Exercices

Exercise 1 *Drone days at EPFL*
Two drone enthusiasts fly their drones over a soccer field. We use a Cartesian coordinate system, with the z axis pointing upwards. Drone A has its position vector, dependent on time t, given by :

$$\vec{r}_A = \begin{pmatrix} 1.2t \\ 0.5t \\ 0.2t \end{pmatrix}$$

Drone B has its position vector, dependent on time t, given by :

$$\vec{r}_B = \begin{pmatrix} 0.2t^2 \\ 5 \\ 0.15t \end{pmatrix}$$

Positions are expressed in meters.

1. What do the two trajectories look like ?
2. Give the velocity and acceleration vectors of each drone as a function of time.
3. What are the scalar velocity and scalar acceleration of each drone ?
4. What is the distance d_{AB} between the drones as a function of time ?

How would you find the minimum distance ?

Exercise 2 *Smooth gliding*
A glider is released by the aircraft towing it in an updraft. From the moment of release ($t = 0$), it follows the following trajectory (Cartesian coordinates, z-axis pointed upwards) :

$$\vec{r} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 200\cos(0.2t) + 100 \\ 200\sin(0.2t) + 500 \\ 3t + 600 \end{pmatrix}$$

with the values given in meters.

1. Give the velocity vector $\vec{v}$ and acceleration vector $\vec{a}$ as well as the scalar velocity v and acceleration a (velocity and acceleration magnitudes) as functions of time.
2. At what time does the glider reach an altitude of 2000 meters ?
3. What does the trajectory look like ? What do you think of the values found for a and v ?
4. We want to express the position using cylindrical coordinates. What should be the new origin of this coordinate system so that the equations of motion become simpler ? How are these coordinates expressed as a function of time ?

Exercise 3 *Spin the records*
A centrifuge is a device used in biology laboratories to separate certain components in a fluid (e.g., blood or plasma). It consists of a disc that can be spun at very high speeds, with test tubes containing the liquid to be centrifuged attached to its outer edge.

Consider a centrifuge with radius $R = 20$ cm that can rotate at a maximum speed of $N_m = 15,000$ revolutions per minute. A test tube is placed on the periphery (i.e., 20 cm from the center), and we are interested in the motion of this tube, which we will consider as a point particle.

Initially, the centrifuge runs at its maximum speed.

1. Find the maximum angular velocity of the test tube.
2. Find the maximum scalar velocity of the test tube.
3. Find the normal acceleration of the test tube.
4. Compute numerical values for the three previous quantities.

   Now let's look at how the centrifuge starts up. It takes $N_f = 5000$ revolutions to reach its maximum speed from a standstill. The angular acceleration is constant during this phase.

5. Find the expression for the angular acceleration of the centrifuge as a function of N_f and N_m, and calculate its numerical value.
6. Find the expression for tangential acceleration during the acceleration phase, and calculate its numerical value.
7. Provide the vector expressions for the velocity and acceleration in the Frenet frame.

Exercise 4 *Agent Logan has a train to catch*
In pursuit of a criminal, FBI agent Logan must cross a river 1600 m wide flowing at 0.80 m·s^{-1} in the shortest possible time and get directly opposite his starting point. Knowing that he can row at 1.50 m·s^{-1} and run at 3.00 m·s^{-1}, describe the route he should take (by boat and on foot along the bank) to cross this river as quickly as possible. Determine the minimum time required for this crossing.

Reminder : if a boat moves at speed $\vec{v}$ relative to the water in a river, and the river flows at $\vec{v}_c$ relative to the bank, then the boat moves at $\vec{v} + \vec{v}_c$ relative to the bank.

Note : This exercise is difficult. You have the mathematical tools to solve it, but it requires understanding the problem as a whole, breaking it down properly, and making the projections correctly !