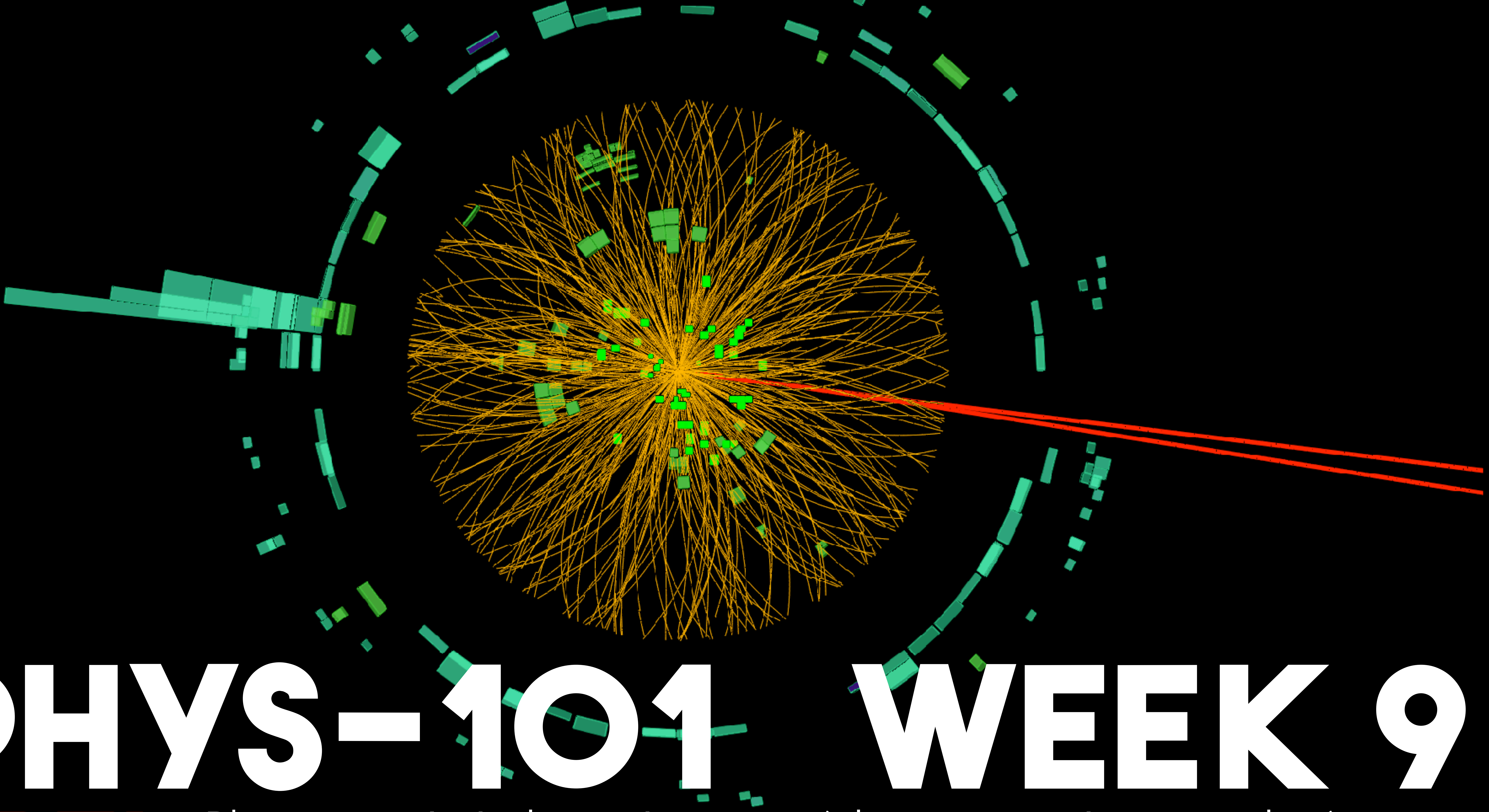


PHYS-101 WEEK 9

EPFL

Physique générale : mécanique (classe inversée en anglais)

Prof. Emma Tolley, 3 November 2025

WEEK 9: COLLISIONS & VARIABLE MASS

- **Mass** is always conserved

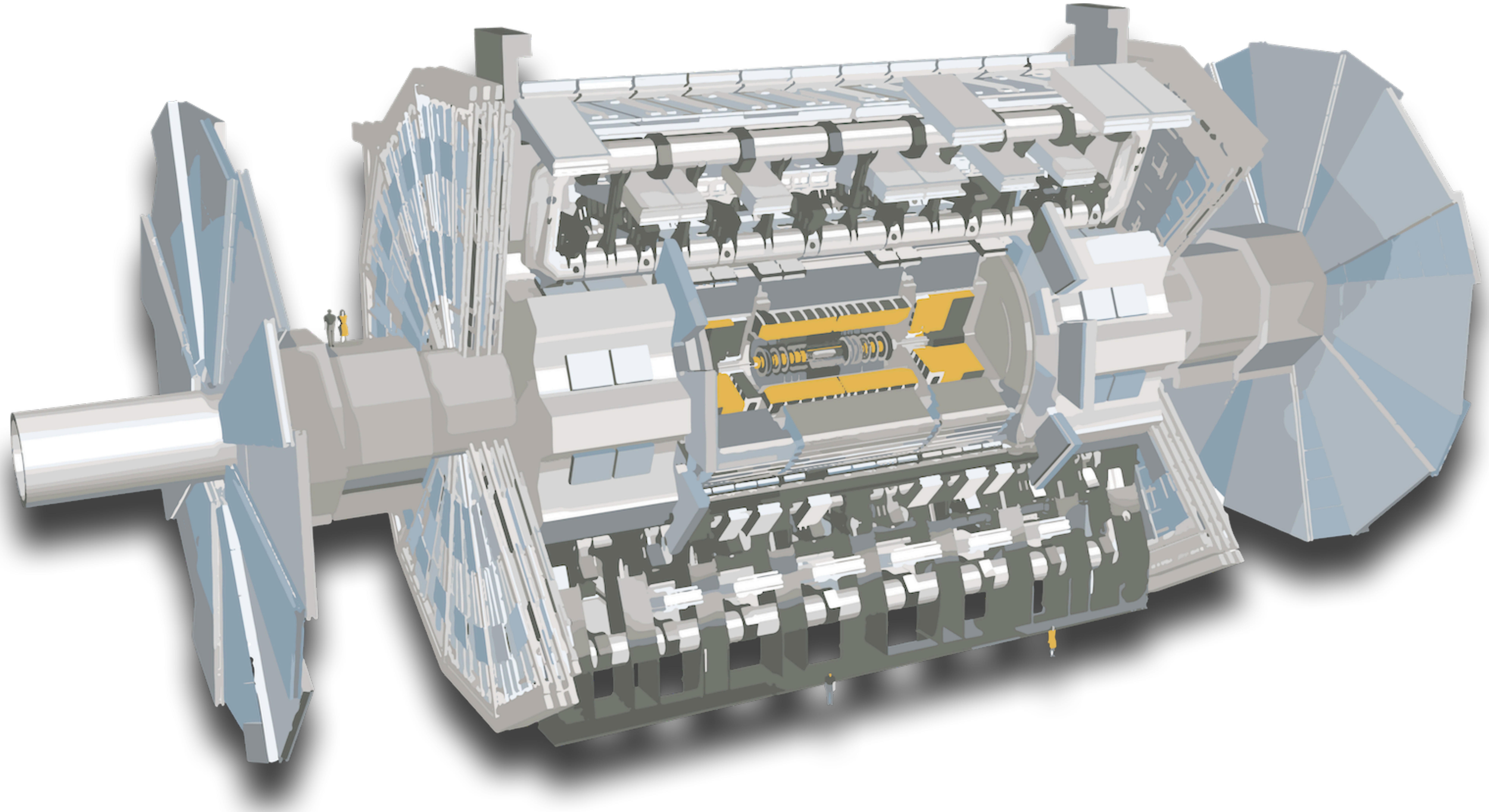
$$m^i_{\text{total}} = m^f_{\text{total}}$$

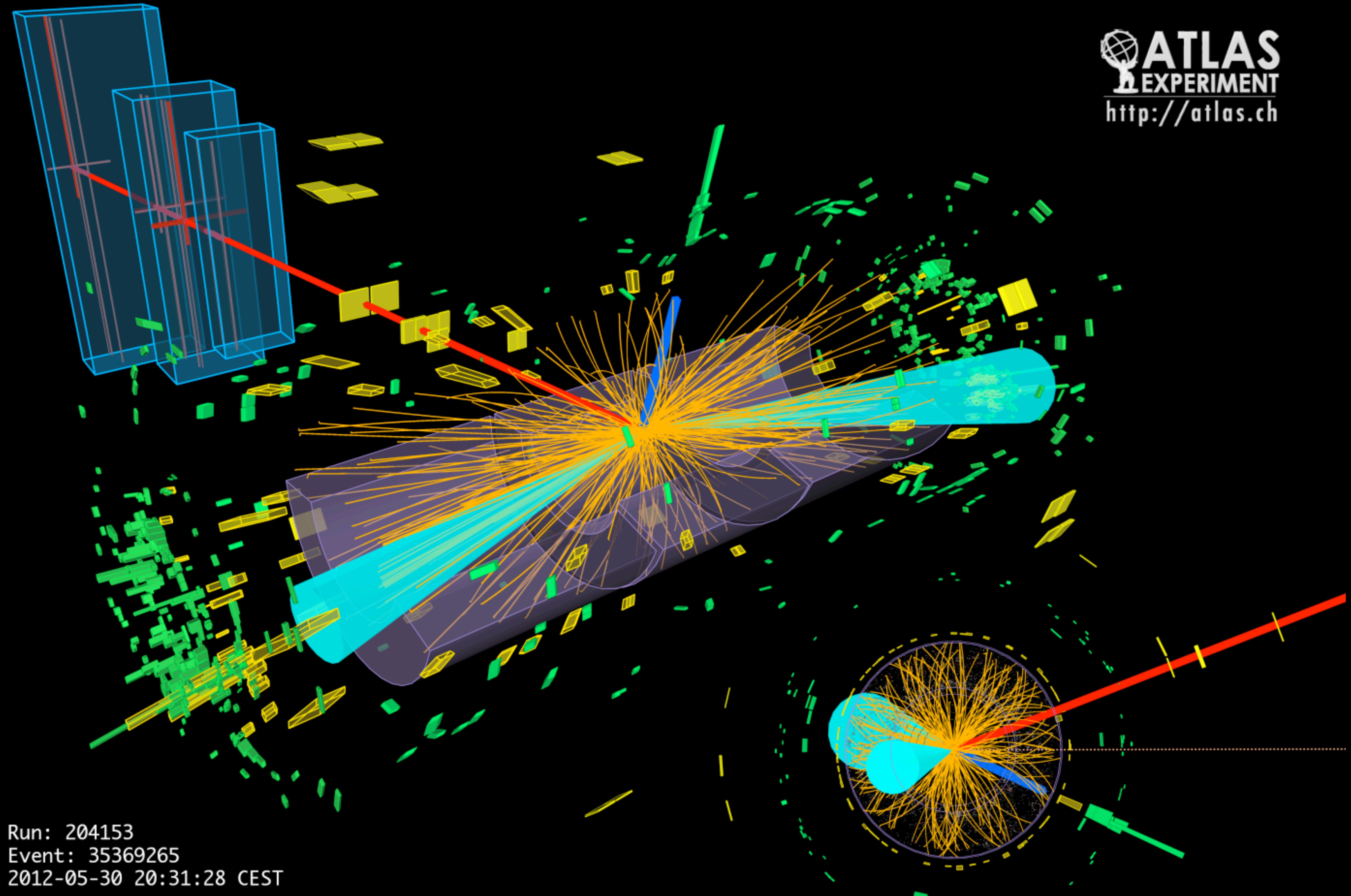
- **Total momentum** is always conserved in a collision

$$\vec{p}^{\,i}_{\text{total}} = \vec{p}^{\,f}_{\text{total}}$$

- **Mechanical Energy** is only conserved in perfectly elastic collisions

$$E^i_{\text{total}} = E^f_{\text{total}}$$

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EXPERIMENT
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DEMO: NEWTON'S CRADLE

