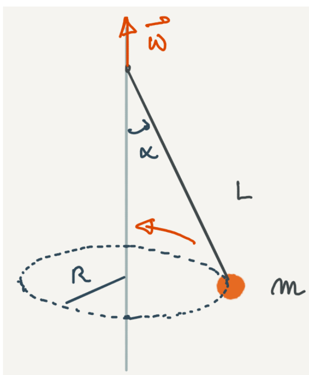
Exercises

Exercise 1 *Un exercice à rebondissements*
Let there be two springs with stiffness constants k_1 and k_2.
Provide the stiffness constant of the equivalent spring if they are mounted :

1. in parallel ;
2. in series.

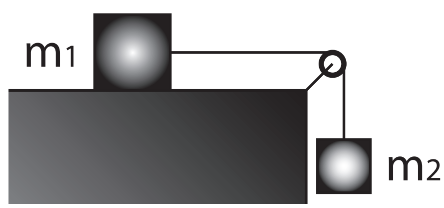
Exercise 2 *Et on fait tourner les serviettes*



A mass m is attached to a string of length L and is rotated in a horizontal plane at a constant angular velocity ω.

Calculate the radius R of the trajectory based on L, ω, and g.
Show that ω must be greater than a minimum angular velocity in order for R to be nonzero.

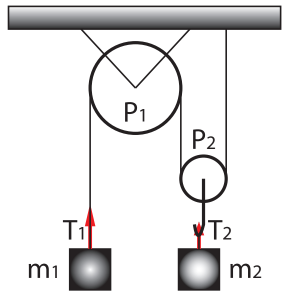
Exercise 3 *The Fall of Constantiblock*
Consider the following setup :



The pulley is massless and frictionless. The block of mass m_1 has a static friction coefficient μ_s and a dynamic friction coefficient $\mu_c < \mu_s$ with the table.

1. What is, depending on m_1, the maximum value of m_2 such that the system can remain stationary ?
2. For this limit value, we assume that a small jolt sets the system in motion. What is the acceleration of m_1 and the tension in the string as a function of m_1 ?
3. If we assume the system is frictionless, and we take $\mu_s = \mu_c = 0$, we find $T = 0$. Why ?

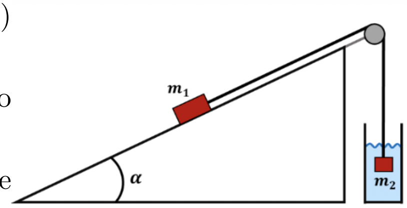
Exercise 4 *Two pulleys are better than one*
The following pulley system is given :



The pulleys are massless and operate without friction. $m_1 = 20$ kg and $m_2 = 30$ kg. Calculate the tensions T_1 and T_2 in the strings and the accelerations a_1 and a_2 of the two masses.

Exercise 5 *Armichedes, give me the strength to carry on*
A mass m_1 is placed on an inclined plane making an angle α with the horizontal. This mass experiences dry friction with the surface of the inclined plane, its static and dynamic dry friction coefficients being μ_s and μ_d, respectively. It is attached by means of an inextensible string and a pulley without mass to a second mass m_2 immersed in a liquid (see diagram opposite). This second mass experiences a force $\vec{F_a}$ due to Archimedes' thrust and directed upwards, as well as a viscous friction force $\vec{F_v} = -\beta\vec{v}$, where $\vec{v}$ is the velocity of mass m_2 and β is the viscous friction coefficient. The system is subject to gravity.

1. Draw a diagram of the system at equilibrium, indicating the forces present and the reference point(s) chosen.
2. Determine the minimum value of mass m_1 for it to slide down the inclined plane.
3. We now assume that the mass m_1 is greater than the value found in the previous point :



a) Determine the equation of motion for mass m_1.

b) Calculate the value of the velocity of mass m_1 as a function of time. What will be its limiting velocity, assuming the ramp is long enough for it to be reached, and this before mass m_2 leaves the liquid ? We will consider the origin of time to be the moment when the system starts moving : $v_0 = v(t = 0) = 0$.

Note : An equation of the form $\frac{dx}{dt} + \lambda x = C$ has a solution of the form $A + Be^{-\lambda t}$ where λ, A, B, C are constants.