^2 - \Omega_0^2}$$

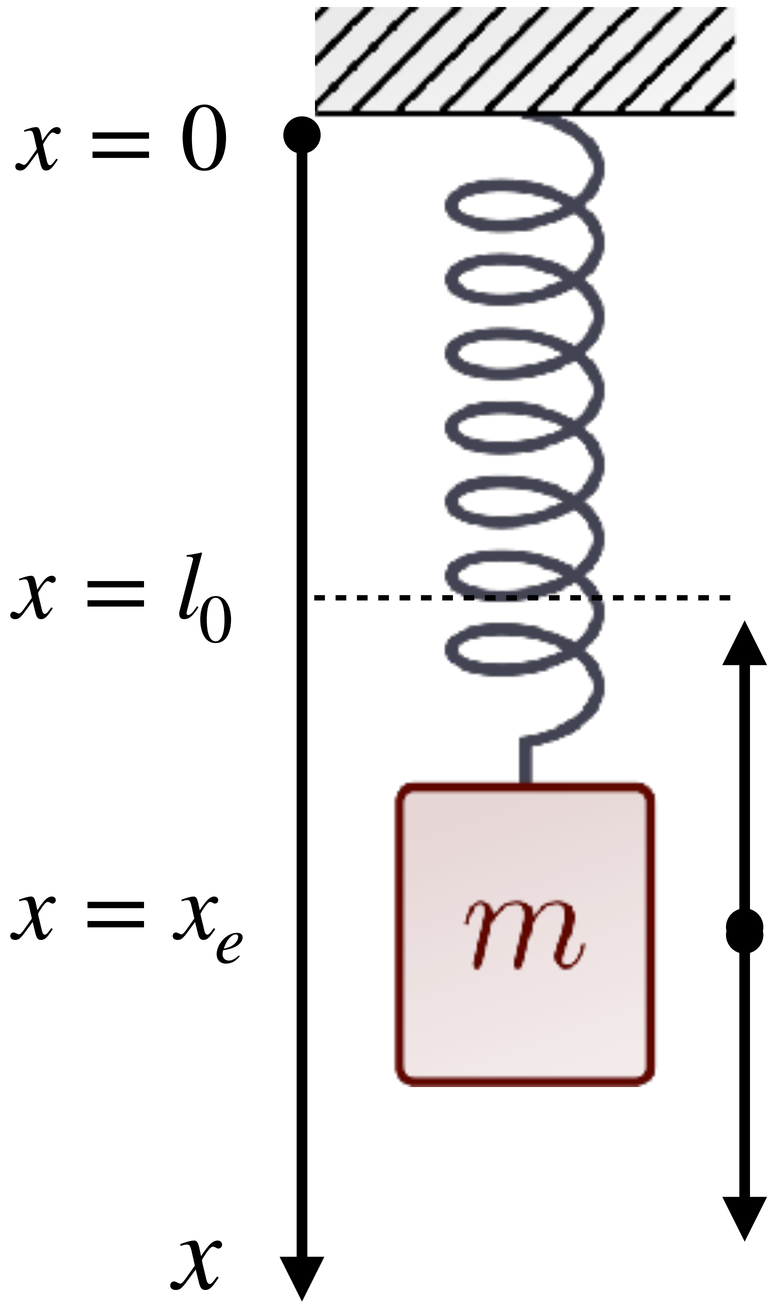
DEMO: FREE OSCILLATOR



$$\vec{F}_k = -k(x - l_0)\vec{e}_x$$

$$m\vec{g} = mg\vec{e}_x$$

DEMO: FREE OSCILLATOR



$$\vec{F}_k = -k(x - l_0)\vec{e}_x$$

$$m\vec{g} = mg\vec{e}_x$$

Newton's 2nd Law

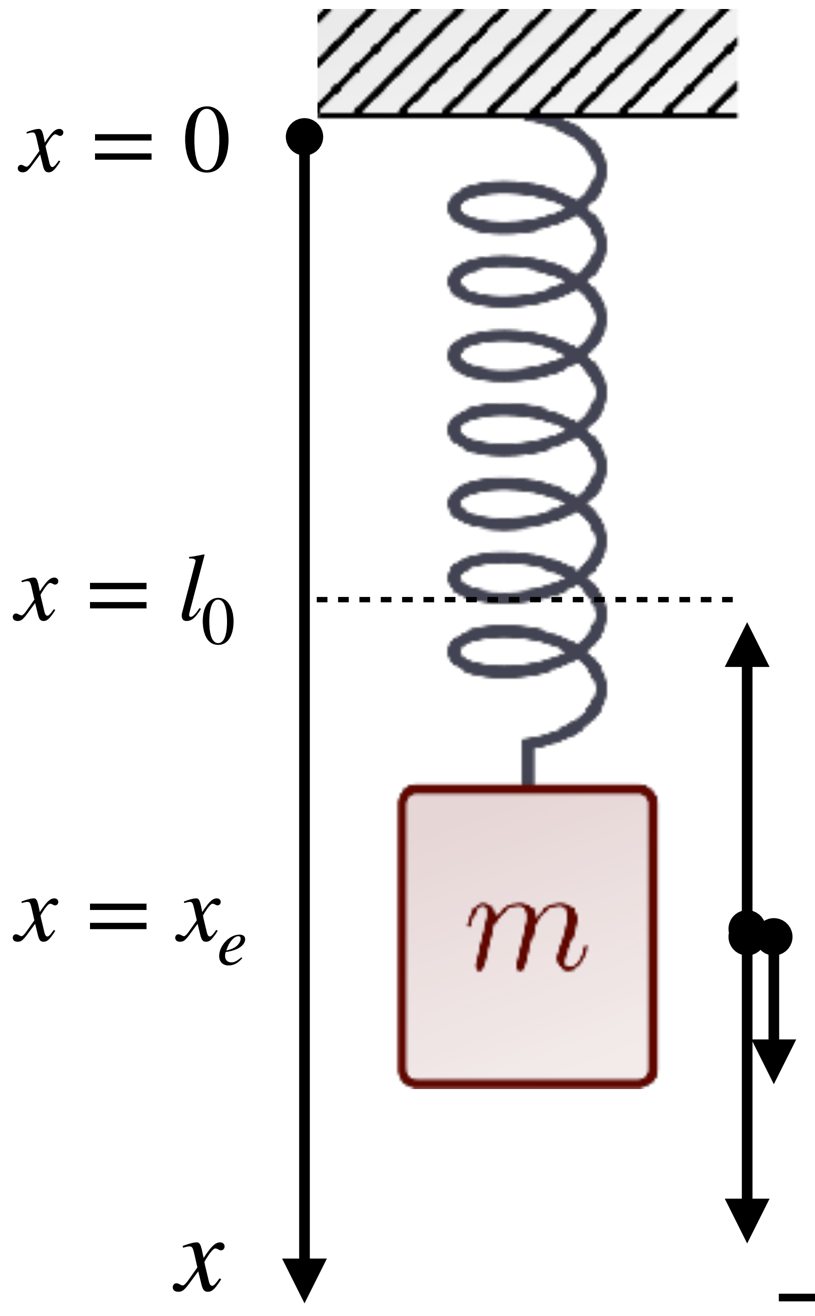
$$m\vec{a} = \vec{F}_k + m\vec{g}$$

$$m\ddot{x} = mg - k(x - l_0)$$

$$\ddot{x} + \frac{k}{m}x = g + \frac{kl_0}{m}$$

$$\Omega_0 = \sqrt{\frac{k}{m}}$$

DEMO: DAMPED OSCILLATOR

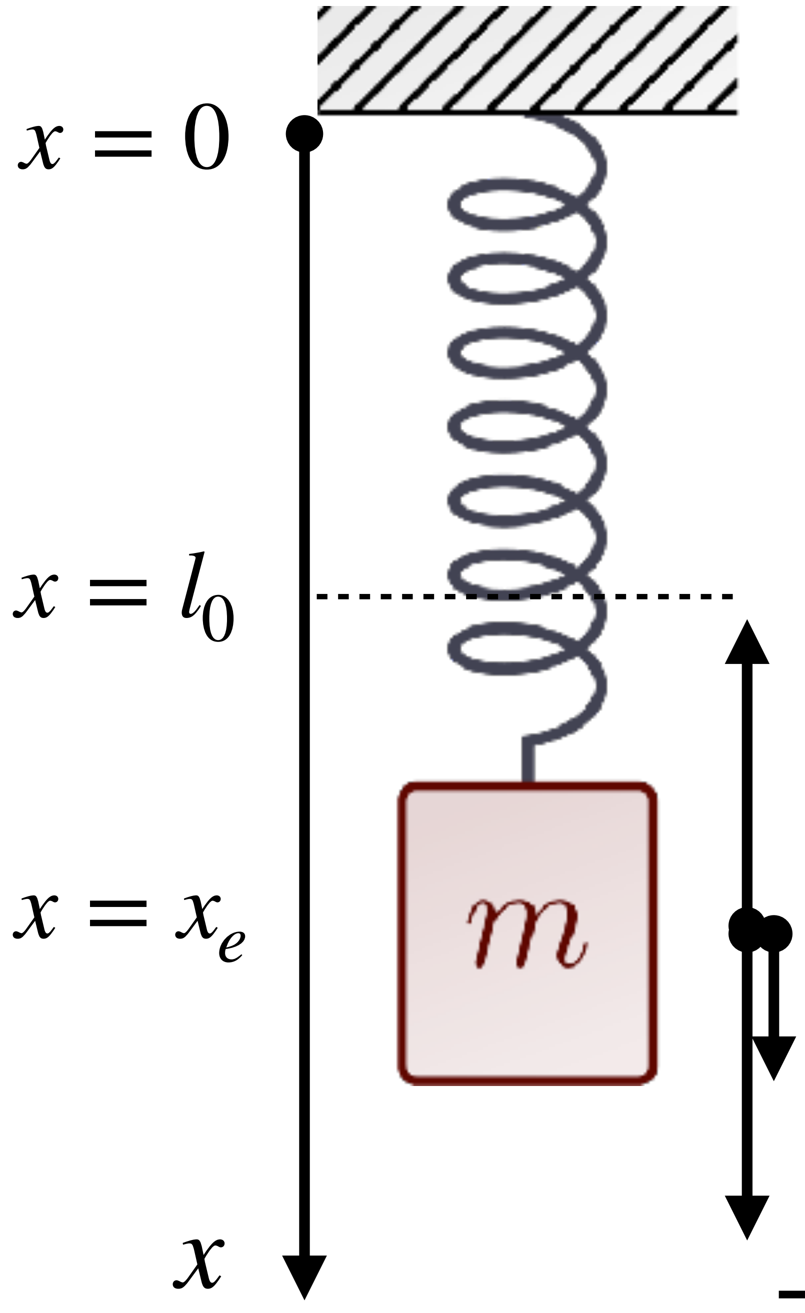


$$\vec{F}_k = -k(x - l_0)\vec{e}_x$$