WEEK 9: COLLISIONS & VARIABLE MASS

- For head-on elastic collisions:

$$\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_1 + m_2} \qquad \vec{v}_2' = \frac{(m_2 - m_1)\vec{v}_2 + 2m_1\vec{v}_1}{m_1 + m_2}$$

WEEK 9: COLLISIONS & VARIABLE MASS

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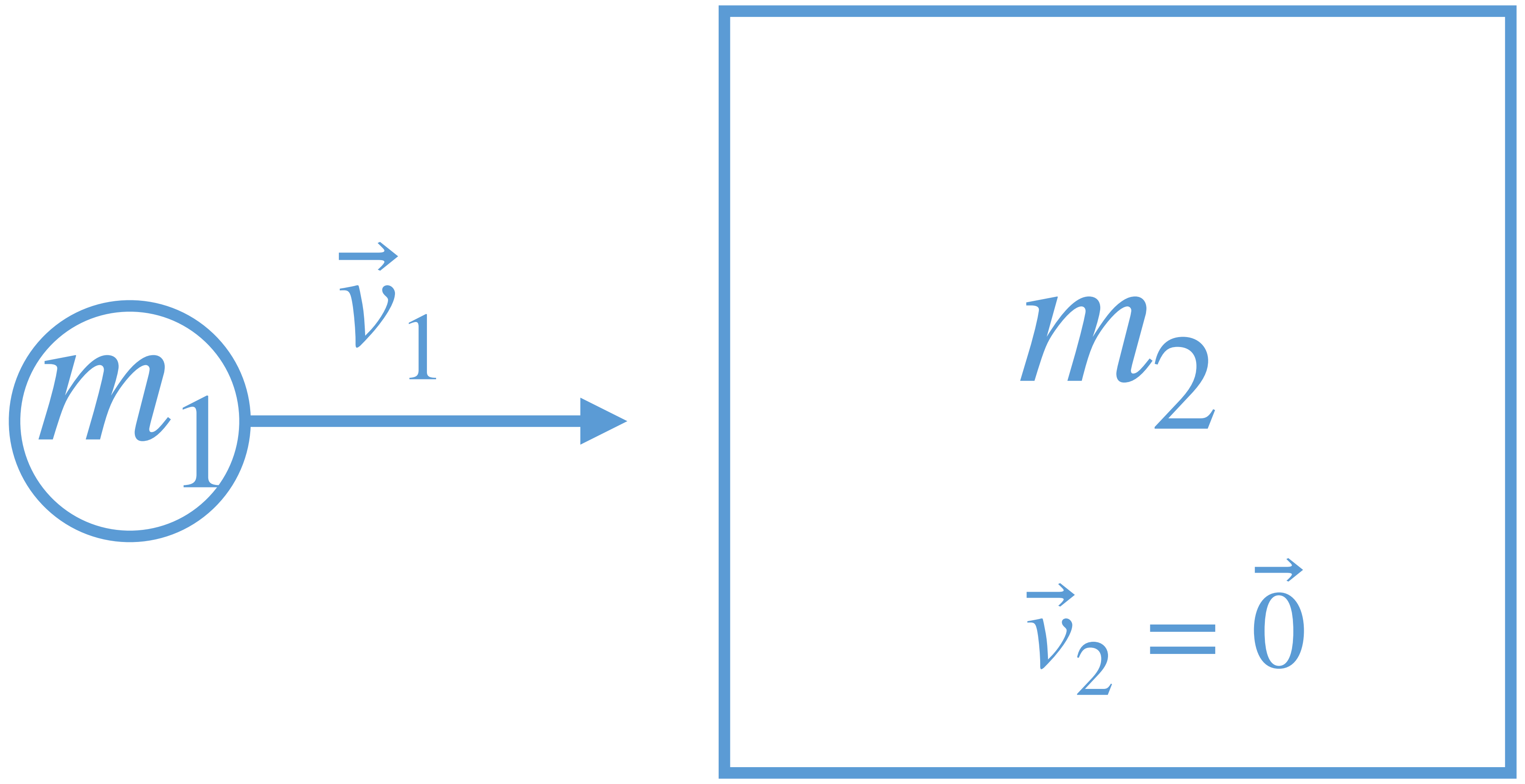


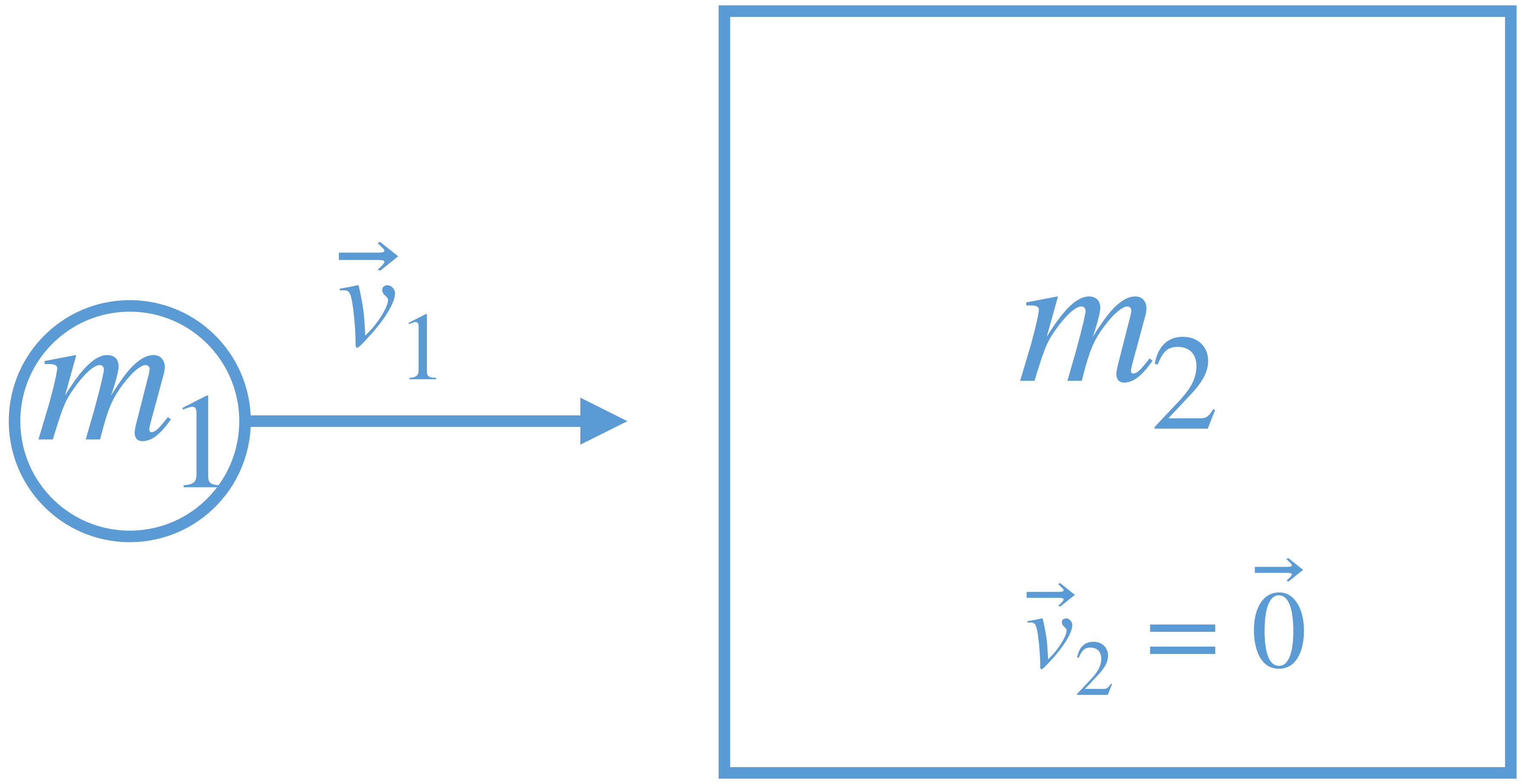
$$m_2 \gg m_1$$

WEEK 9: COLLISIONS & VARIABLE MASS

- For head-on elastic collisions:

$$\vec{v}_1' = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{m_1 + m_2} \qquad \vec{v}_2' = \frac{(m_2 - m_1)\vec{v}_2 + 2m_1\vec{v}_1}{m_1 + m_2}$$

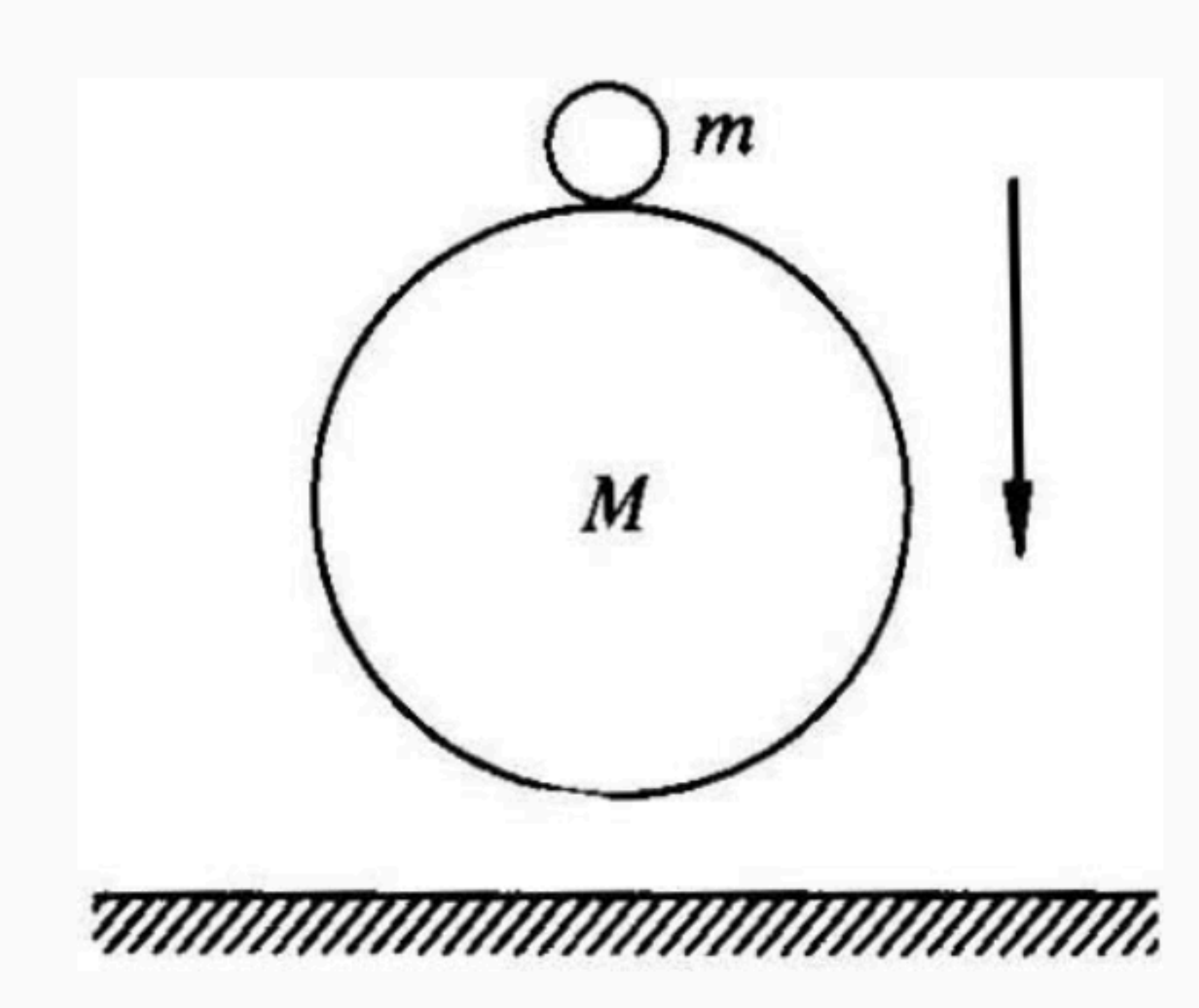
$$m_2 \gg m_1, \quad m_2 + m_1 \approx m_2$$



$$\vec{v}_1' \approx \frac{-m_2\vec{v}_1 + 2m_2\vec{0}}{m_2} = -\vec{v}_1$$

$$\vec{v}_2' \approx \frac{m_2\vec{v}_2 + 2m_1\vec{v}_1}{m_2} = \vec{v}_2 + 2\vec{v}_1\frac{m_1}{m_2} \approx \vec{v}_2 = \vec{0}$$

