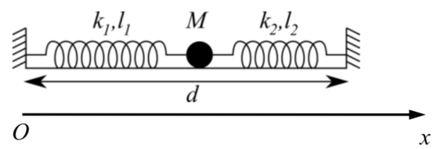
Exercises

Exercise 1 *Caught between two sides*

A point particle with mass M is attached on each side by two springs with spring constants k_1 and k_2 and rest lengths l_1 and l_2. Let d be the distance between the two walls to which the springs are attached. We neglect any friction in this problem.



a) Initially, only the first spring (k_1, l_1) is considered ; the second spring is not attached to the mass. Determine the equation of motion for the mass M.

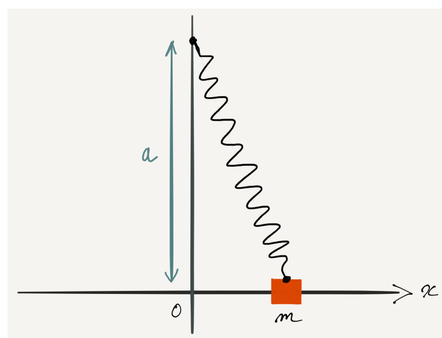
b) We now attach the second spring. Determine the equilibrium position of the mass M.

c) Determine the equation of motion of the mass M.

d) At what frequency does the mass oscillate in both cases ?

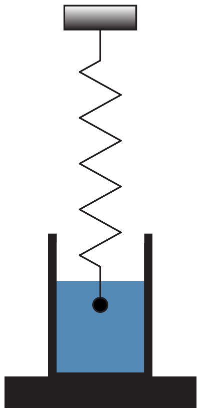
Exercise 2 *In search of rest*

We have a spring R_1 with stiffness k_1, length l_0 at rest, and a point mass m connected to the spring placed vertically, the fixed end of the spring being at a distance a from the axis (Ox). The mass slides without friction on a horizontal rail.



1. First, assume that $a > l_0$. Determine the differential equation of motion and the oscillation frequency. Assume a displacement of $x \ll a$.
2. Let us now consider the case $a = l_0$. What happens to the differential equation obtained previously ? What could explain this result ?

Exercise 3 *Immerse yourself in the problem*



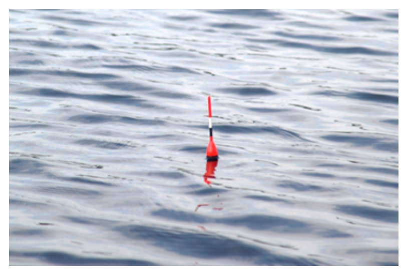
A sphere with radius r and mass m is suspended from a spring with stiffness k and length l_0 at rest. When moved in a liquid with viscosity coefficient η, the sphere is subjected to a frictional force given by Stokes' formula :

$$\vec{F}_f = -6\pi\eta r\vec{v}$$

with $\vec{v}$ the velocity of the sphere.

Establish the equation of motion of the sphere immersed in the liquid and deduce the expression for the period of oscillation T. Make reasonable assumptions about the initial conditions.In air, where fluid friction is negligible, the period of oscillation is T_0. Determine the viscosity coefficient of the liquid as a function of m, r, T, and T_0.

1. **Exercise 4** *Fishing for the right solution*



The float (or bobber) of a fishing rod floats on the surface of the water. It is cylindrical in shape with radius r, height h, and uniform mass. The float remains vertical in the water and moves up and down while remaining partially submerged. In addition to its weight, the float is subject to Archimedes' force $\vec{P_A}$ and a viscous friction force $\vec{F} = -k\eta\vec{v}$. The density of the float is two-thirds that of water : $\rho_f = \frac{2}{3}\rho_{water}$.

a) Calculate the height h' of the float that is submerged at equilibrium.

b) Determine the differential equation of the float's motion. Express the undamped angular frequency Ω_0 and the damping coefficient γ as functions of the problem data.

c) We press on the float and it begins to oscillate vertically until it returns to its equilibrium position. What can you say about the type of damping ? Draw the amplitude of the oscillation as a function of time.