Exercises

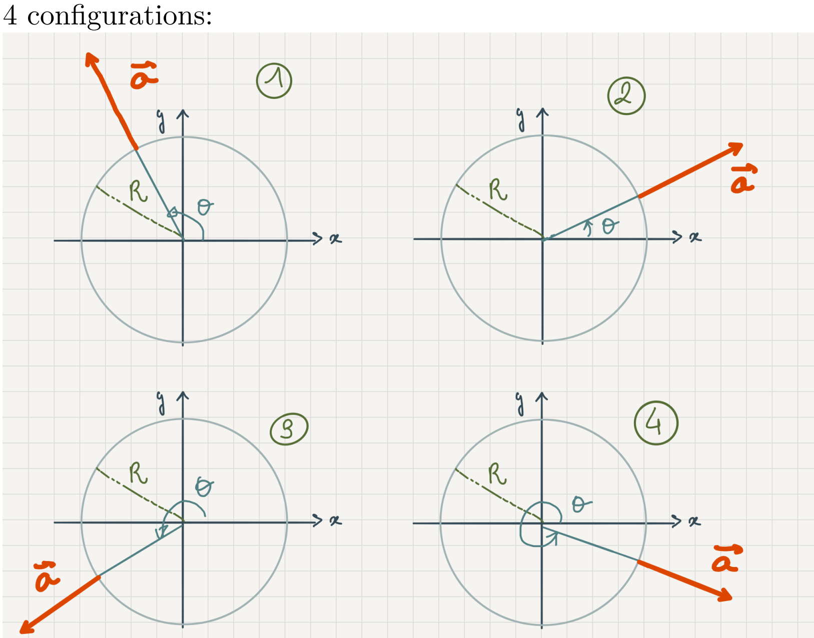
Exercise 1
An automobile company offers cars with a body molded under high pressure, made of 9.35 kg of iron. To celebrate its 100th anniversary, the company plans to cast a car body—but in gold. What mass of gold is required?

Data: The density of iron is $\rho_{Fe} = 7.9\ g/cm^3$ *and the density of gold is* $\rho_{Au} = 19.3\ g/cm^3$

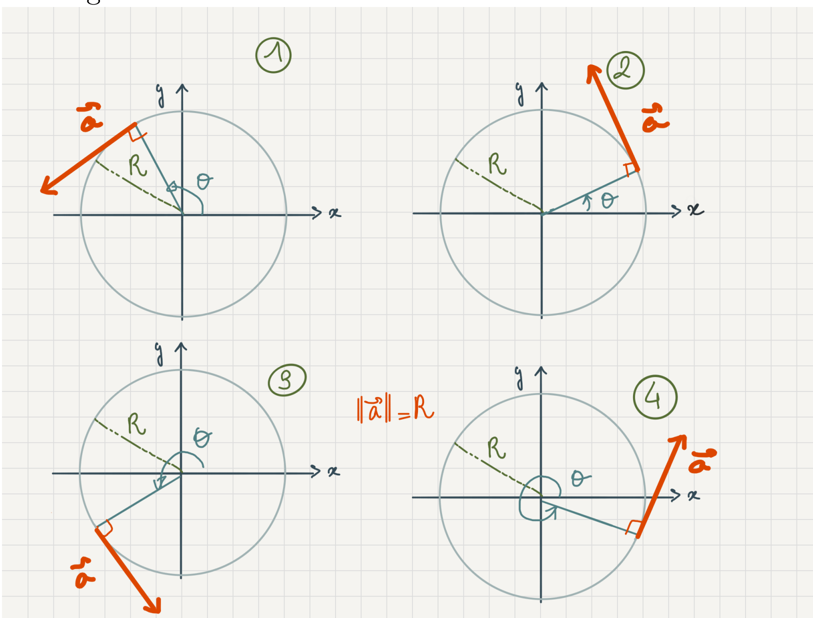
Exercise 2 The *International Prototype of the Kilogram* is an alloy of 90% platinum and 10% iridium. It is a cylinder measuring precisely 39.0 mm in height and 39.0 mm in diameter. What is the density of the material, given in kg/m^3?

Exercise 3 A nerve signal in the human body travels at a speed of approximately 100 $m{\cdot}s^{-1}$. If you stub your toe on the table leg at home in the evening, estimate the time it takes for the nerve impulse to travel from your toe to your brain.