EXERCISE: DROP THE BALL



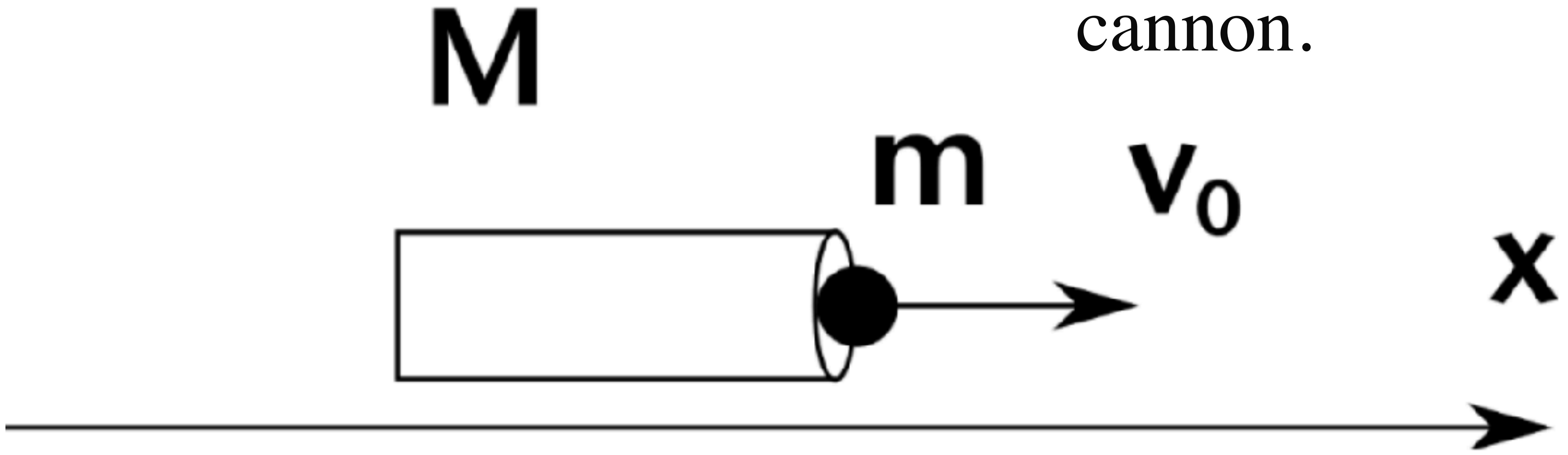
A small ball of mass m is placed on top of a larger ball of mass M, and the two balls are dropped to the floor from height h. How high does the small ball rise after the collision?

Neglect air resistance, assume that the collisions are elastic, that $m \ll M$, and that h is much larger than the radii of either of the balls.

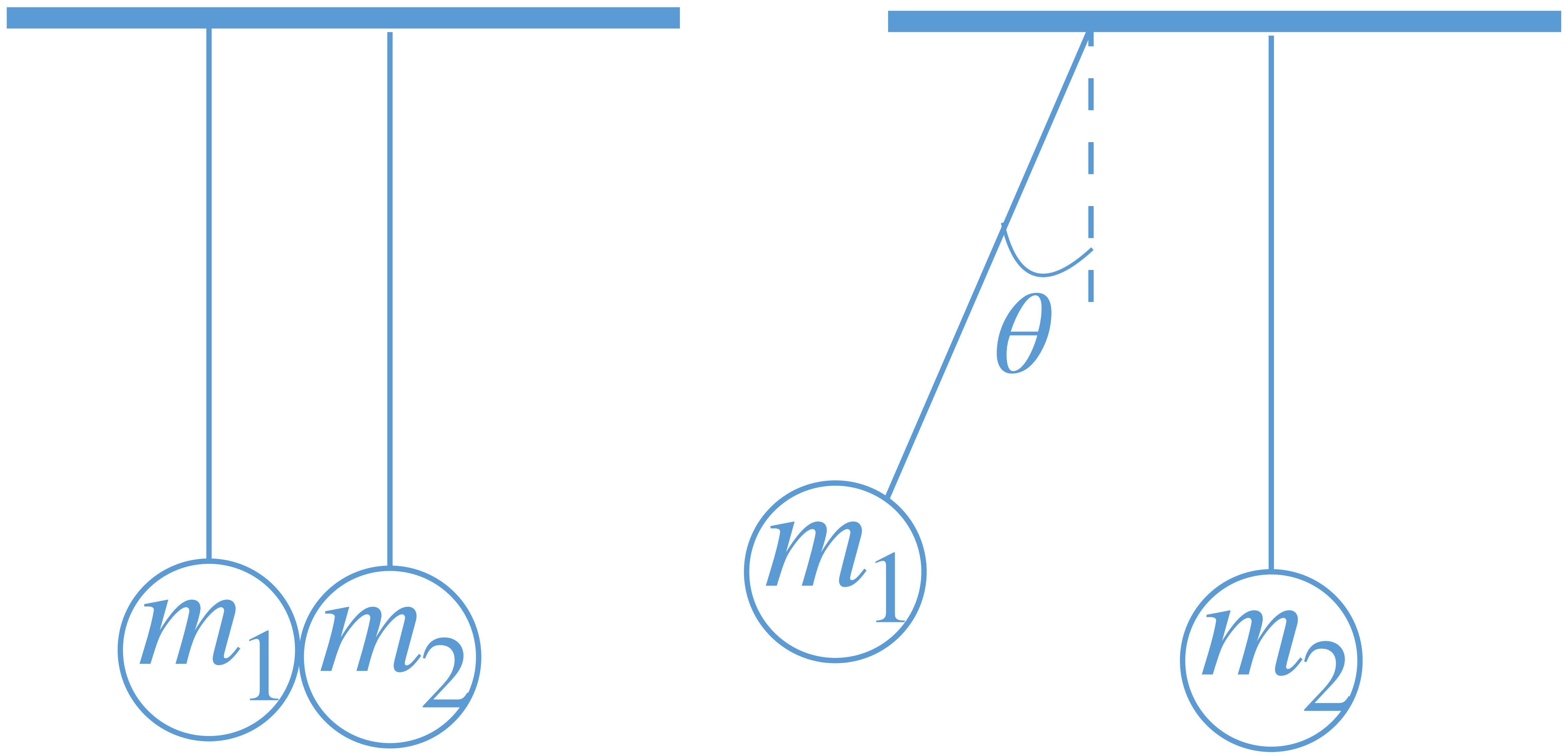
To help visualize the problem, assume that the balls are slightly separated when the larger ball hits the floor.

EXERCISE: CANNONBALL

A canon with mass M is initially immobile on a frictionless air rail. It shoots a bullet with mass m and velocity v0. Calculate the recoil velocity of the cannon.

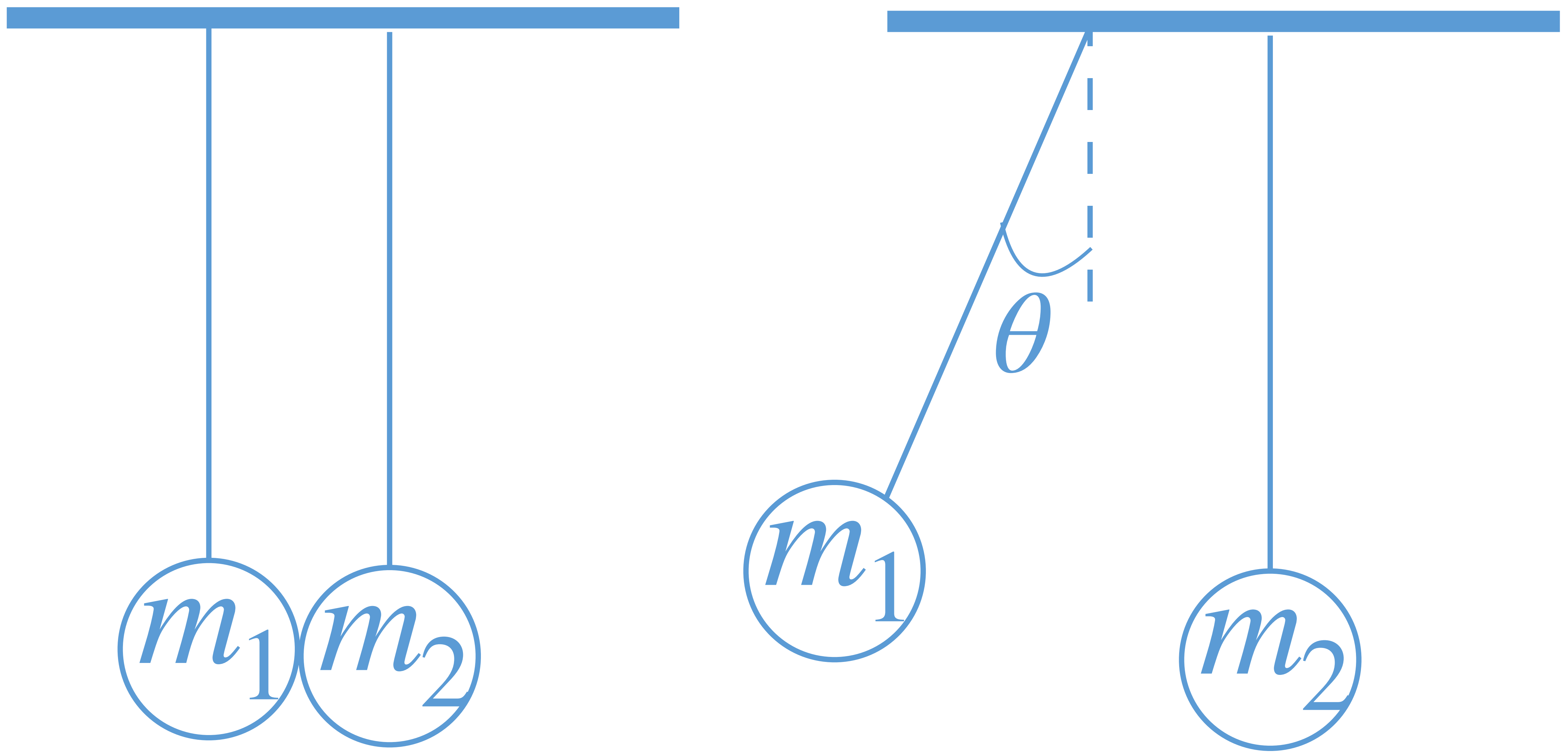


EXERCISE: 2 PENDULUMS



We move m_1 by an angle θ and release it with zero initial velocity. Assuming that the collision is elastic, to what angles do m_1 and m_2 move if $m_1 = 2\,m_2$?

