# Exercises

**Exercise 1** *Spending all of one's energy*
Consider a harmonic oscillator in sinusoidal motion driven by a force $F_0 \cos \omega_e t$. The oscillator consists of a spring with stiffness $k$ attached to a mass $m$ and damped by fluid friction with a friction coefficient $b_l = K\eta$.

1. Write the expression for the position of the mass $x(t)$ as a function of the problem data and the excitation frequency $\omega_e$, in the steady state regime.
2. Calculate the energy dissipated during a period $T = \frac{2\pi}{\omega_e}$.
   We give $\int_0^{2\pi} \sin^2(\theta) d\theta = \pi$. This is found using : $\int_0^{2\pi} \sin^2(\theta) d\theta = \int_0^{2\pi} \cos^2(\theta) d\theta = I \Rightarrow 2I = \int_0^{2\pi} (\cos^2(\theta) + \sin^2(\theta)) d\theta \Rightarrow 2I = 2\pi \Rightarrow I = \pi$.
3. Deduce the power dissipated in steady state.
4. A damped spring oscillator, excited at resonance, requires power $P_r$ to compensate for the damping. The damped mass is denoted by $m$. If the machine is turned off, the amplitude decreases by half within one second.

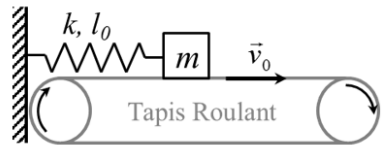
   What is the amplitude when the machine is running ?
   Note : $f_{res} = 20$ Hz ; $P_r = 800$ W ; $m = 100$ kg.
5. *(optional)* Show that $P_{diss}$ is maximal for $\omega_e = \Omega_0$.

**Exercise 2** *Chauve qui peut !*
The Joker has managed to trap Batman ! Batman finds himself suspended at the end of a rope of length $l$ between two giant propellers as sharp as razor blades. These two propellers rotate at variable speeds, so that the resulting wind exerts a total force $\vec{F}_h = F_0 \cos(\omega_e t) \vec{e}_\varphi$ (perpendicular to the rope) on Batman. We consider Batman to be a point particle of mass $m$ oscillating in a plane containing the axes of rotation of the propellers, and we consider the rope to be rigid. We also ignore friction.

a) Give the equation of motion for Batman in the small oscillations approximation.
b) Express the amplitude of Batman's oscillations and the resonance frequency $\omega_{res}$.
c) The rotation of the propellers is controlled by the Joker, so that he can adjust the pulsation $\omega_e$ of the force exerted on Batman. Furthermore, the propellers are far enough apart that the small angle approximation no longer holds true when Batman approaches them. The Joker is a gambler, so he offers the hero a choice between a frequency slightly higher or lower than the resonance frequency $\omega_{res}$ (calculated in point b). Which of these choices will save Batman ? Justify your answer without calculation.

**Exercise 3** *Stay fit*

A mass $m$ is placed on a conveyor belt on which it undergoes a dry friction force. We denote the coefficient of static friction as $\alpha_s$ and the coefficient of dynamic friction as $\alpha_d$. In addition, the mass is attached to the wall by a spring with stiffness $k$, rest length $l_0$, and negligible mass. Let $g$ be the acceleration due to gravity.




a) At the start ($t = 0$), the length of the spring is equal to $l_0$ (i.e., the spring is at rest) and the conveyor belt starts moving. It thus drives the mass $m$ at a constant speed $v_0$, under the effect of static friction. Calculate the time $t_d$ at which the mass slips*, as well as the distance $d$ traveled. Express them in terms of the data given in the problem. The length of the belt is considered to be much greater than the displacement of the mass.

b) Give the equation of motion for mass $m$ immediately after it slips.

c) The previous equation is valid as long as the mass slides on the belt. Give the condition for the mass to stick back to the belt. Qualitatively plot the velocity as a function of time from $t = 0$, assuming $\alpha_d = 0$.

**Exercise 4** *A fragmented exercise*

A shell of mass $m$ explodes into several fragments. The explosion is characterized by a positive factor $Q$. $Q$ is the difference between the kinetic energy of the system after and before the explosion $Q = E^f_{kin} - E^i_{kin}$.

1. Show that if the shell explodes into two fragments, they move in opposite directions in the center of mass (CoM) reference frame.
2. Show that if the shell explodes into three fragments, their momentum relative to the center of mass $G$ lies in the same plane.
3. Show that if the shell splits into two equal fragments, the norm of their momentum and their velocity in the CoM reference frame are equal to $\sqrt{\frac{mQ}{2}}$ and $\sqrt{\frac{2Q}{m}}$, respectively.
4. Show that if the shell splits into three equal fragments, the first two of which have the same velocity in the CoM reference frame and form a right angle, their momentum in the CoM reference frame is respectively equal to $P_1 = P_2 = \sqrt{\frac{mQ}{6}}$ and $P_3 = \sqrt{\frac{mQ}{3}}$.

*. Slipping refers to the moment the mass stops sticking to the conveyor belt, that is, when static friction is no longer sufficient to hold it and sliding begins.