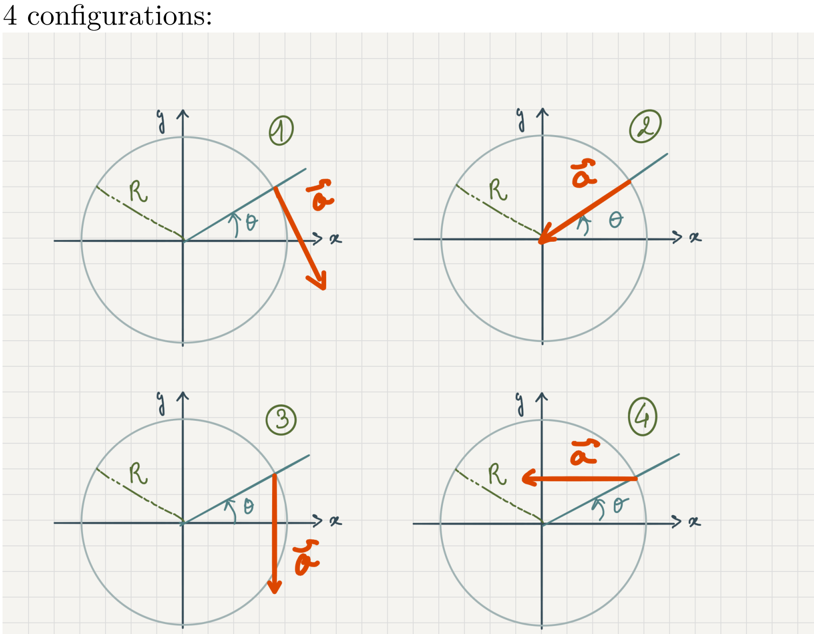
Exercise 4 Express the components of the vector $\vec{a}$ as functions of R and θ in the following 4 configurations:



Exercise 5 Express the components of the vector $\vec{a}$ as functions of R and θ in the following 4 configurations:



Exercise 6 Express the components of the vector $\vec{a}$ as functions of R and θ in the following 4 configurations:



Réponses

1. 22.8 kg
2. 21500 kg·m^{-3}
3. $t \simeq 17$ ms
4. $\vec{a} = (R\cos\theta, R\sin\theta)$ dans les quatre cas.
5. $\vec{a} = (-R\sin\theta, R\cos\theta)$ dans les quatre cas. 1– $\vec{a} = (R\sin\theta, -R\cos\theta)$; 2– $\vec{a} = (-R\cos\theta, -R\sin\theta)$; 3– $\vec{a} = (0, -R)$; 4– $\vec{a} = (-R, 0)$.

Solutions

Solution 1 We know that $m = \rho V$, and for gold or iron the volume is the same. So:

$$m_{Fe} = \rho_{Fe} V \tag{1}$$

$$m_{Au} = \rho_{Au} V \tag{2}$$

By equating the two expressions for the volume, we deduce:

$$m_{Au} = m_{Fe} * \frac{\rho_{Au}}{\rho_{Fe}} = 22.8kg \tag{3}$$

Note : Always perform an analytical solution before plugging in numbers.

Solution 2

$$V_{cylindre} = h\pi R^2 \tag{4}$$

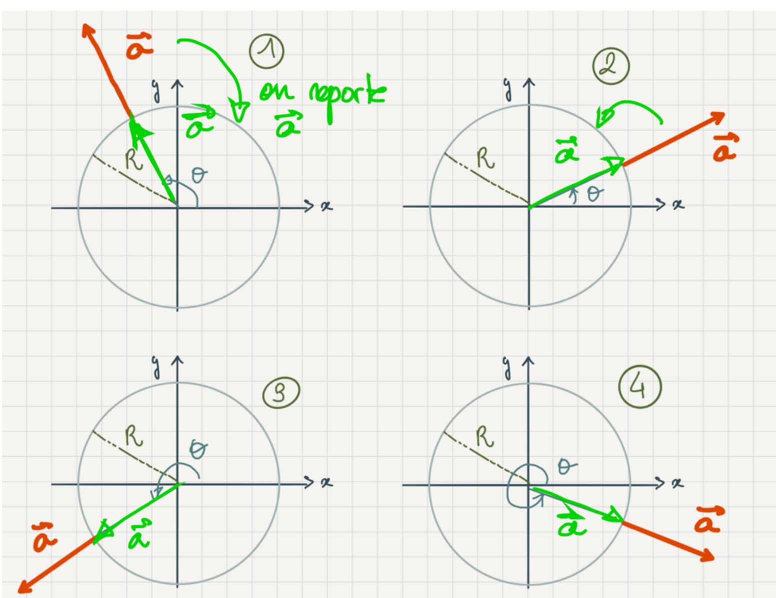
$$\rho = \frac{m}{V_{cylindre}} = \frac{1}{3.9 * 10^{-3} * \pi * \frac{3.9}{2}^2 * 10^{-6}} = 21.5 * 10^3 kg \cdot m^{-3} \tag{5}$$

Note: Some information may be superfluous (in this case, the percentage of each metal).