EXERCISE: 2 PENDULUMS



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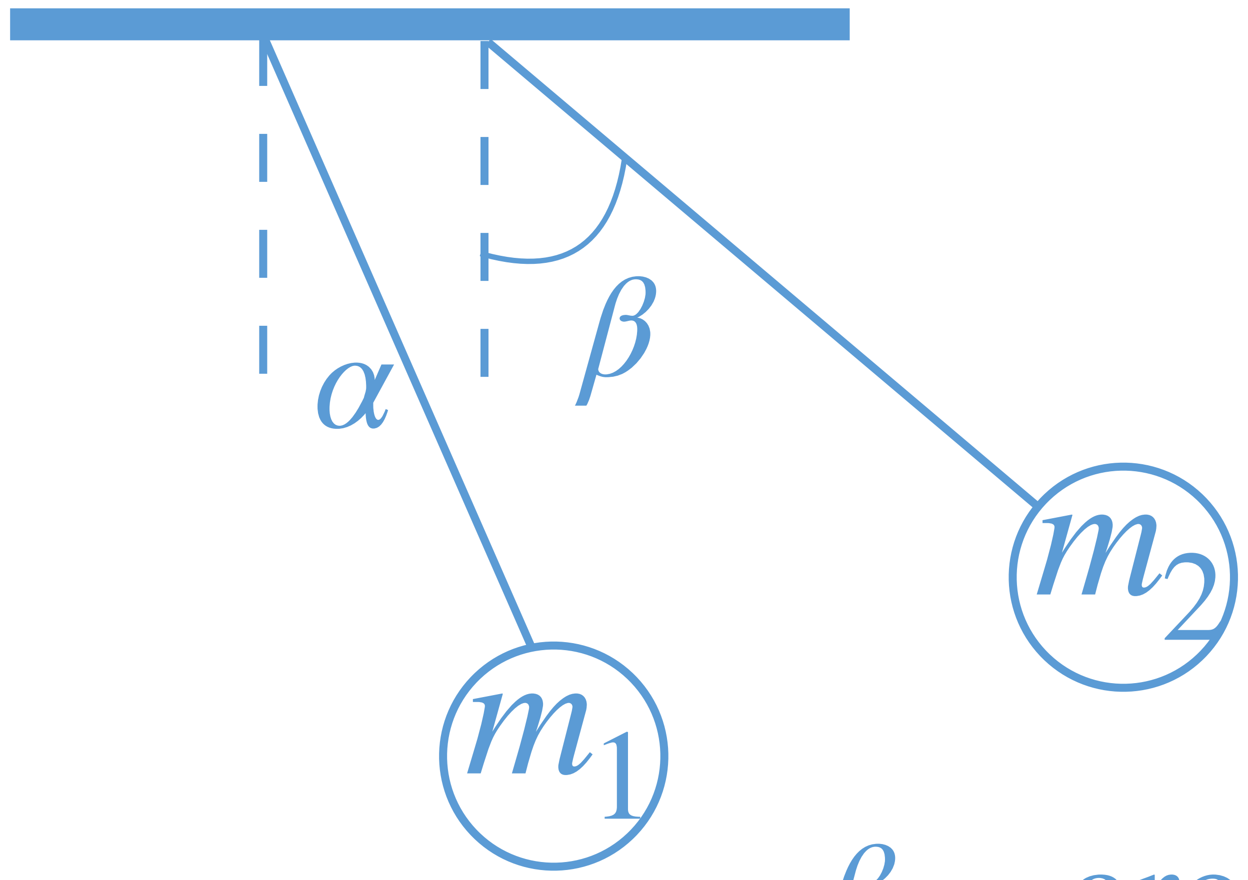
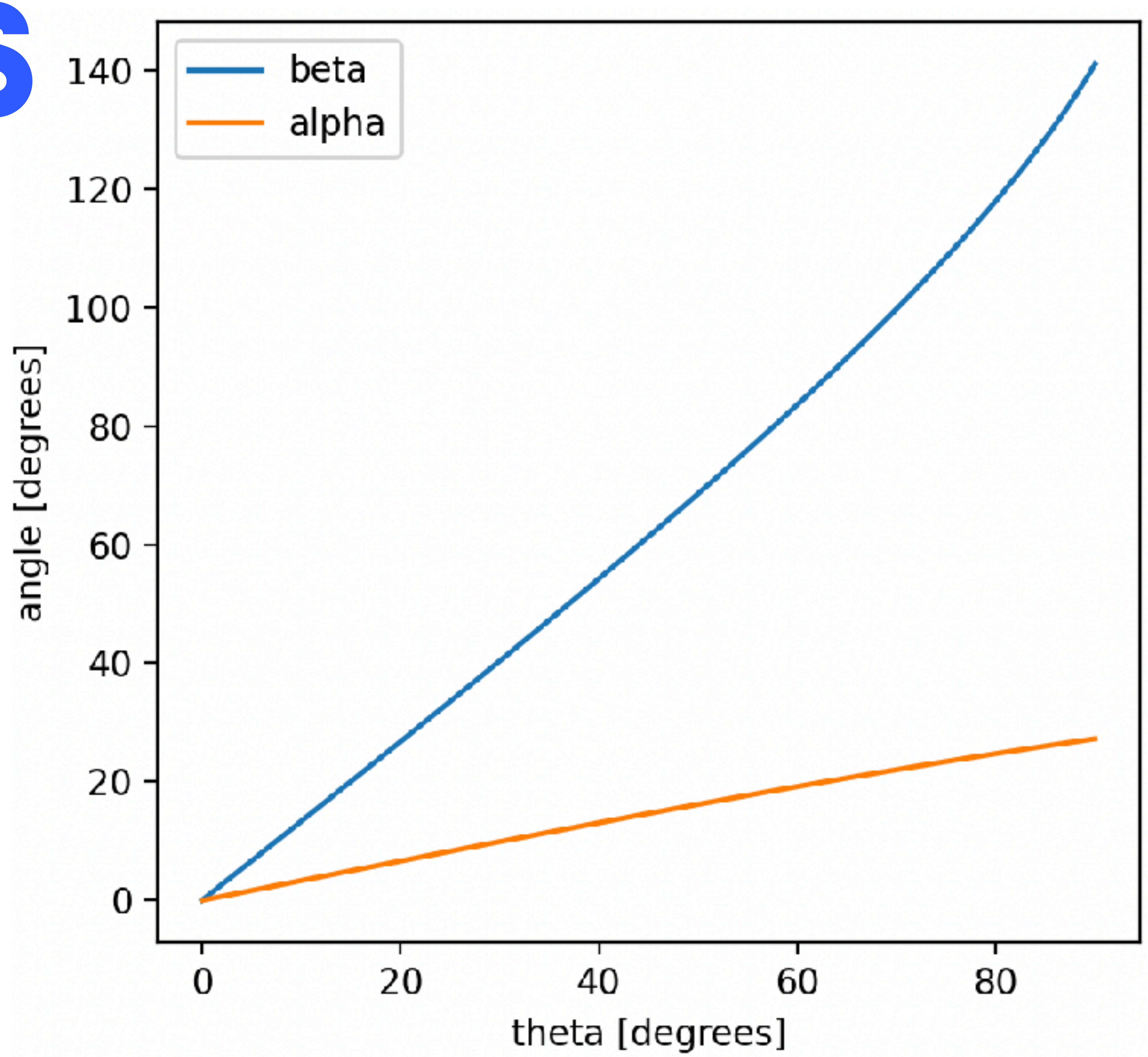
EXERCISE: 2 PENDULUMS