$$\vec{F}_f = -b\dot{x}\vec{e}_x$$

$$m\vec{g} = mg\vec{e}_x$$

DEMO: DAMPED OSCILLATOR



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$$\vec{F}_f = -b\dot{x}\vec{e}_x$$

$$m\vec{g} = mg\vec{e}_x$$

Newton's 2nd Law

$$m\vec{a} = \vec{F}_k + \vec{F}_f + m\vec{g}$$

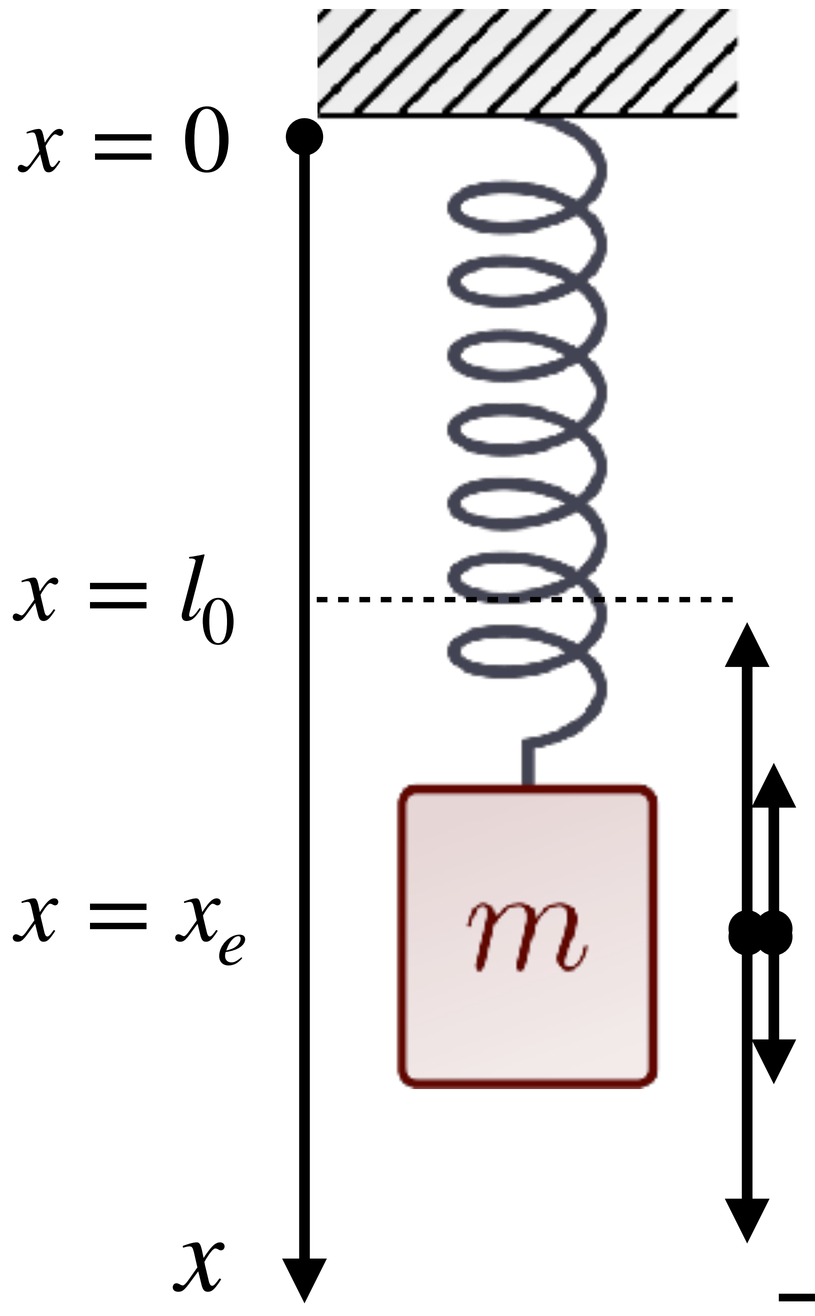
$$m\ddot{x} = mg - b\dot{x} - k(x - l_0)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = g + \frac{kl_0}{m}$$

$$\gamma = \frac{b}{2m} \qquad \Omega_0 = \sqrt{\frac{k}{m}} \qquad \omega = \sqrt{\Omega_0^2 - \gamma^2}$$

