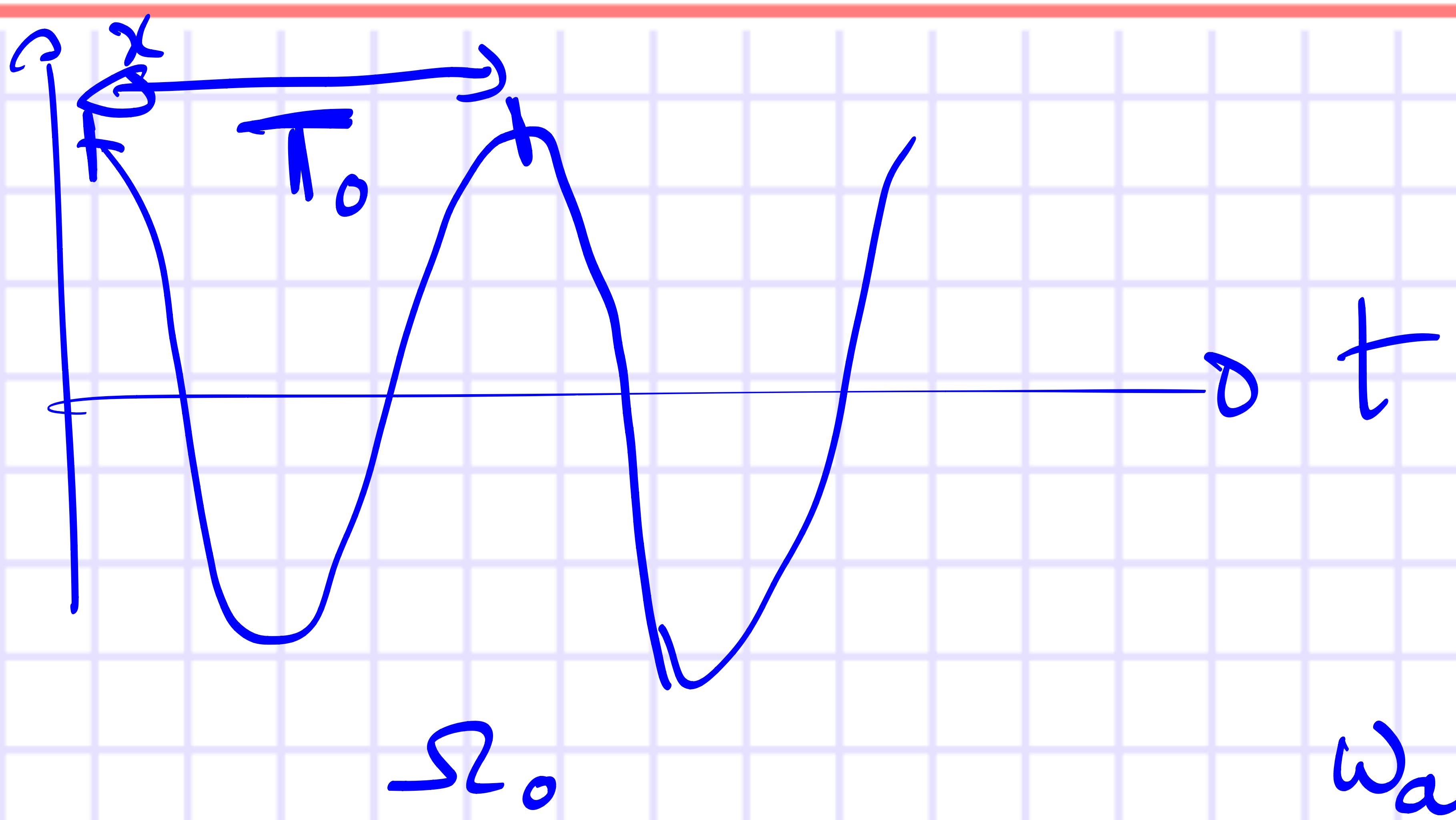
