

0.1 Units

In this course we will use the International System of Units, abbreviated SI, the modern form of the metric system. In mechanics, we will need three fundamental units:

1. To measure **time**, we use **seconds** (s). One second is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom ($\Delta\nu_{Cs}$).
2. To measure **length**, we use **meters** (m). The meter is the length of the path traveled in vacuum by light during a time interval of $1/299,792,458$ of a second. The definition changed in 1983! The book by Alonso and Finn was published in 1980 and contains the previous definition.
3. To measure **mass**, we use **kilograms** (kg). The kilogram is defined by fixing the numerical value of the Planck constant, h , to $6.62607015 \times 10^{-34}$ kg m² s⁻¹.

0.2 Dimensional analysis

When we say “ h has units of length” we can write this mathematically as:

$$\dim h = L \tag{1}$$

where use L represents length. We will also use T for time and M for mass.

Dimensional analysis is using units to check the consistency of a result, to recover a formula, or to try to guess one. The idea is that an equality can only be valid if the units are the same on both sides of the “equals” sign.

For example, a stone is dropped from a height h . The calculated fall time is:

$$t_f = \sqrt{\frac{2h}{g}} \tag{2}$$

with h the height (a length) and g the acceleration due to gravity in m/s^2 .

Is this formula correct? The first thing we can do is check that the units are consistent. This does not guarantee that the result is correct, but if the units do not match, we know that it is wrong. To start, we can write the dimension of our variables:

$$\dim h = L, \quad \dim g = L/T^2 \tag{3}$$

Then plug this into our equation for t_f :

$$\dim t_f = \dim \sqrt{\frac{2h}{g}} = \sqrt{\frac{L}{LT^{-2}}} = \sqrt{T^2} = T \tag{4}$$

So we find that t_f has units of time, as we expect. See exercise set 0 for more examples.