$$x(t) = Ce^{-\gamma t}\cos(\omega t + \phi) + l_0 + \frac{mg}{k}$$

DEMO: DRIVEN OSCILLATOR



$$\vec{F}_k = -k(x - l_0)\vec{e}_x$$

$$\vec{F}_{ext} = F_0 \cos(\omega_e t)\vec{e}_x$$

$$\vec{F}_f = -b\dot{x}\vec{e}_x$$

$$m\vec{g} = mg\vec{e}_x$$

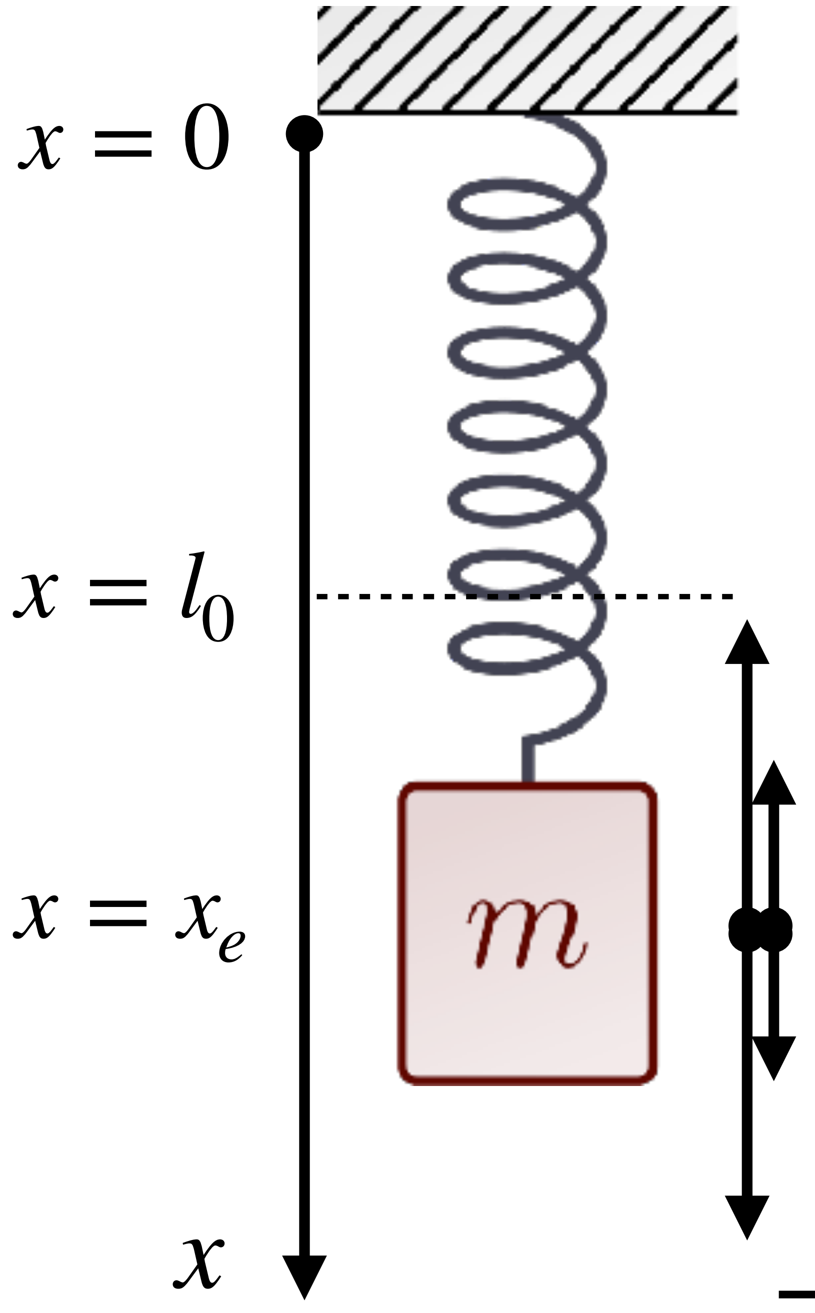
Newton's 2nd Law

$$m\vec{a} = \vec{F}_k + \vec{F}_f + m\vec{g} + \vec{F}_{ext}$$

$$m\ddot{x} = mg - b\dot{x} - k(x - l_0) + F_0 \cos(\omega_e t)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos(\omega_e t) + g + \frac{kl_0}{m}$$

DEMO: DRIVEN OSCILLATOR



$$\vec{F}_k = -k(x - l_0)\vec{e}_x$$

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$$m\vec{g} = mg\vec{e}_x$$

Newton's 2nd Law

$$m\vec{a} = \vec{F}_k + \vec{F}_f + m\vec{g} + \vec{F}_{ext}$$

$$m\ddot{x} = mg - b\dot{x} - k(x - l_0) + F_0 \cos(\omega_e t)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos(\omega_e t) + g + \frac{kl_0}{m}$$

$$y = x - \frac{mg}{k} - l_0$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{F_0}{m}\cos(\omega_e t)$$

$$y_1(t) = A(\omega_e)\cos(\omega_e t + \phi(\omega_e))$$

MORE GENERAL CASES

$$\ddot{x} + 2\gamma\dot{x} + \Omega_0^2 x = f(t)\,?$$

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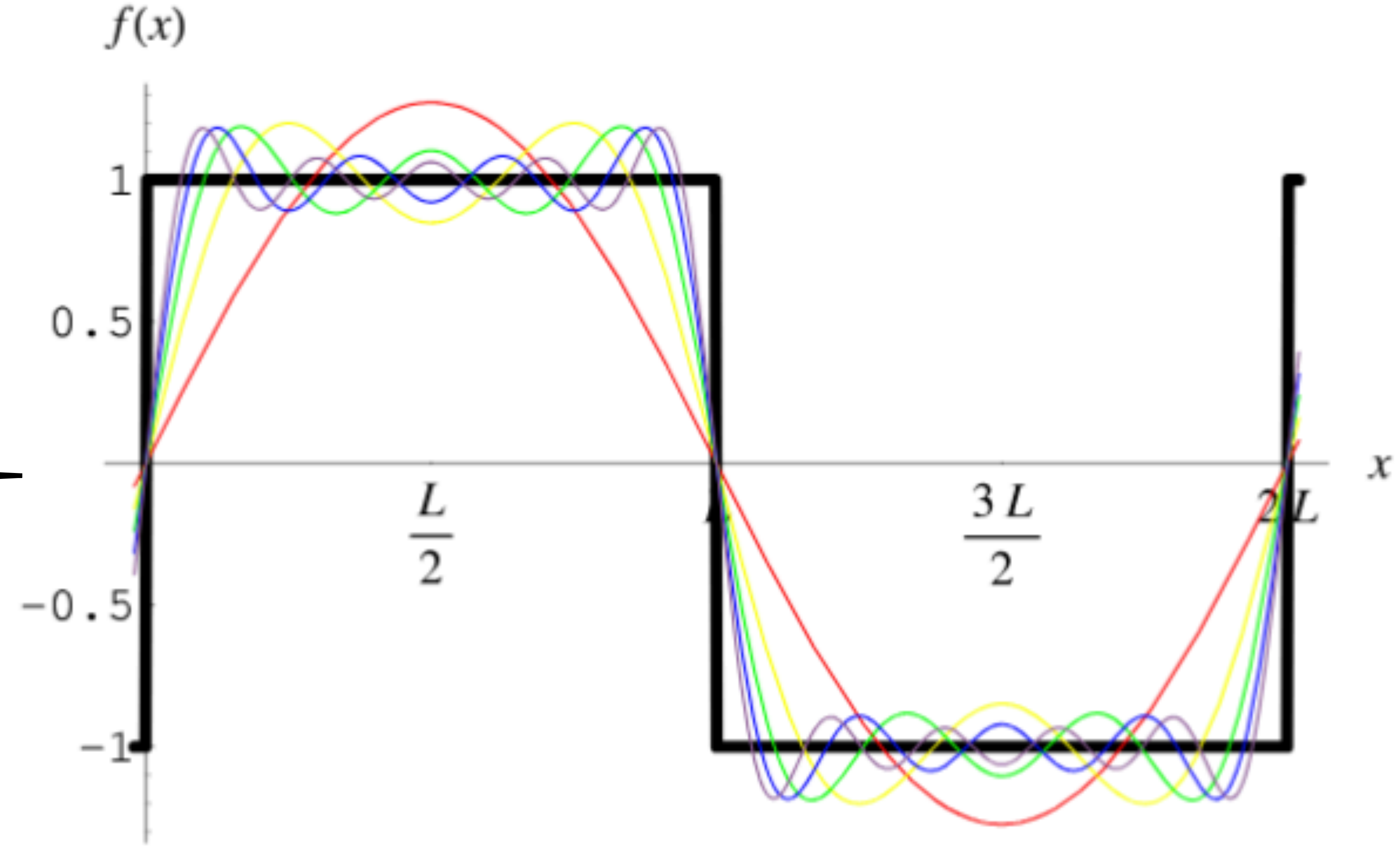