Solution 3 Toe-to-brain distance $\simeq 1.7$ m, so:

$$t = \frac{d}{v} = \frac{1.7}{100} = 17ms \tag{6}$$

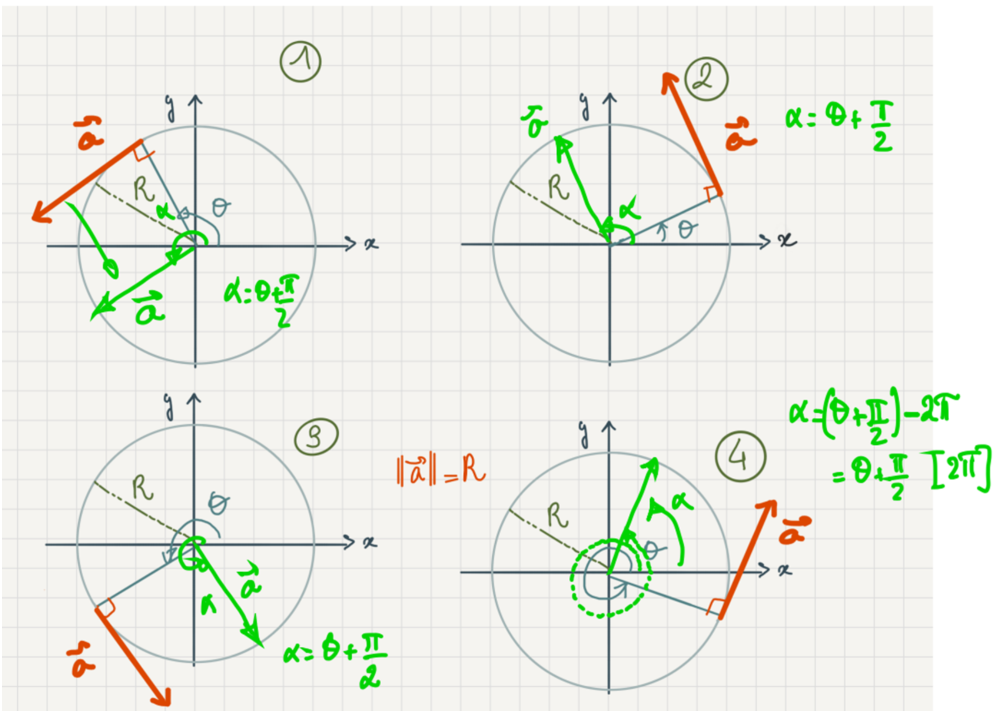
Solution 4 We reference $\vec{a}$ to the center of the circle; in all cases, $\vec{a}$ starts from the center, points to the circle, and θ measures the angle between Ox and $\vec{a}$.



Thus, $\vec{a} = (R\cos\theta, R\sin\theta)$ in all four cases.

Solution 5 We reference $\vec{a}$ to the center of the circle; it will then make an angle α with Ox: $\vec{a} = (R\cos\alpha, R\sin\alpha)$
By relating the angles, we get: $\alpha = \theta + \pi/2$
But we know that:

$$\cos(\theta + \pi/2) = -\sin\theta \quad (7)$$
$$\sin(\theta + \pi/2) = \cos\theta \quad (8)$$



Therefore, $\vec{a} = (-R\sin\theta, R\cos\theta)$ in all four cases.

Note: We observe that the relationships remain the same regardless of the quadrant of the unit circle.
The vector $\vec{a}$ is always tangent to the circle and points in the positive trigonometric direction. A diagram drawn for a vector in the first quadrant also works in the others—thankfully!

Solution 6
(1 and 2): Here too, we reference $\vec{a}$ to the center of the circle, identify its position using an angle α, and determine the relation between θ and α:

1: $\vec{a} = (R\cos\alpha, R\sin\alpha) = (R\cos(\pi/2-\theta), R\sin(\pi/2-\theta)) = (R\sin\theta, -R\cos\theta)$ (9)

2: $\vec{a} = (R\cos\alpha, R\sin\alpha) = (R\cos(\pi+\theta), R\sin(\pi+\theta)) = (-R\cos\theta, -R\sin\theta)$ (10)

3: Only one component along $\vec{e}_y$: $\vec{a} = (0, -R)$ (11)

4: Only one component along $\vec{e}_x$: $\vec{a} = (-R, 0)$ (12)