Exercises

Exercise 1 *Derivatives*
Calculate derivatives as a function of time (t) for the following functions :

1. $\cos(t)$
2. $\sin(t)$
3. $\tan(t) = \frac{\sin(t)}{\cos(t)}$
4. $\ln(t)$
5. $\sqrt{t} = t^{\frac{1}{2}}$
6. t^α ($\alpha \neq 0$)
7. $\sin(t)\cos(t)$
8. $t\cos(t)$
9. $t\cos(t)\sin(t)$
10. $\sin(t^2)$

Exercise 2 *Return of the circle*

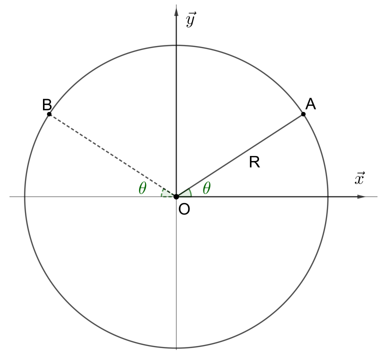
We consider the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, where points A and B lie on a circle of radius R:

a) Write the Cartesian components of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of R and θ.

b) Draw the vectors $\vec{u} = \overrightarrow{OA} + \overrightarrow{OB}$ and $\vec{v} = \overrightarrow{OA} - \overrightarrow{OB}$.

c) Write the Cartesian components of $\vec{u}$ and $\vec{v}$.

d) Redraw the diagram for $\theta = \frac{3\pi}{4}$ and $\theta = -\frac{\pi}{3}$.



Exercise 3 *Derivatives are derivative*
Let $\theta(t) = \omega t$ be a function of time. Calculate the time derivatives of the following functions:

1. $\cos(\theta)$
2. $\sin(\theta)$
3. $\tan(\theta)$
4. $e^{i\theta}$
5. $\sin(\theta)\cos(\theta)$

Exercise 4 *Vector basics*
We have vectors $\overrightarrow{u}\begin{vmatrix}\cos\theta \\ \sin\theta\end{vmatrix}$ and $\overrightarrow{v}\begin{vmatrix}-\sin\theta \\ \cos\theta\end{vmatrix}$ in Cartesian coordinates.

a) Draw $\vec{u}$ and $\vec{v}$ for $\theta = \frac{\pi}{6};\ \frac{\pi}{4};\ \frac{\pi}{3};\ \frac{\pi}{2};\ \frac{4\pi}{3};\ \pi;\ -\frac{3\pi}{4}$ and $-\frac{\pi}{4}$

b) Calculate $\|\vec{u}\|$ and $\|\vec{v}\|$

c) Prove that $\vec{u} \perp \vec{v}$

Exercise 5 *Dimensional Analysis*

1. Using dimensional analysis, verify that the following formula is correct:

   The distance x traveled during time t by a point with acceleration a, initial velocity v_0 and initial position x_0 is:

   $$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

2. What is the correct formula for the distance D of a projectile launched with initial speed v_0 at an angle α with respect to the horizontal, with g the acceleration due to gravity?

   (a) $D = \frac{g}{v_0} \sin 2\alpha$

   (b) $D = \frac{v_0}{g} \sin 2\alpha$

   (a) $D = \frac{g^2}{v_0} \sin 2\alpha$

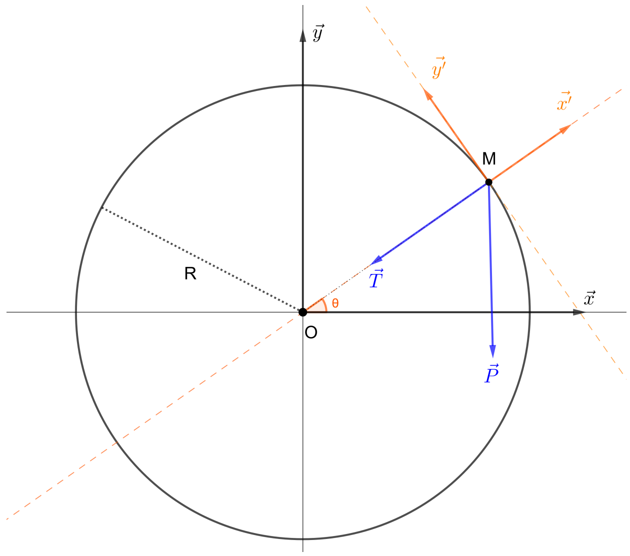
   (b) $D = \frac{v_0^2}{g} \sin 2\alpha$

Exercise 6 *Revenge of the derivatives*
Suppose that θ is a function of time $\theta(t)$. We ill use $\dot{\theta}(t) = \frac{d\theta(t)}{dt}$ for the time derivative of θ. *Note that we have not explicitly written $\theta(t)$, it is implicit that θ is a function of time t*

Calculate the derivative of $f(t)$ with repsect to time for :

1. $\cos(\theta)$
2. $\sin(\theta)$
3. $\tan(\theta)$
4. $\ln(\theta)$
5. $e^{i\theta}$
6. $\sin(\theta)\cos(\theta)$
7. θ^α
8. $\theta\cos(\theta)\sin(\theta)$

Exercise 7 *Learn to project*
Let M be a point on a circle of radius R. Let $\vec{T}$ be a vector pointing toward O and $\vec{P}$ be a vector parallel to O_y, with $|\vec{T}| = T$ and $|\vec{P}| = P$.



a) Give the components of the vectors $\overrightarrow{OM}$, $\vec{P}$, and $\vec{T}$ as functions of R, T, P, and θ.

b) Give the components of $\vec{P}$ and $\vec{T}$ in the coordinate system (M, x', y').

Exercise 8 *Reference frame, distance, and speed*
We want to study the motion of a point P moving across a table.

a) How many parameters are needed to locate the position of a point on the table?

b) How can the motion of point P be described?

c) Let A and B be two points located on the trajectory of point P. Express the distance between A and B: is this the same as the distance traveled by P?

d) What is the speed of P between A and B? What is it called? Is there a relationship between this speed and the speeds of P at A and at B?