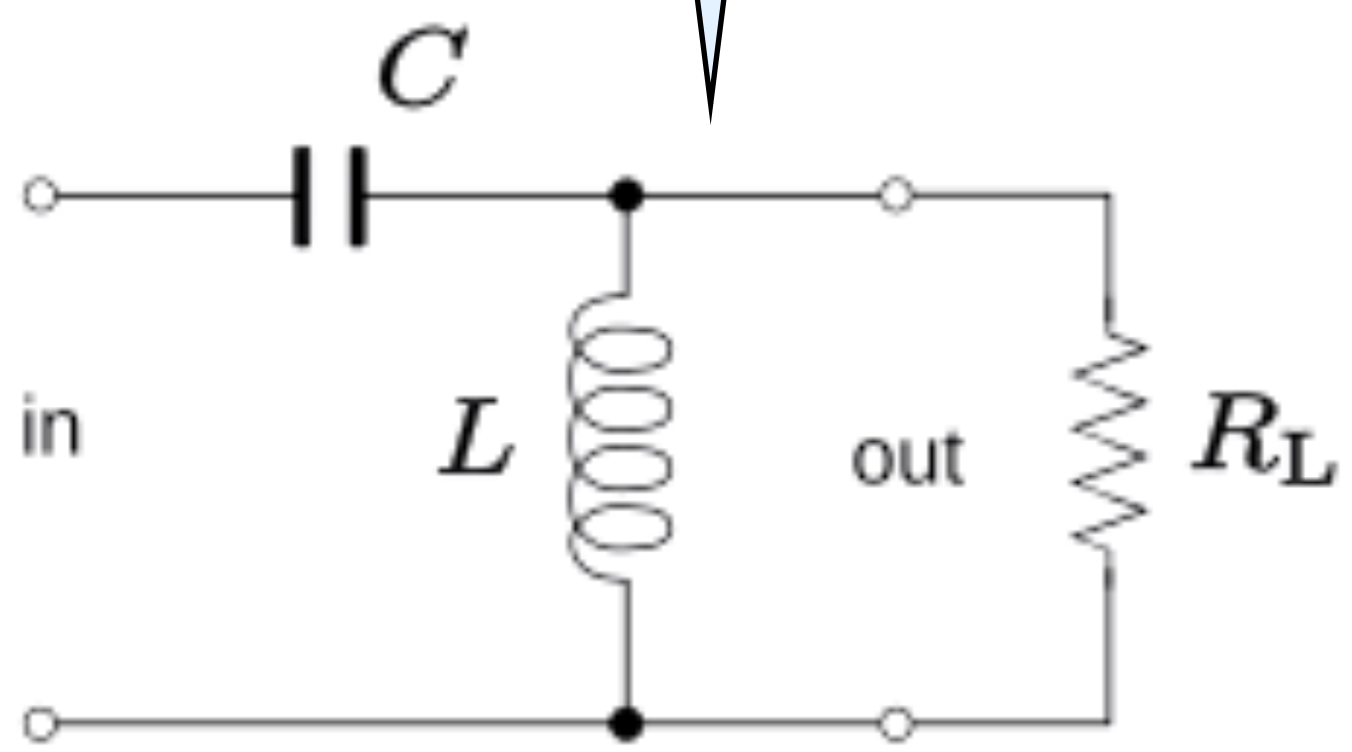
Can decompose any function into sum of sines/cosines using the Fourier transform

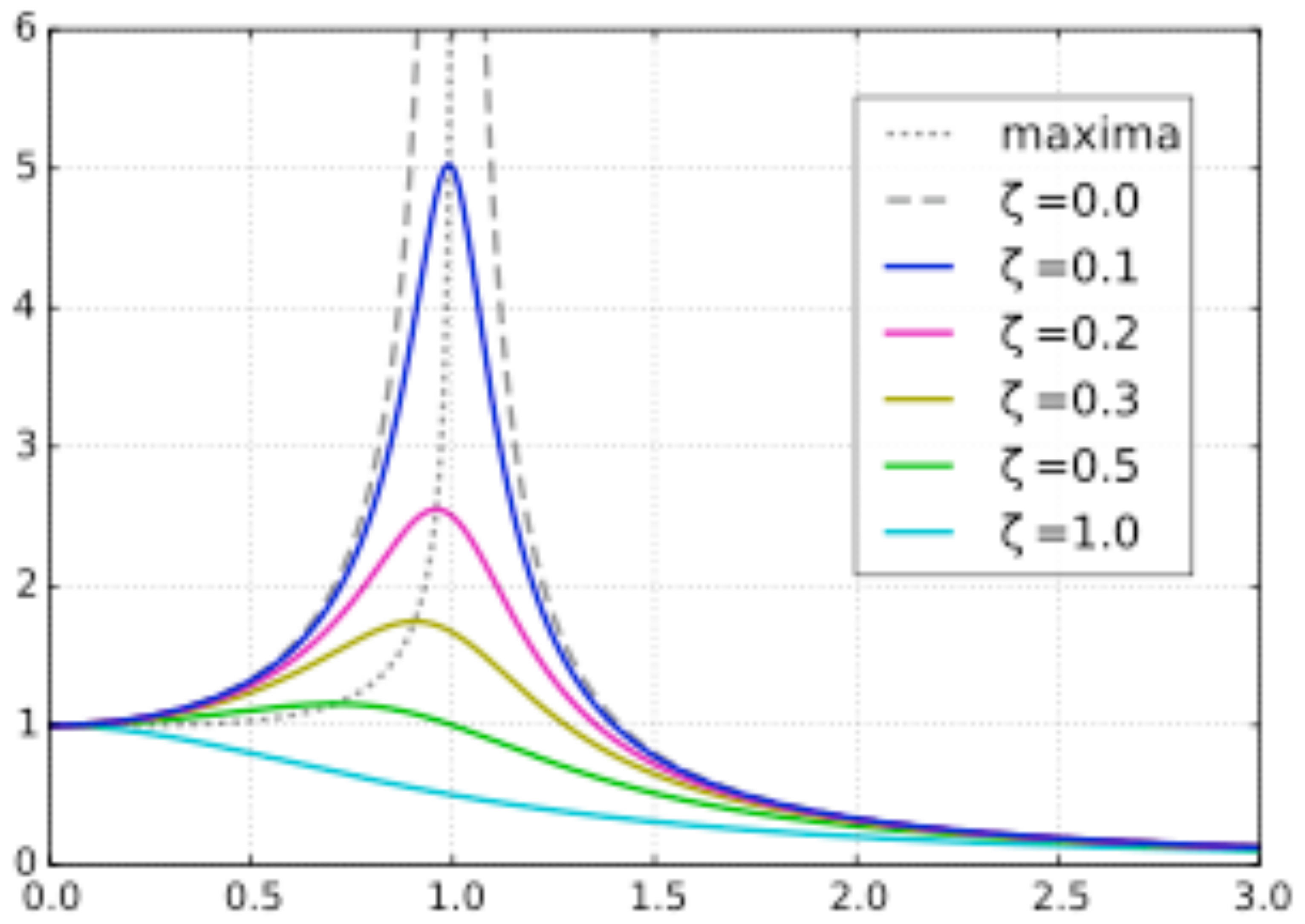
Can then find the particular solution for each of the terms in the Fourier series