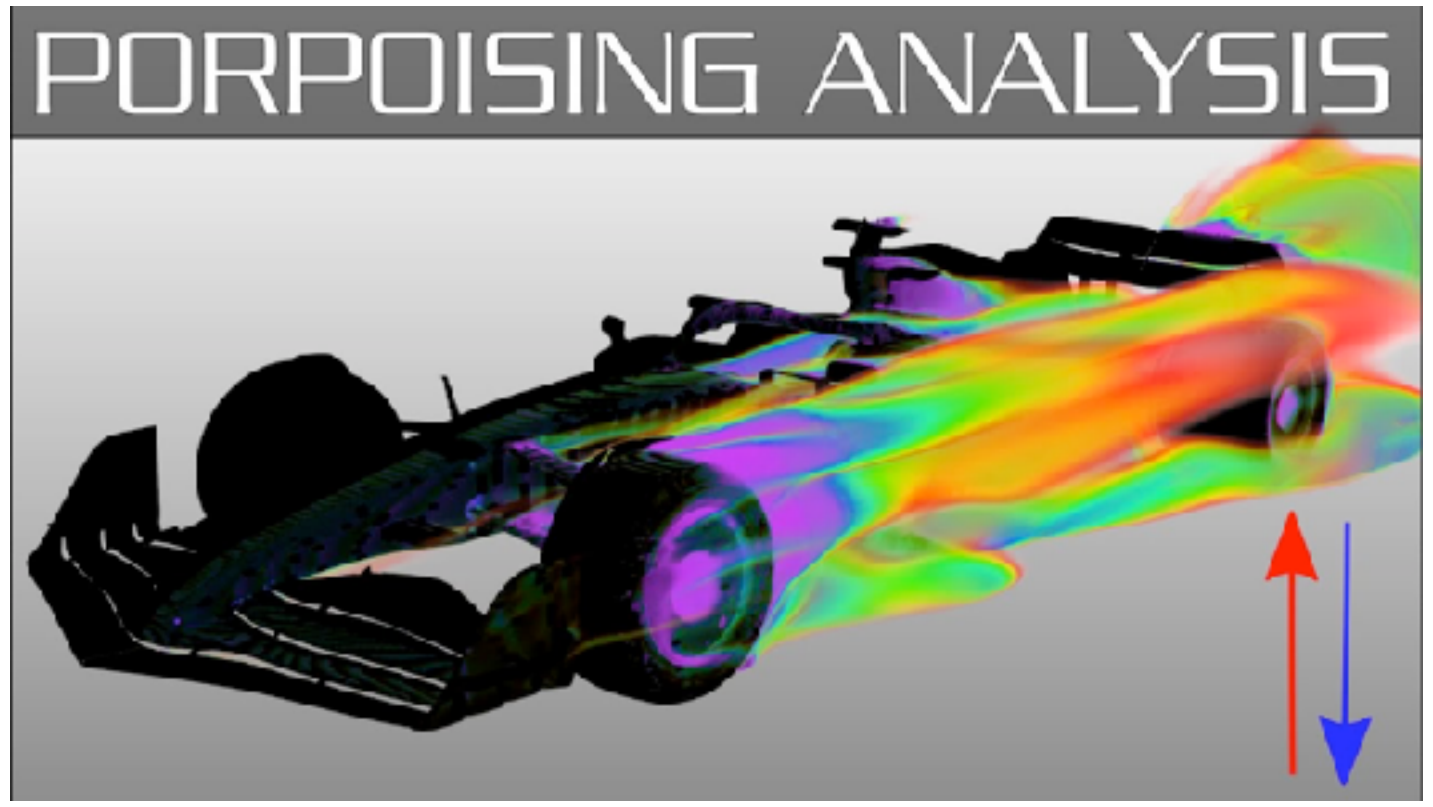


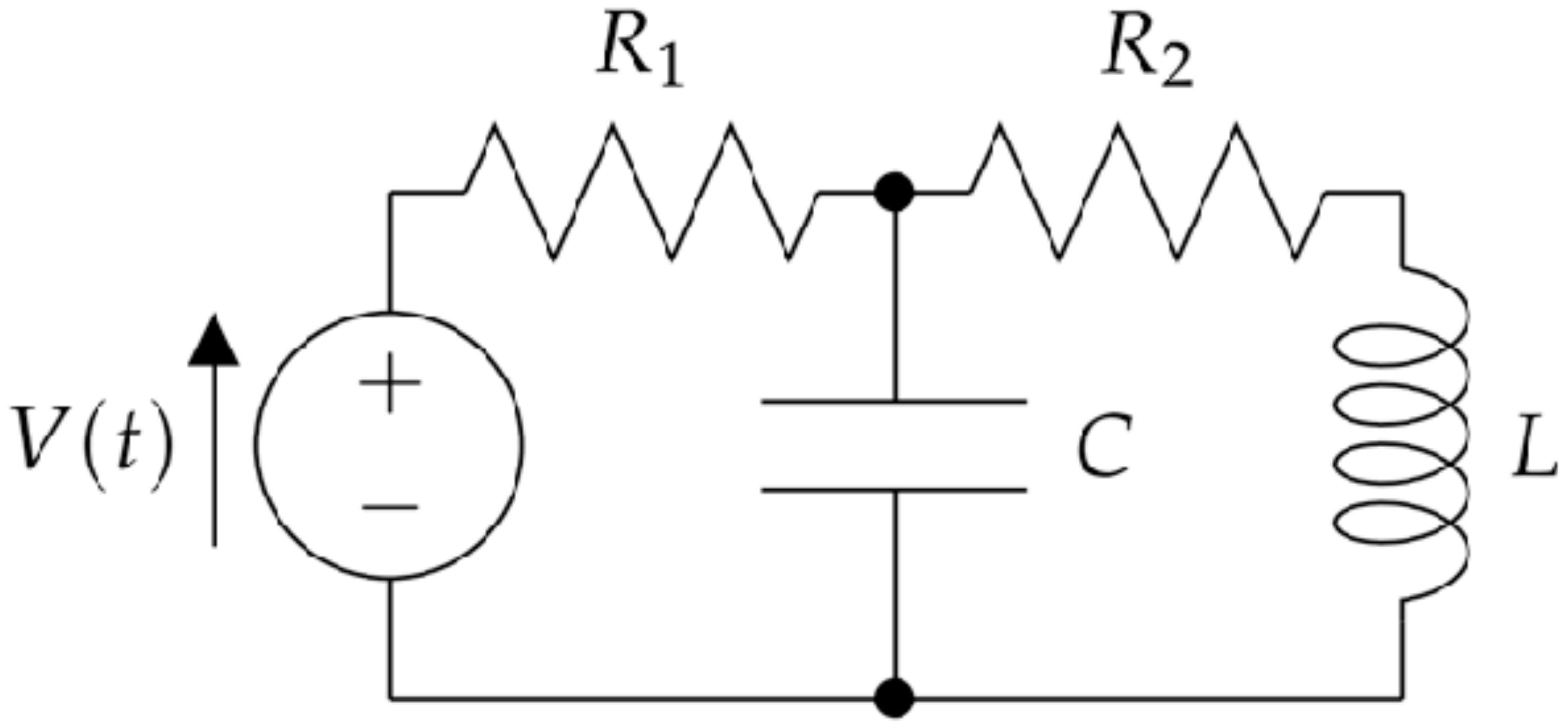
$$\beta = \arccos\left[1 - \frac{16}{9}(1 - \cos\theta)\right]$$

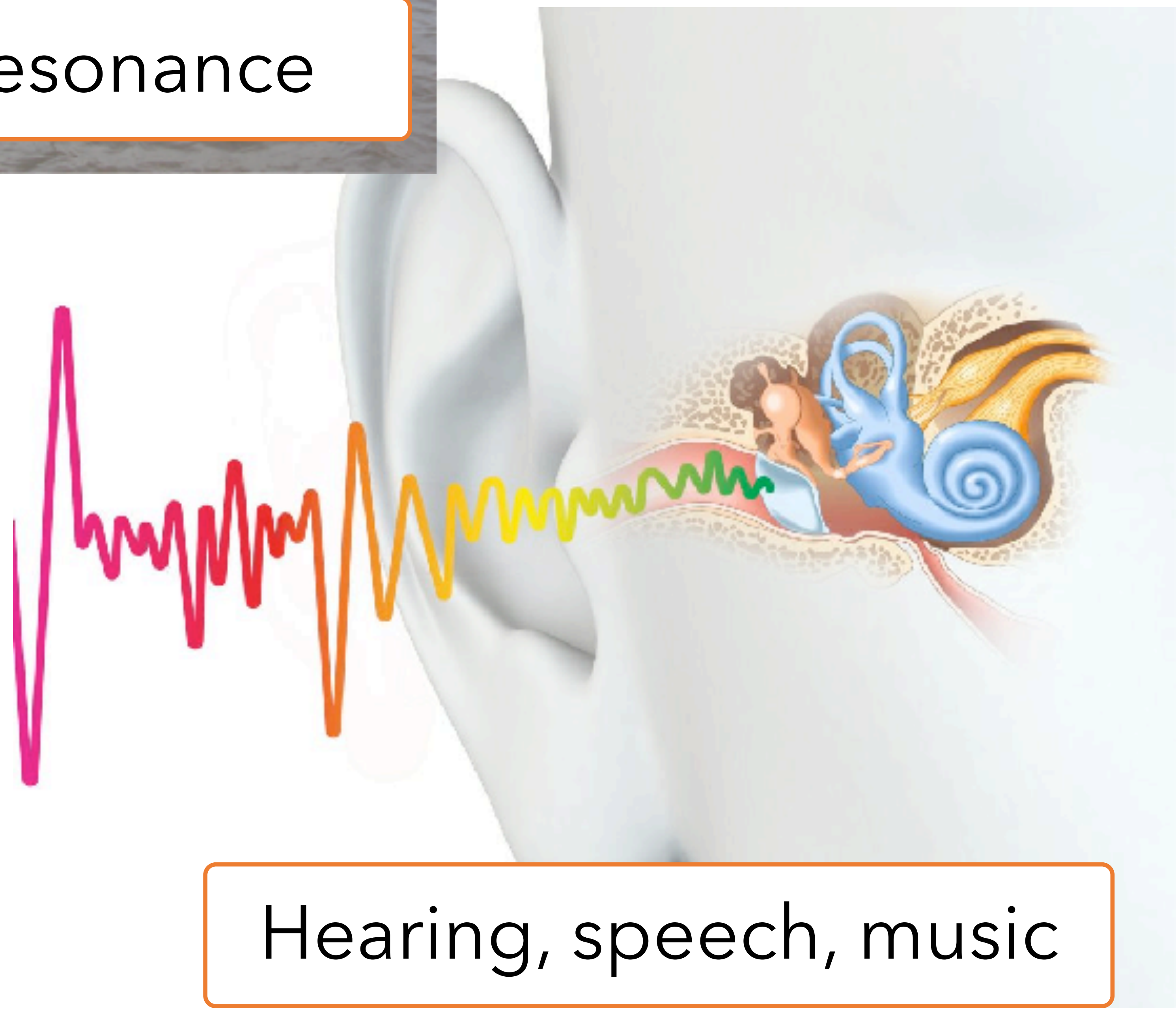
$$\alpha = \arccos\left[1 - \frac{1}{9}(1 - \cos\theta)\right]$$



NEXT 2 WEEKS: HARMONIC OSCILLATOR



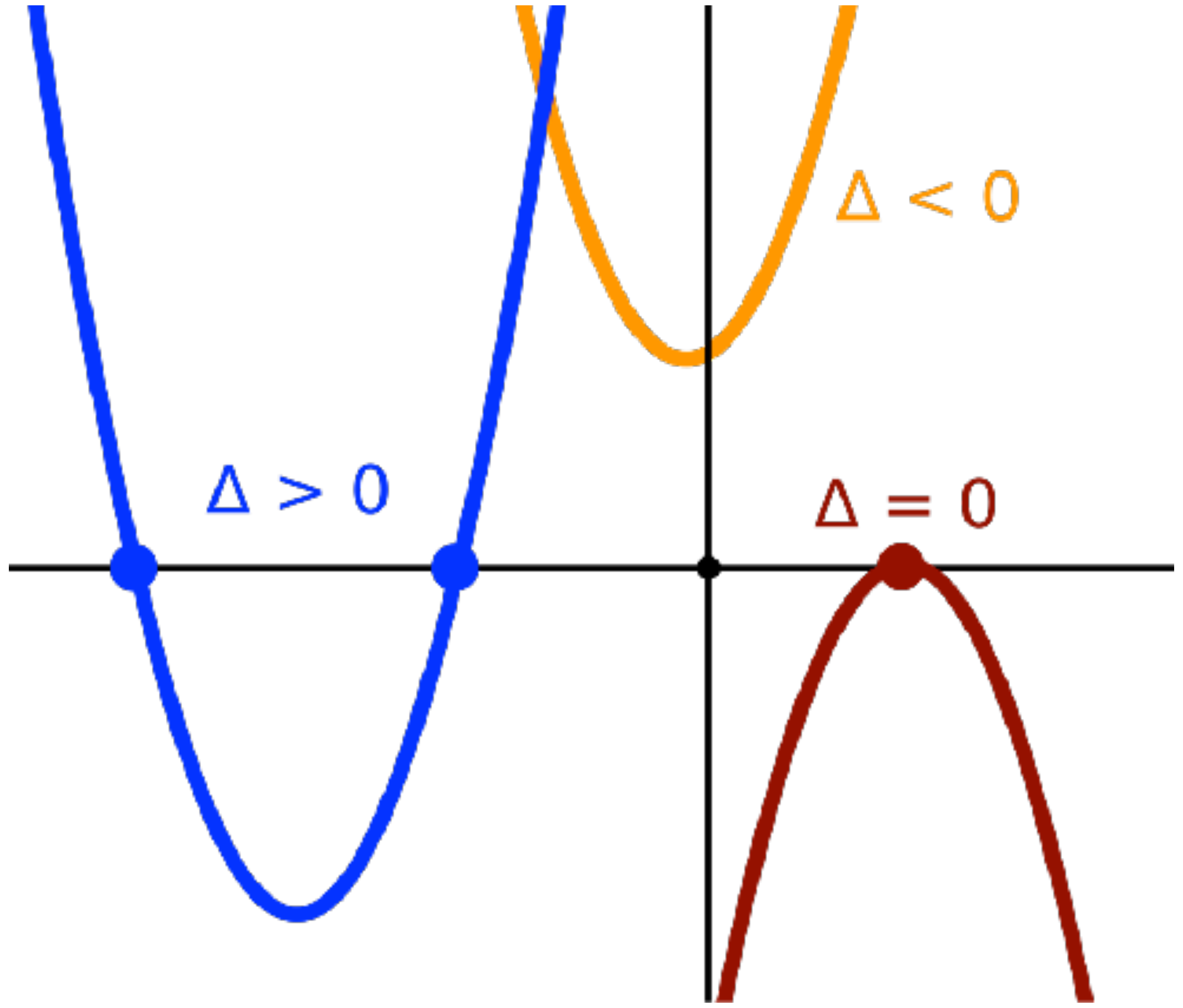




CONCEPTS FOR NEXT WEEK

Quadratic formula:

$$ax^2 + bx + c = 0$$



Solutions:

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Discriminant:

$$\Delta = b^2 - 4ac$$

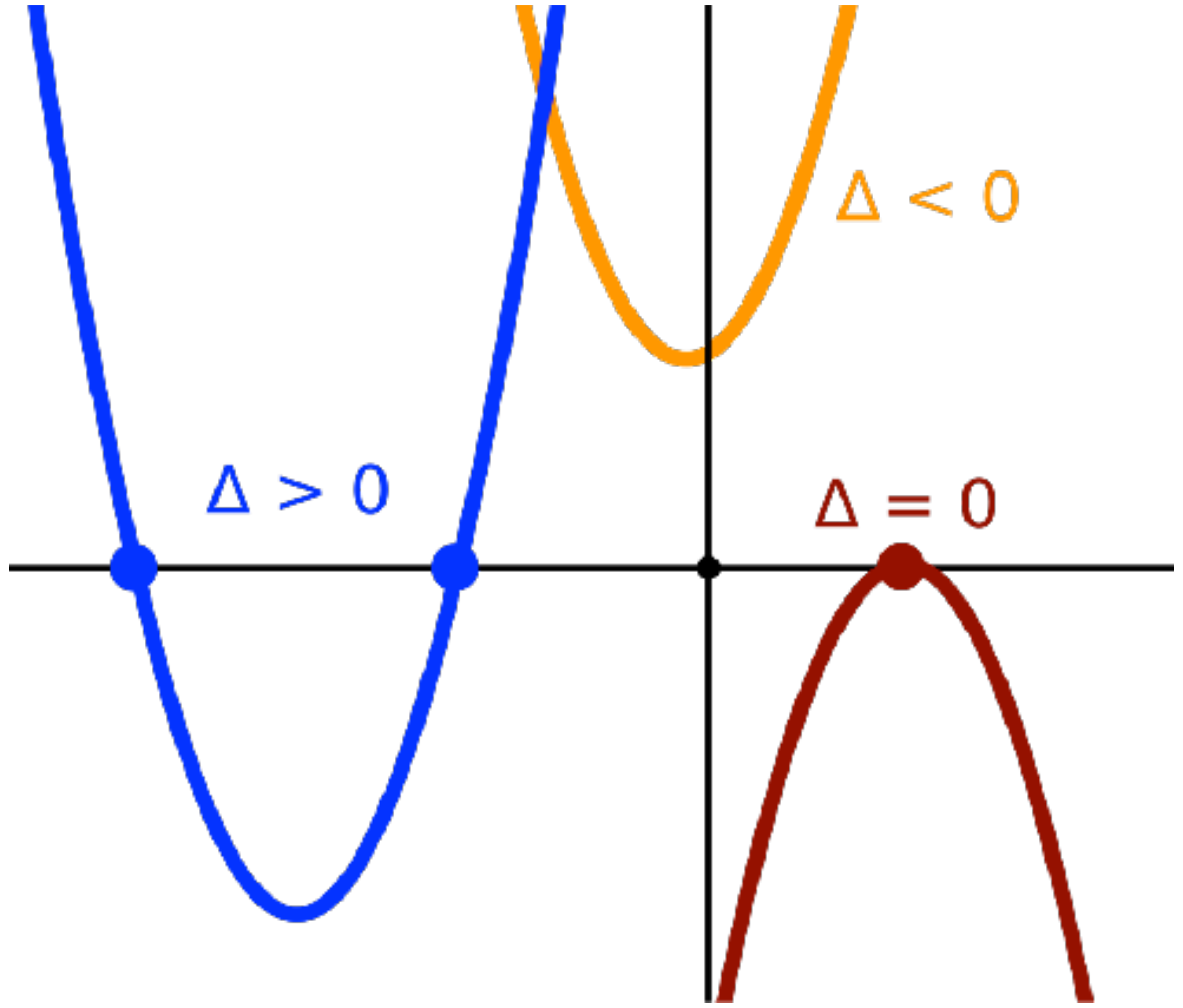
$\Delta = 0$ 1 real solution for x

$\Delta > 0$ 2 real solutions for x

CONCEPTS FOR NEXT WEEK

Quadratic formula:

$$ax^2 + bx + c = 0$$



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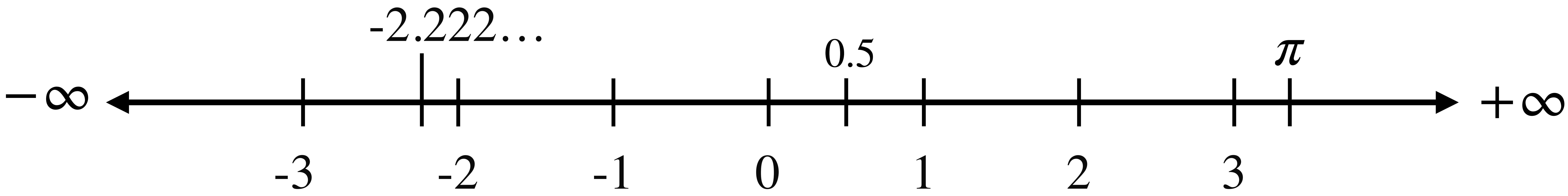
$\Delta = 0$ 1 real solution for x

