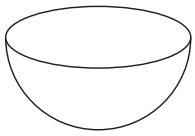
Exercices

Exercice 1 *Knowing how to find your coordinates*
We consider a hemispherical bowl in which a ball of mass m can move freely.



1. What is the coordinate system suited to the description of the system ?
2. Give the expression of $\vec{a}$ and $\vec{v}$ in this coordinate system taking into account the constraints.
3. We now suppose that the ball moves in a horizontal plane. What do the expressions of $\vec{v}$ and $\vec{a}$ become ?
4. The ball still moves in a horizontal plane but now with $v = \text{const}$. What is the nature of the motion ? What do $\vec{v}$ and $\vec{a}$ become ?

Exercice 2 *Be careful not to lose your bearings*
A TGV (high-speed train) travels along a terrestrial meridian from north to south at 300 km/h. It is located at latitude 60° North. We will take as reference frame $\mathcal{R}$ a geocentric reference frame. In Cartesian coordinates, the z-axis is the South–North axis, with positive z pointing to the North.

We begin by neglecting the rotation of the Earth.

1- Draw a diagram.

2- Express the velocity and acceleration of the TGV in spherical coordinates.

3- Calculate the magnitude of the acceleration vector.

Now we take into account the Earth's rotation. We denote by $\vec{\Omega}$ the Earth's rotation vector. We will take as reference frame $\mathcal{R}'$ attached to the Earth the frame with origin at the Earth's center that rotates with the Earth.

4- Express $\vec{\Omega}$ in Cartesian coordinates and in spherical coordinates. Calculate $\Omega = |\vec{\Omega}|$.

5- Calculate the velocity and acceleration of the TGV in $\mathcal{R}'$. Represent the different terms on the diagram.

6- Compute the different terms numerically and comment.

Exercice 3 *A coordination exercise*
A rifle and a target are mounted on a horizontal support rotating around a vertical axis. If the support does not rotate, the bullet hits the center of the target. Determine the deviation of the impact if the support rotates at an angular velocity ω, given that :

— The target is 110 cm from the axis ;

— The muzzle end is 40 cm from the axis ;

— The bullet speed is 250 $\mathrm{m{\cdot}s^{-1}}$;

— The mount rotates making 1 turn in 2 seconds.

Exercice 4 *"Rien ne résiste à un acharnement de fourmi" -Victor Hugo*
An ant moves on a disk rotating at constant angular velocity. Its trajectory is a straight line with respect to a reference frame attached to the disk. It starts from the center at time $t = 0$ and moves at constant speed v_0.

1. Give the position of the ant as a function of time : in the frame attached to the disk and in the laboratory frame, in Cartesian coordinates ;
2. Give the equation of the trajectory in the laboratory frame, in Cartesian coordinates and in polar coordinates ;
3. Determine the velocity in the absolute (laboratory) and relative (disk-attached) frames, in Cartesian coordinates and in polar coordinates ;
4. Calculate its acceleration in both frames, in Cartesian coordinates and in polar coordinates.