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right).$$

MORE GENERAL CASES

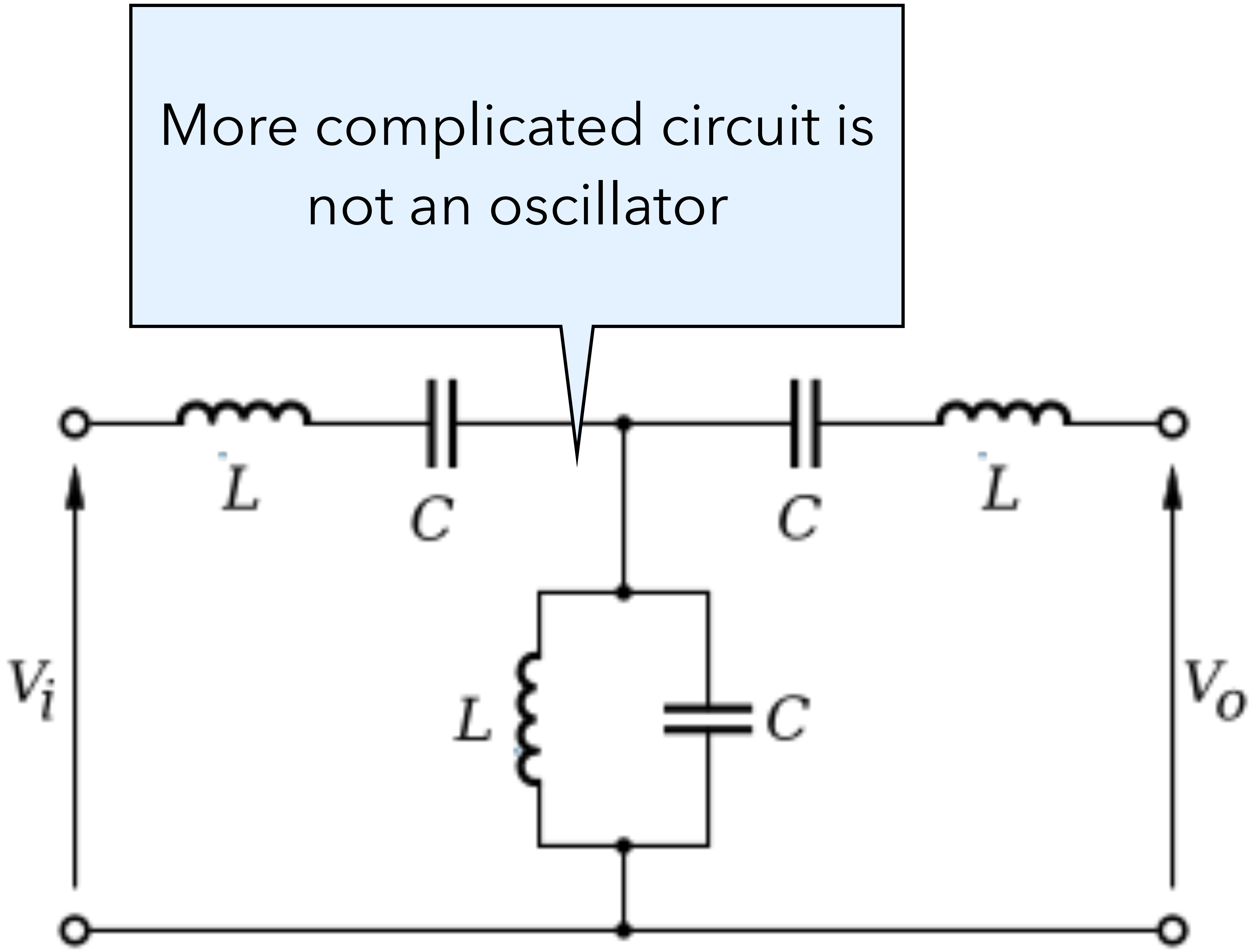
Simple AC-driven RLC circuit: damped harmonic oscillator!