$\Delta > 0$ 2 real solutions for x

$\Delta < 0$ 2 complex solutions for x

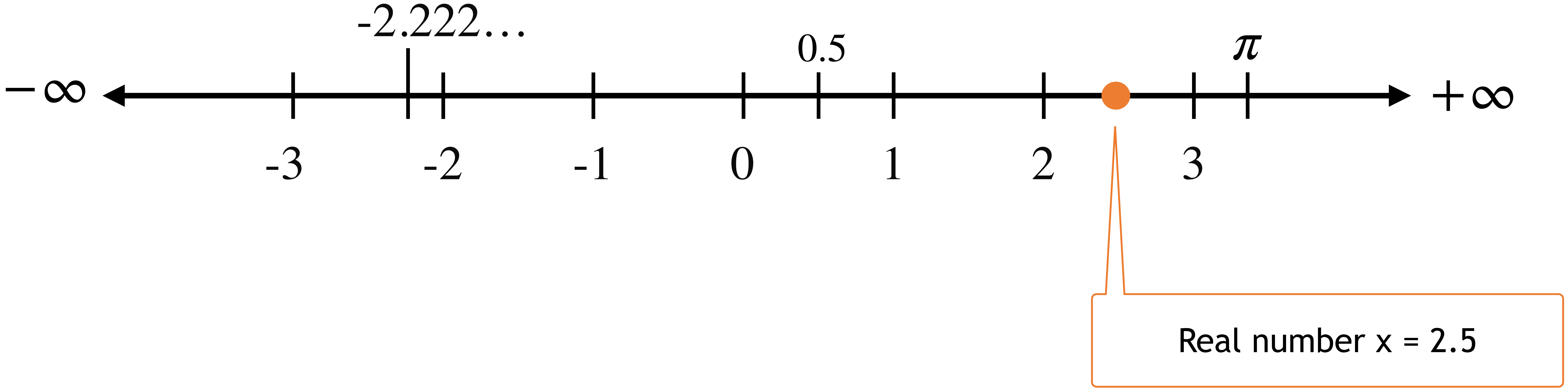
CONCEPTS FOR NEXT WEEK

Real axis



CONCEPTS FOR NEXT WEEK

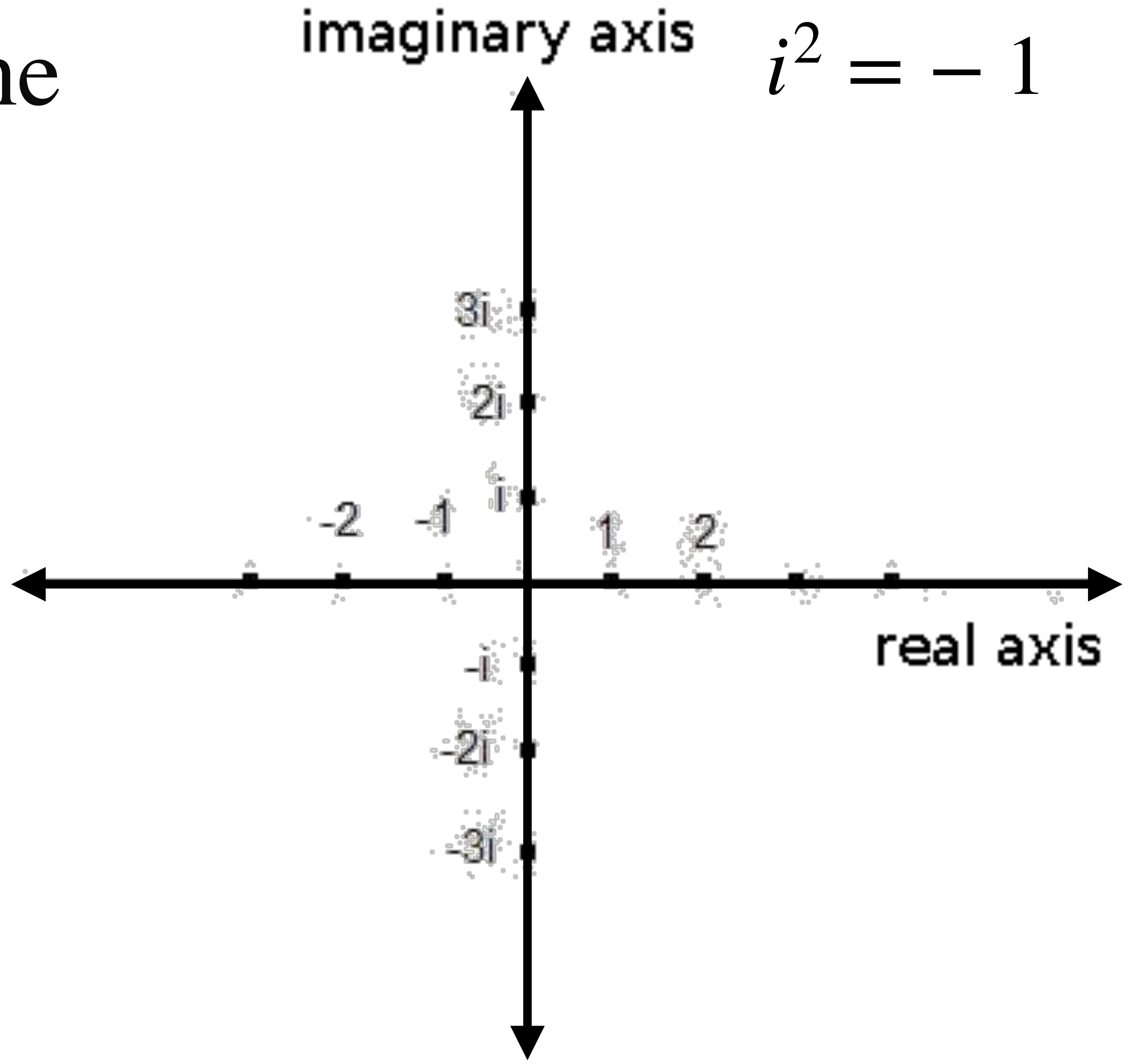
Real axis



CONCEPTS FOR NEXT WEEK

Complex plane

$i^2 = -1$



CONCEPTS FOR NEXT WEEK

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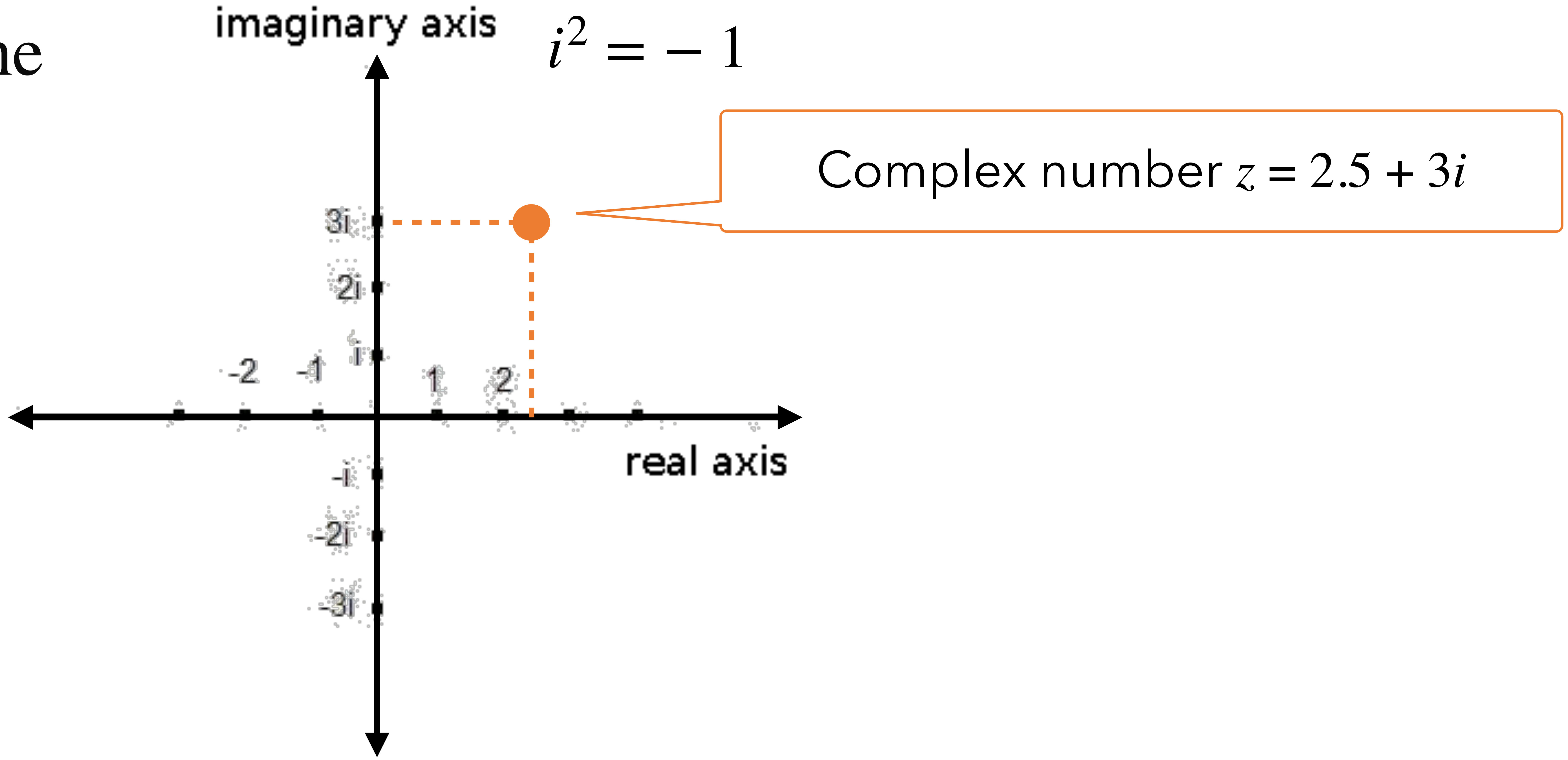

CONCEPTS FOR NEXT WEEK

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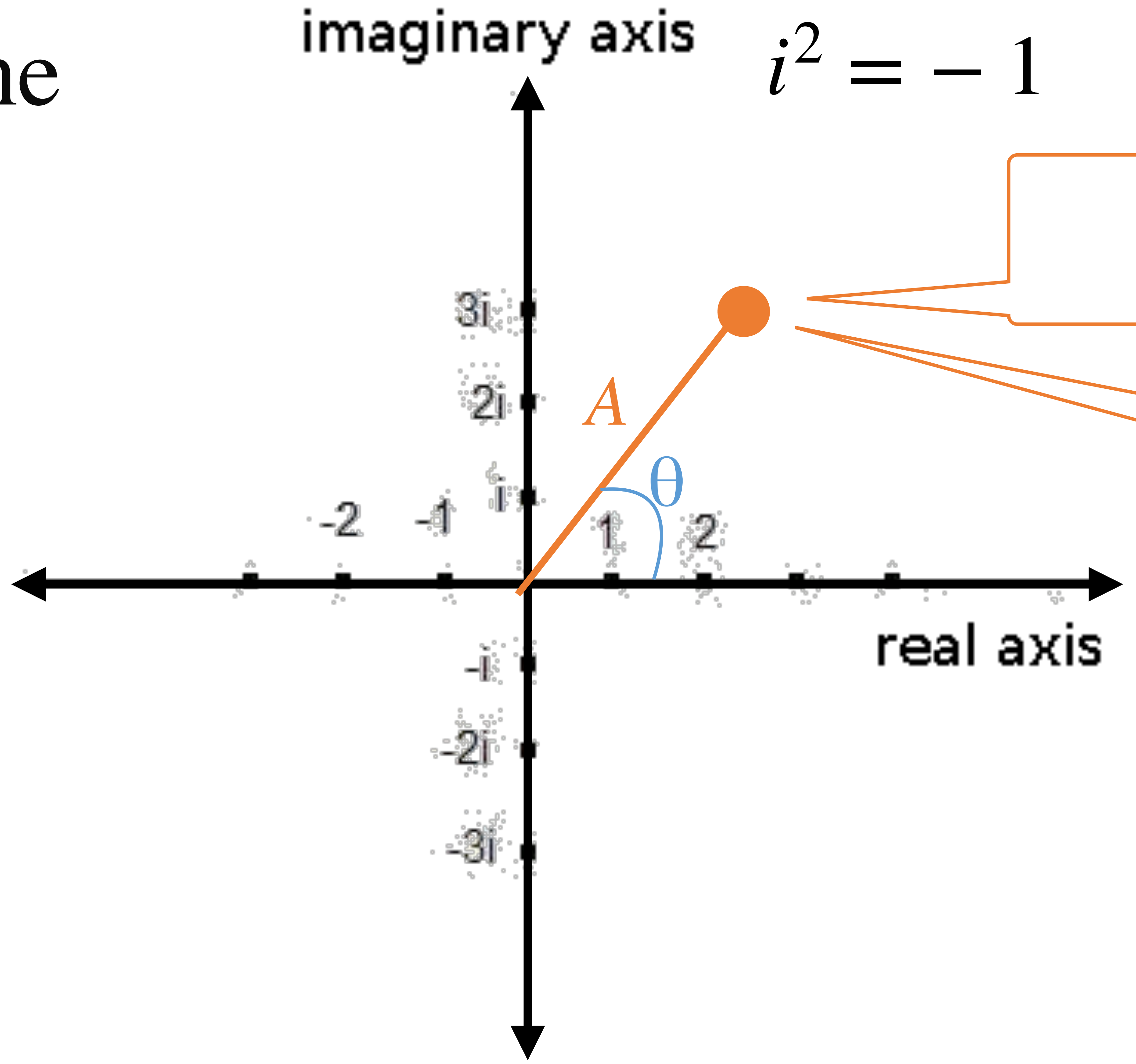
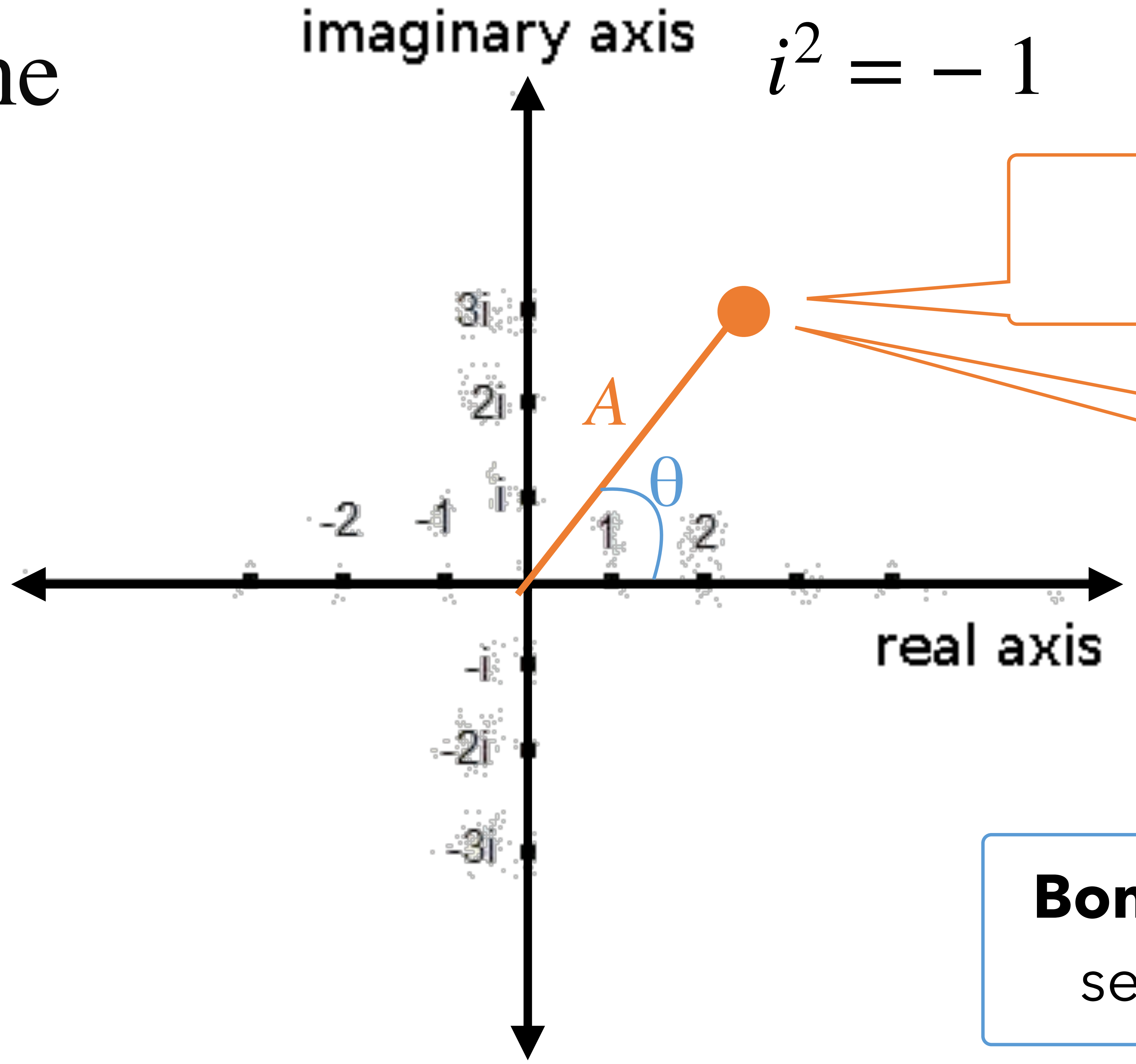