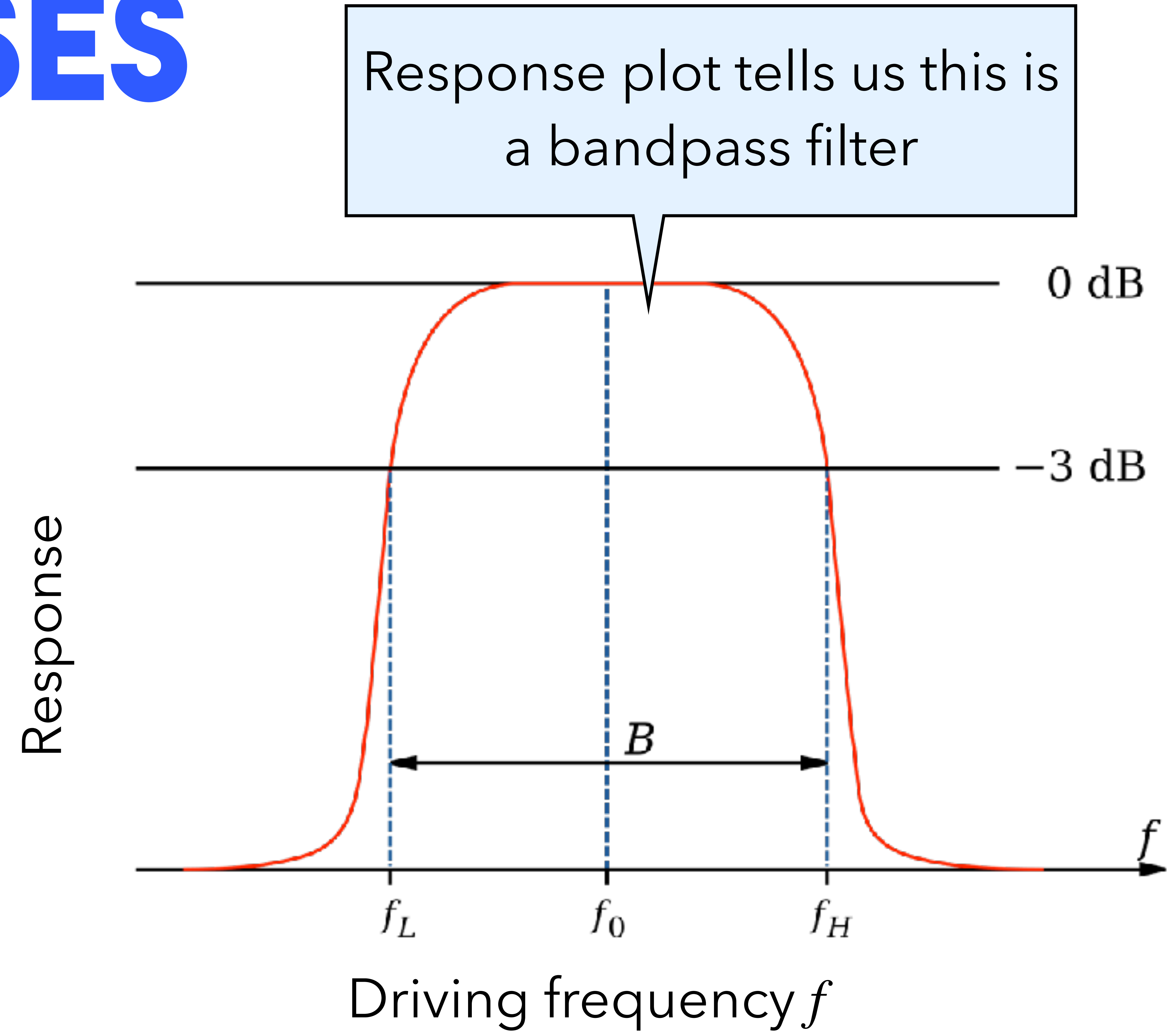




Relative driving frequency ω_e/Ω_0

MORE GENERAL CASES

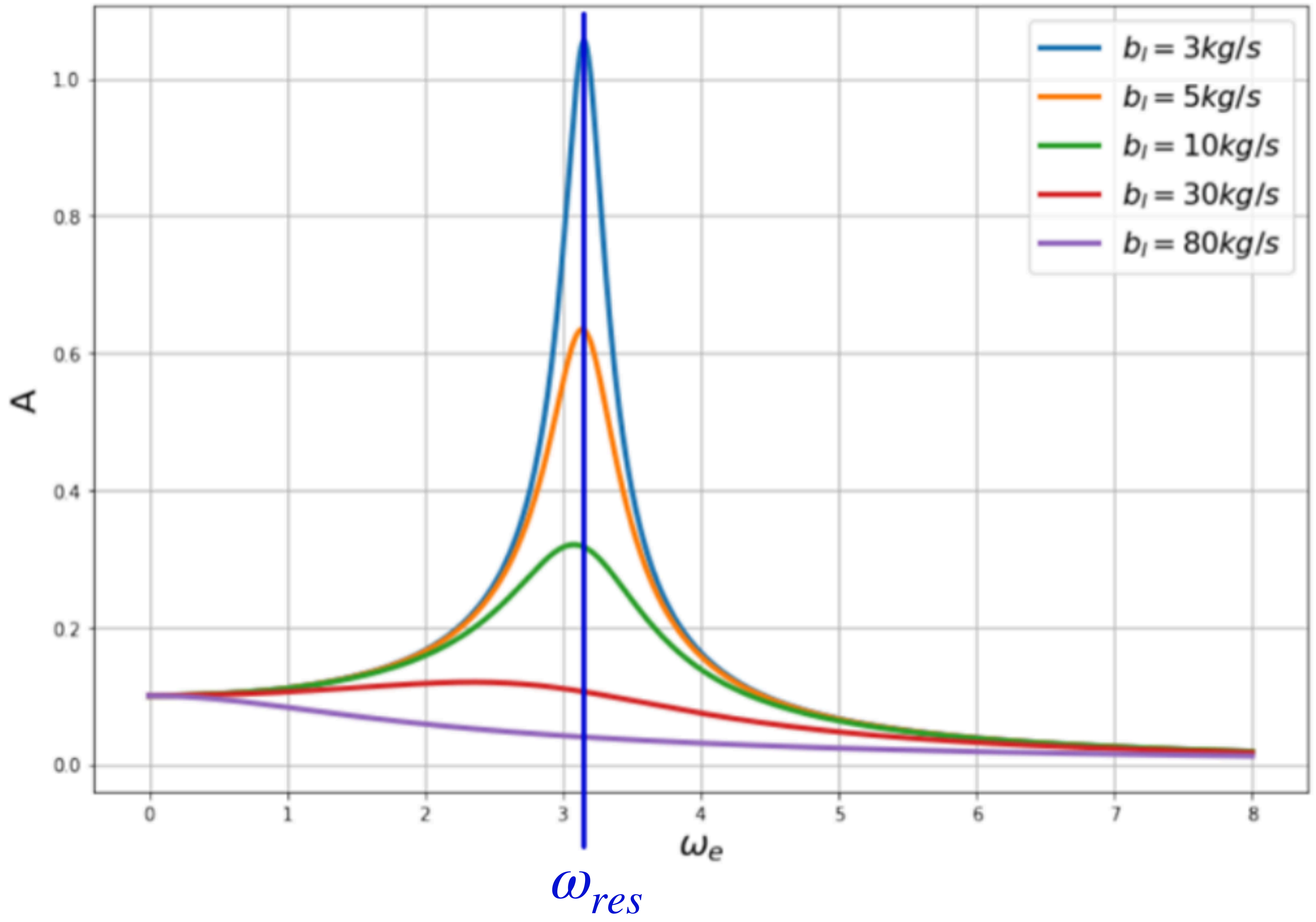





TACOMA BRIDGE
COLLAPSE
PATHE GAZETTE
BRITISH
PATHÉ

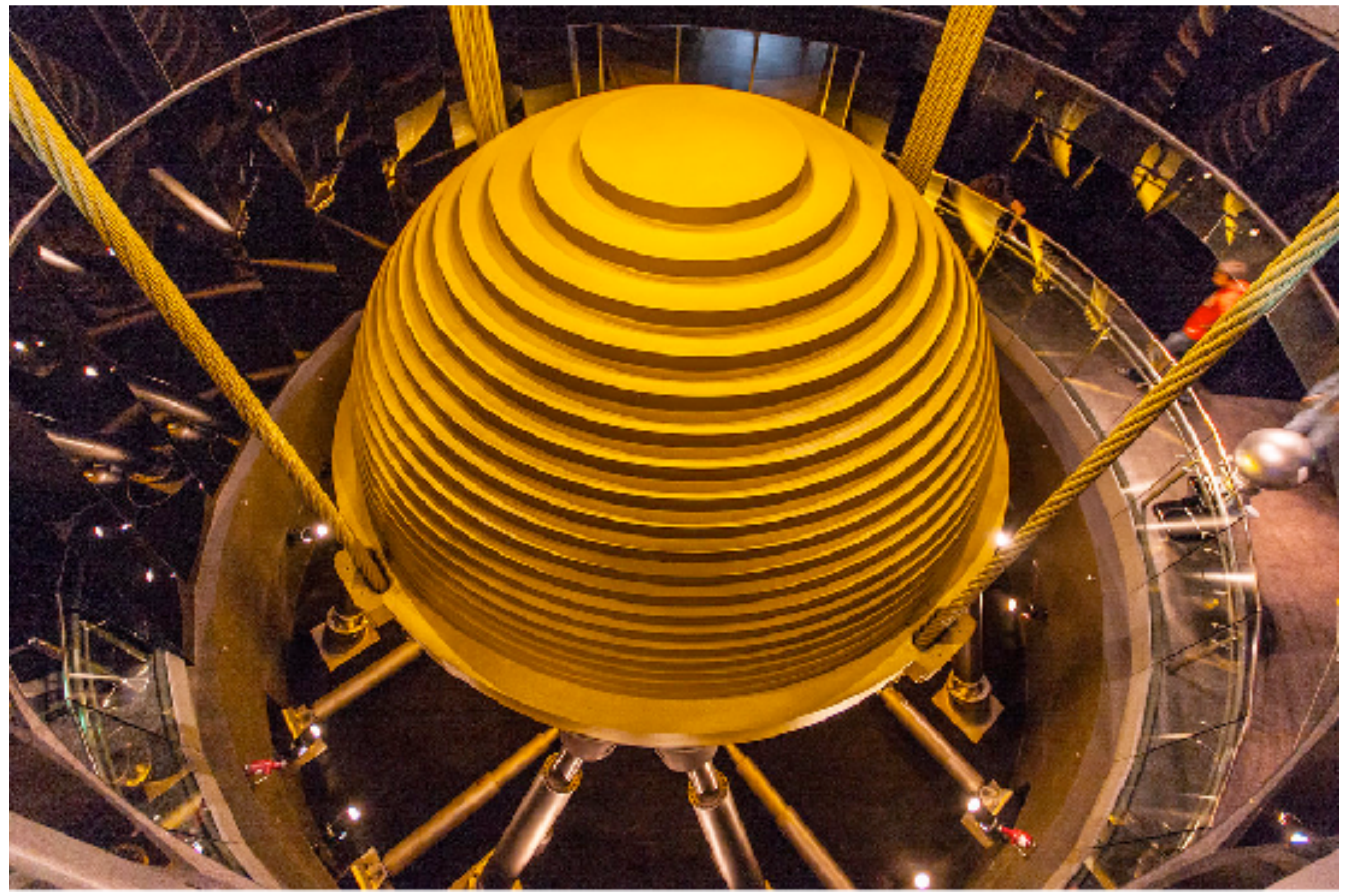
Humen Pearl River Bridge built in 1997

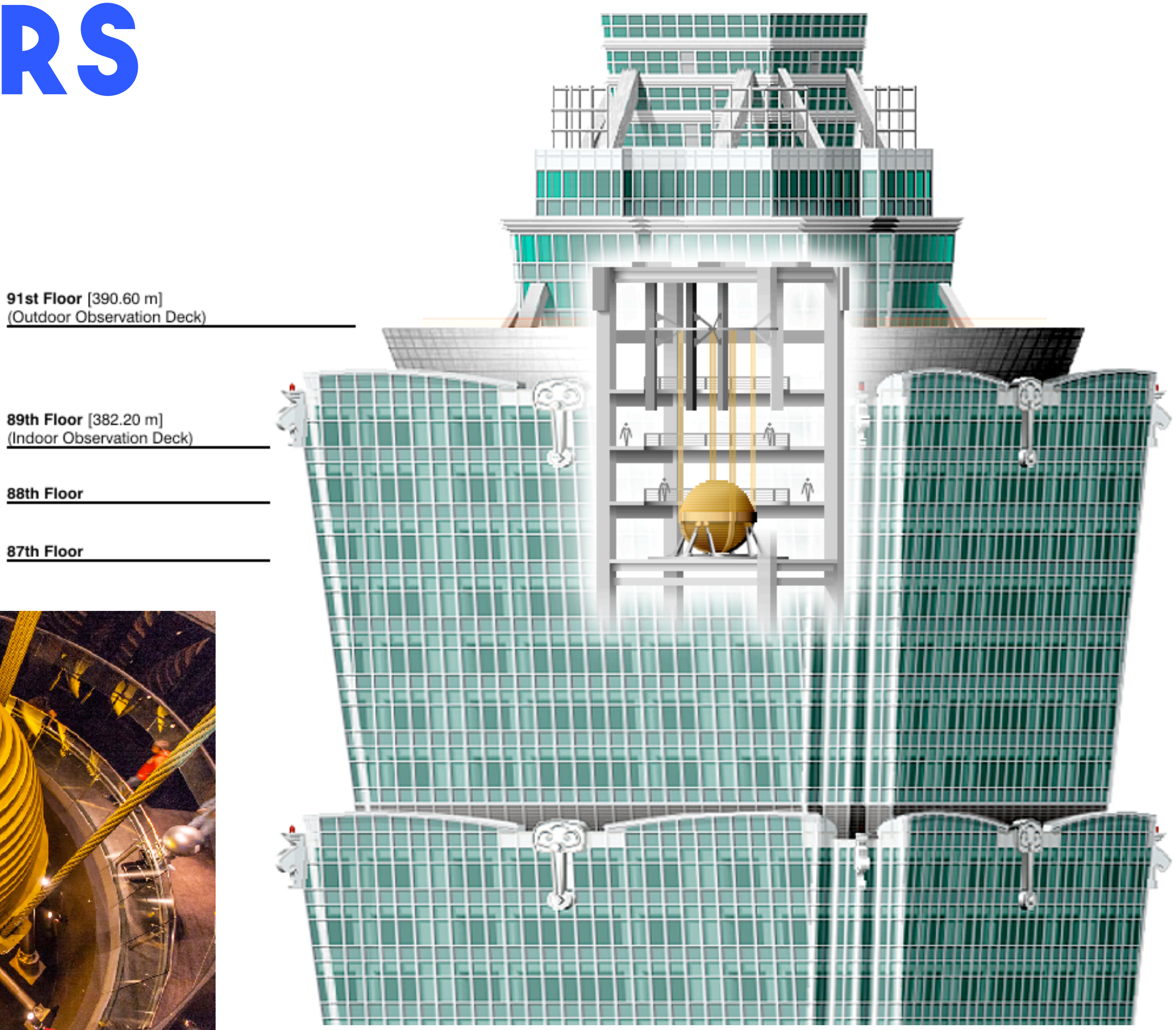
May 2020



$b_l = 3kg/s$
$b_l = 5kg/s$
$b_l = 10kg/s$
$b_l = 30kg/s$
$b_l = 80kg/s$
A
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
ω_e
ω_{res}