Complex number $z = 2.5 + 3i$

Complex number $z = Ae^{i\theta}$

CONCEPTS FOR NEXT WEEK

Complex plane



$i^2 = -1$

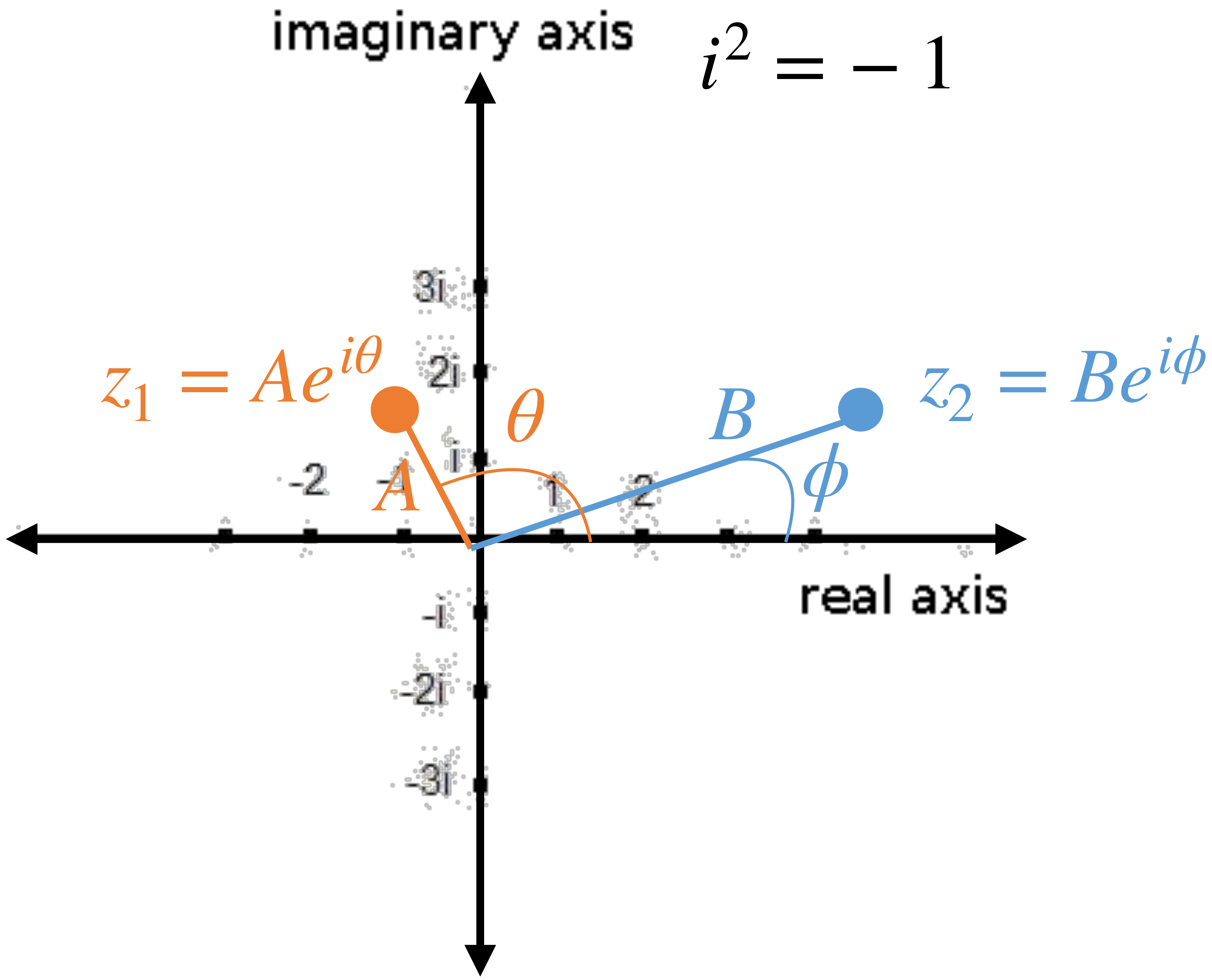
Complex number $z = 2.5 + 3i$

Complex number $z = Ae^{i\theta}$

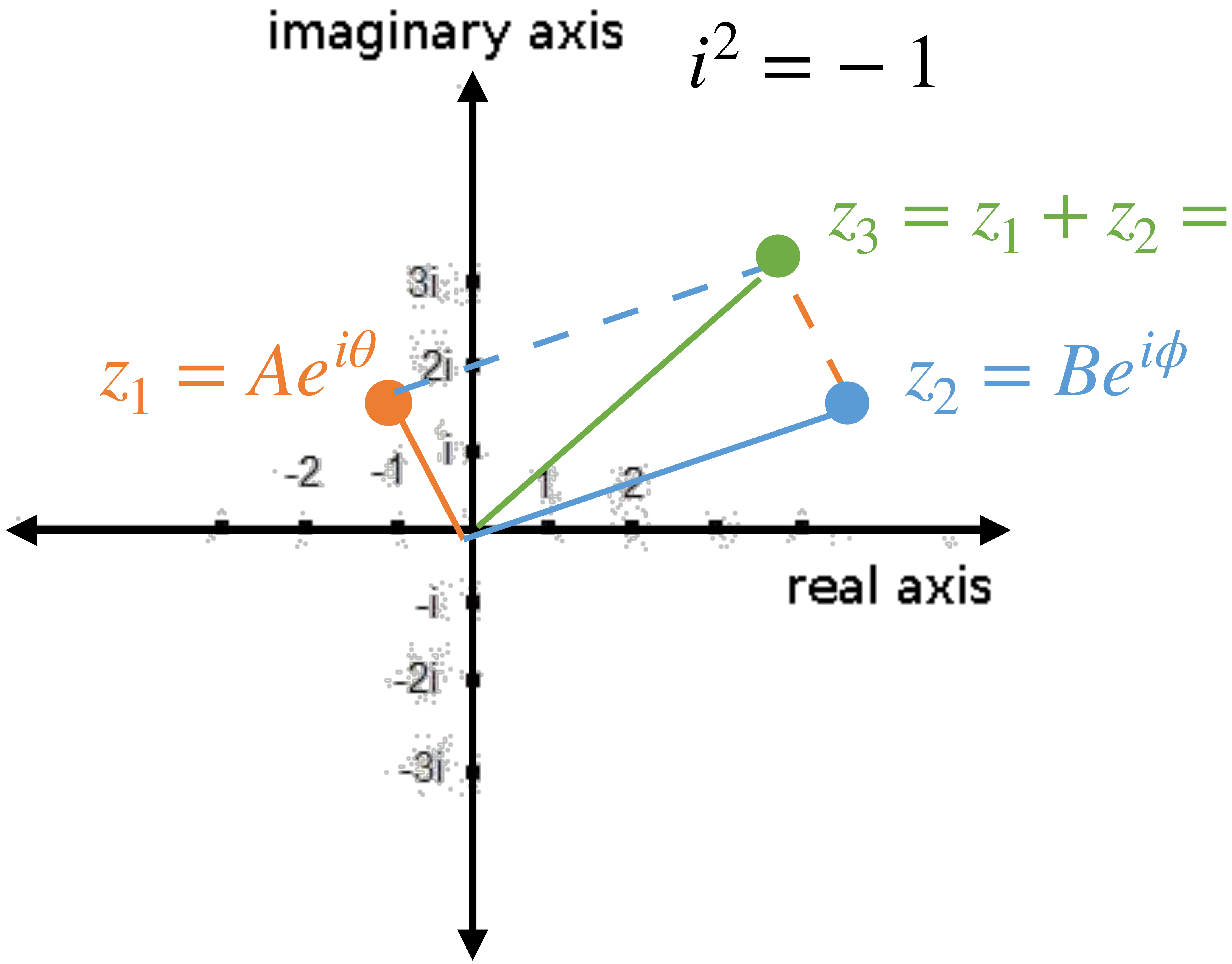
$$Ae^{i\theta} = A\cos\theta + i\sin\theta$$

Bonus exercise: prove this using the using the Taylor series expansion of these three different functions

CONCEPTS FOR NEXT WEEK



CONCEPTS FOR NEXT WEEK

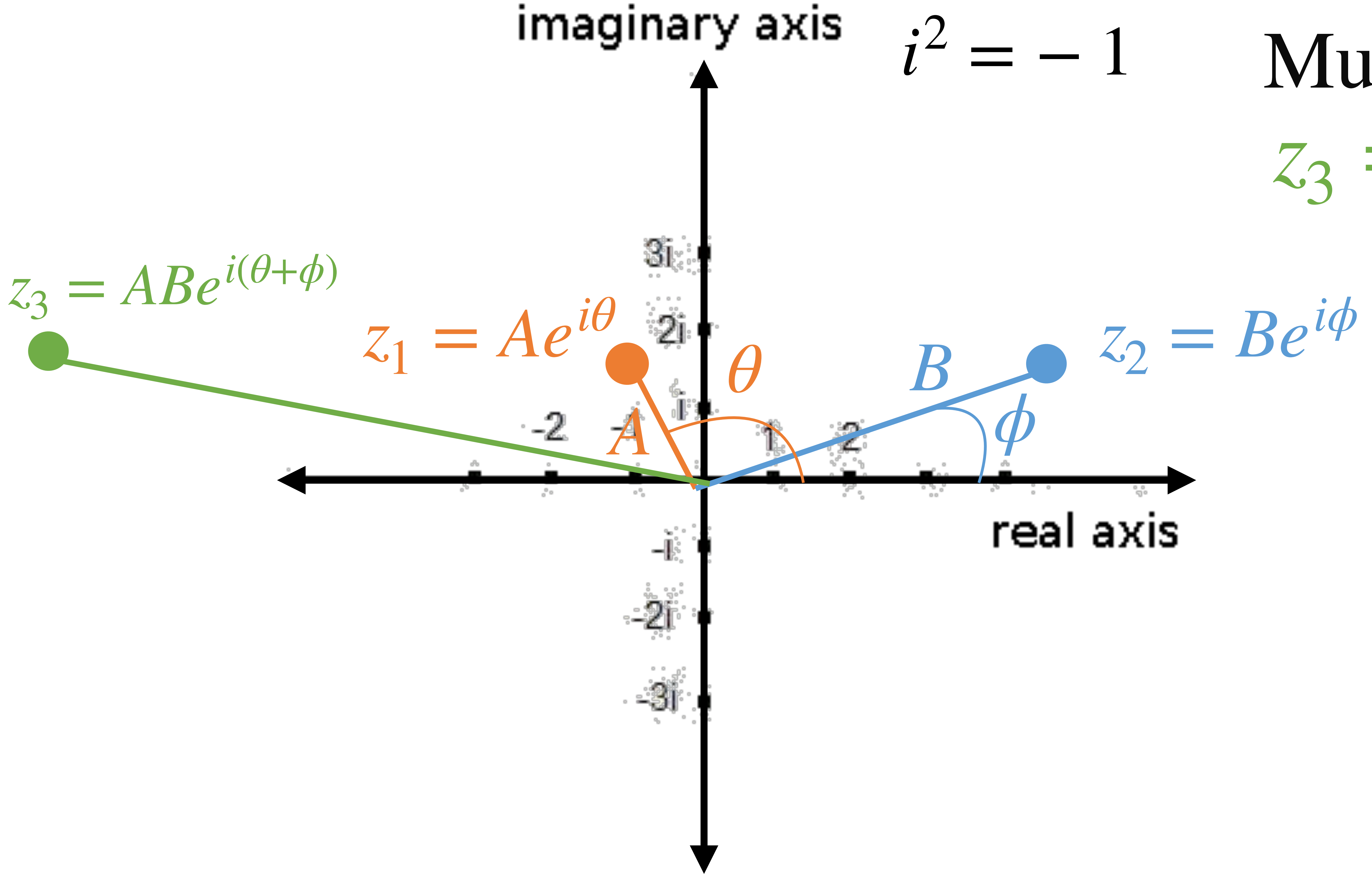


$$i^2 = -1$$

Addition

$$z_3 = z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$$

CONCEPTS FOR NEXT WEEK



Multiplication

$$z_3 = z_1 z_2 = Ae^{i\theta}Be^{i\phi} = ABe^{i(\theta+\phi)}$$

Scale by B

Rotate by ϕ