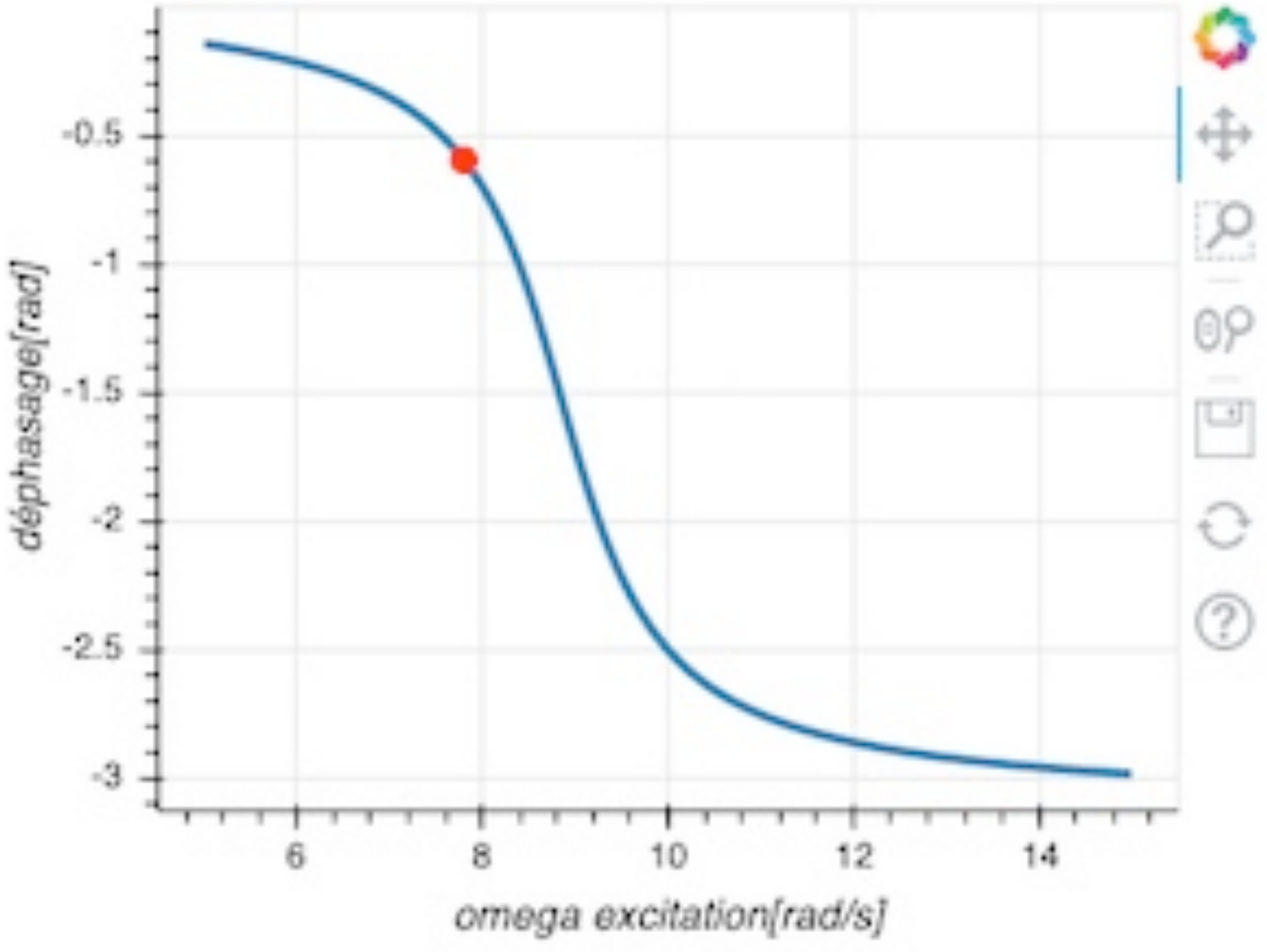
TUNED MASS DAMPERS

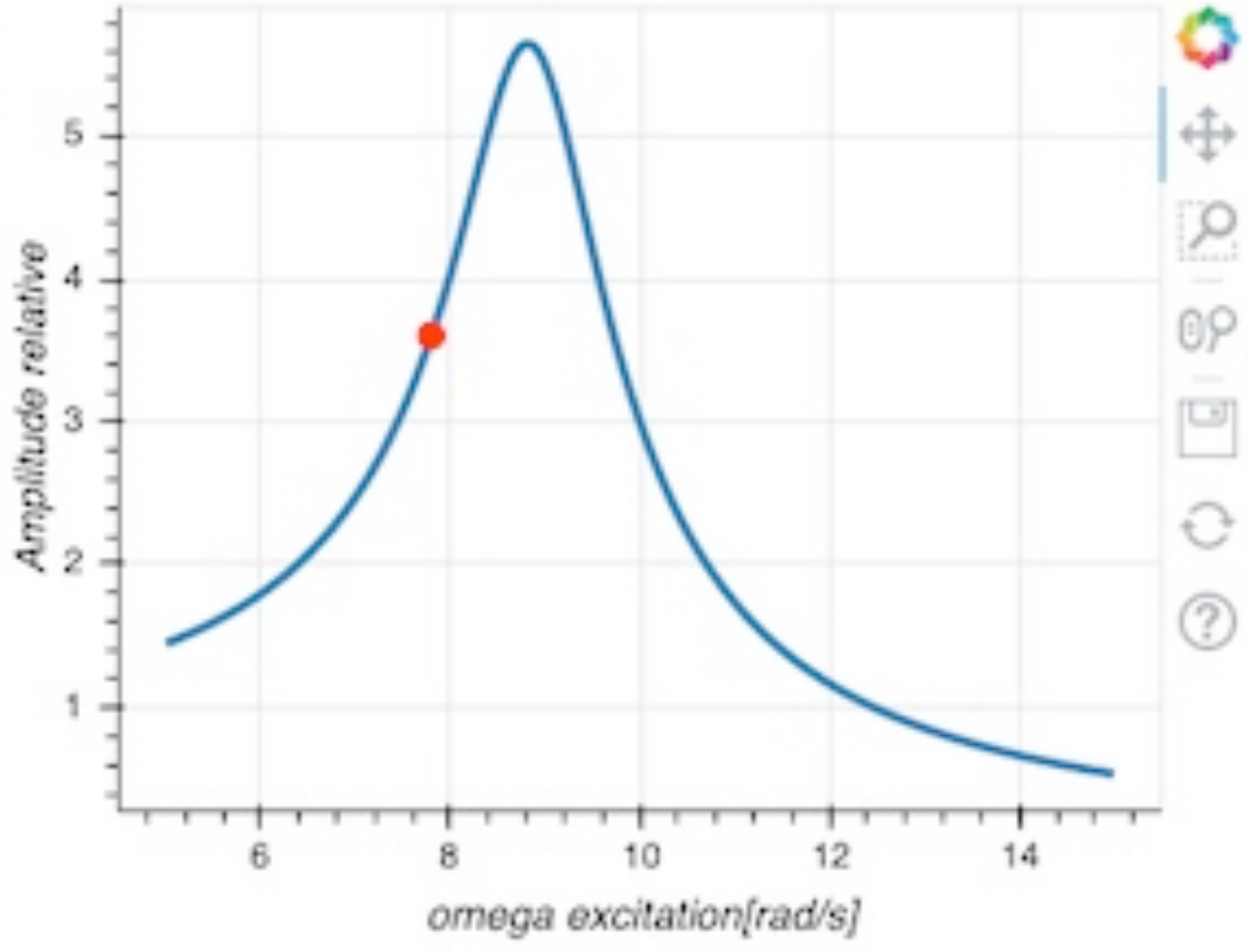




TUNED MASS DAMPERS

QUIZ: DRIVEN HARMONIC OSCILLATOR





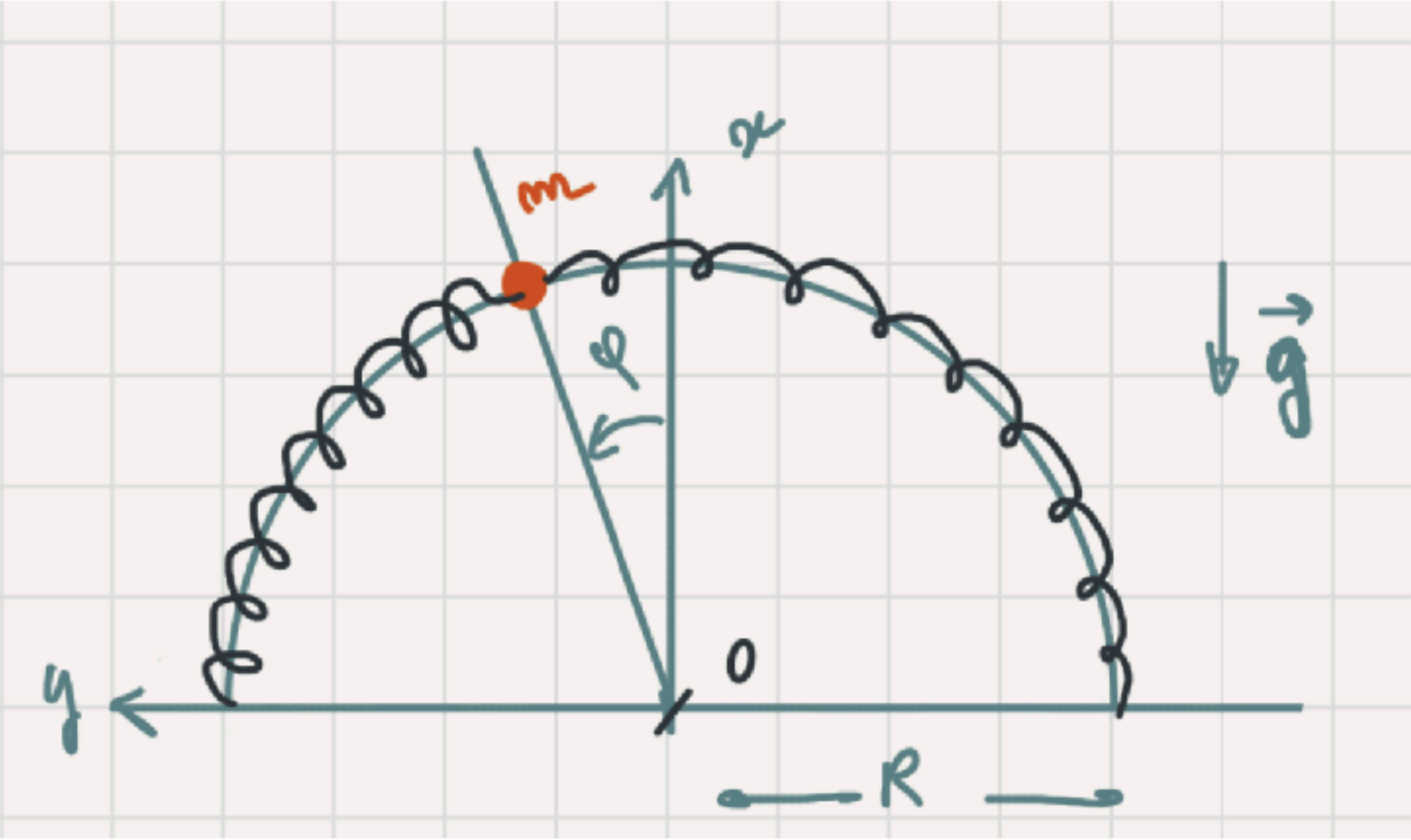
Regarding the driven oscillator, and in particular the graph of amplitude and phase shift as a function of the excitation frequency:

- I have absolutely no idea what it represents — 0%
- I understand that these are the function graphs A(we) and φ(we), but I don't see the connection to the experiment. — 0%
- OK, I see what it represents and the link to the experiment, but I don't see how to use it. — 0%
- It's very clear; it's a very concise representation of the oscillator. — 0%

EXERCISE: METRONOME

(Question from an old exam)

We want to study a device intended to serve as a metronome (an instrument used in music to keep time). It consists of a rigid semicircular rail of radius R, fixed vertically, a mass m that slides on the rail, and two springs on the rail. All friction is neglected. The springs have a spring constant k and a rest length $l_0 = R\pi/2$



a) List the forces acting on the system and represent them on the diagram.

b) Express the acceleration of mass m in the polar coordinates of the drawing, taking into account the characteristics of the device.

c) Establish the equations of motion in polar coordinates.

d) Show that position $\phi = 0$ is an equilibrium position.

e) Establish the differential equation of motion for ϕ in the case where ϕ remains small.

f) Still in the case where ϕ remains small, what condition linking k, m, and R ensures that the system behaves as a harmonic oscillator?

g) Express the potential energy of the mass as it moves along the rod, as a function of the ϕ-coordinate, first in the general case, then in the case of small angles. Recall that for small angles, $\cos(\phi) \approx 1 - \phi^2/2$

h) In the approximation where ϕ remains small, (re)discover, using the potential energy, the condition on k for the system to behave as a harmonic oscillator.

i) Calculate the expression for the mechanical energy in the general case (no assumptions about ϕ), then use it to derive the differential equation of motion of the mass

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