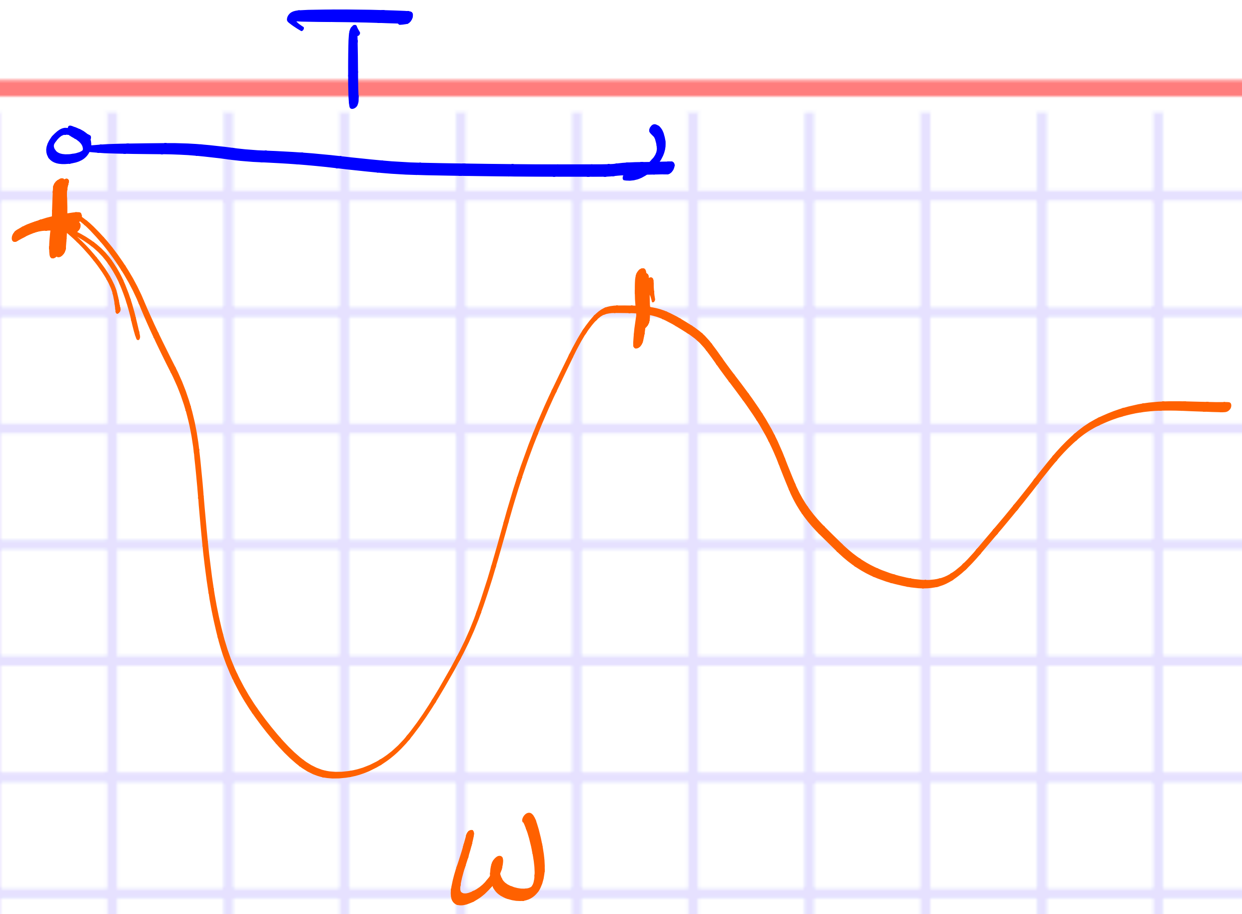
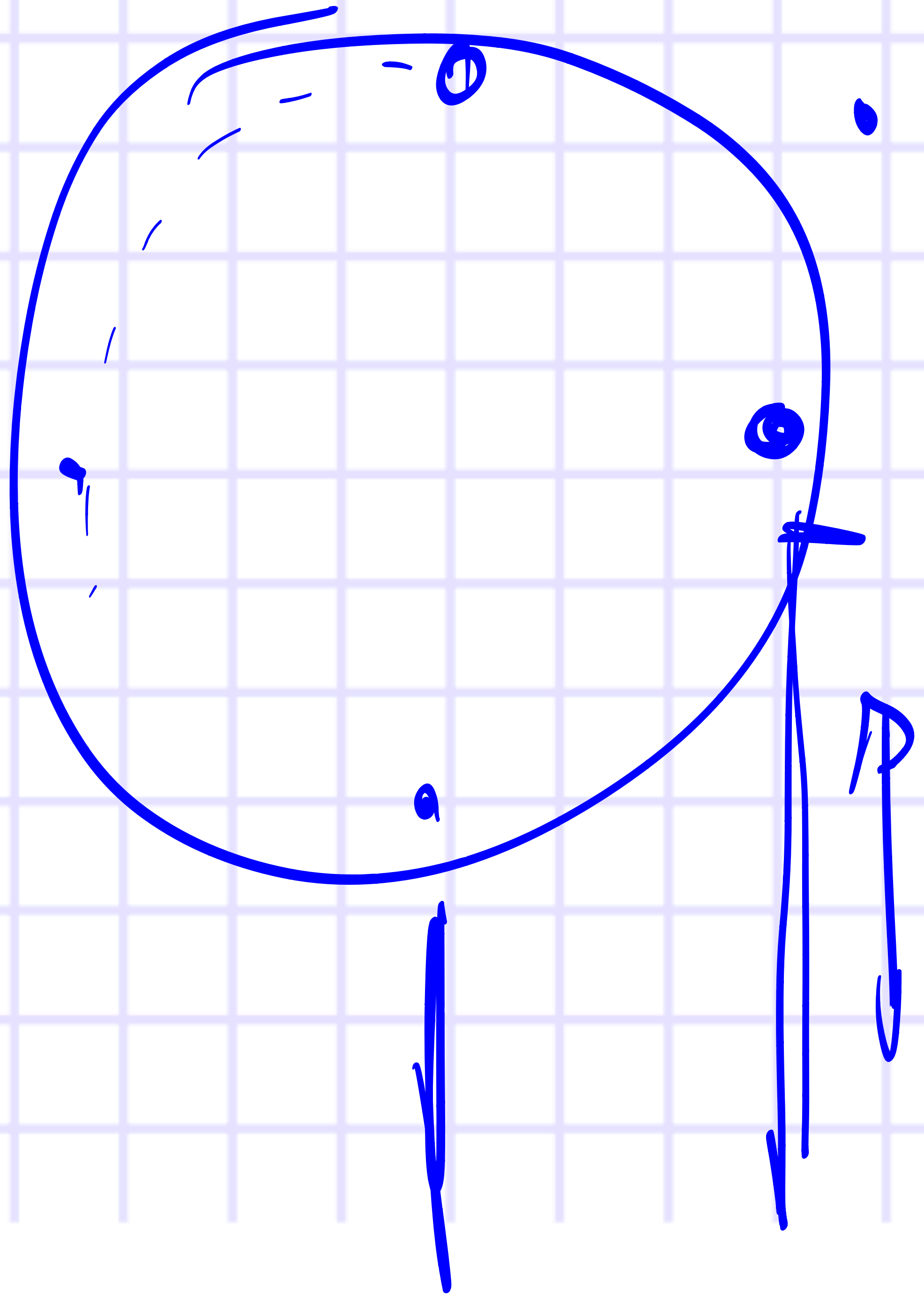
Mécanique générale, classe inversée.

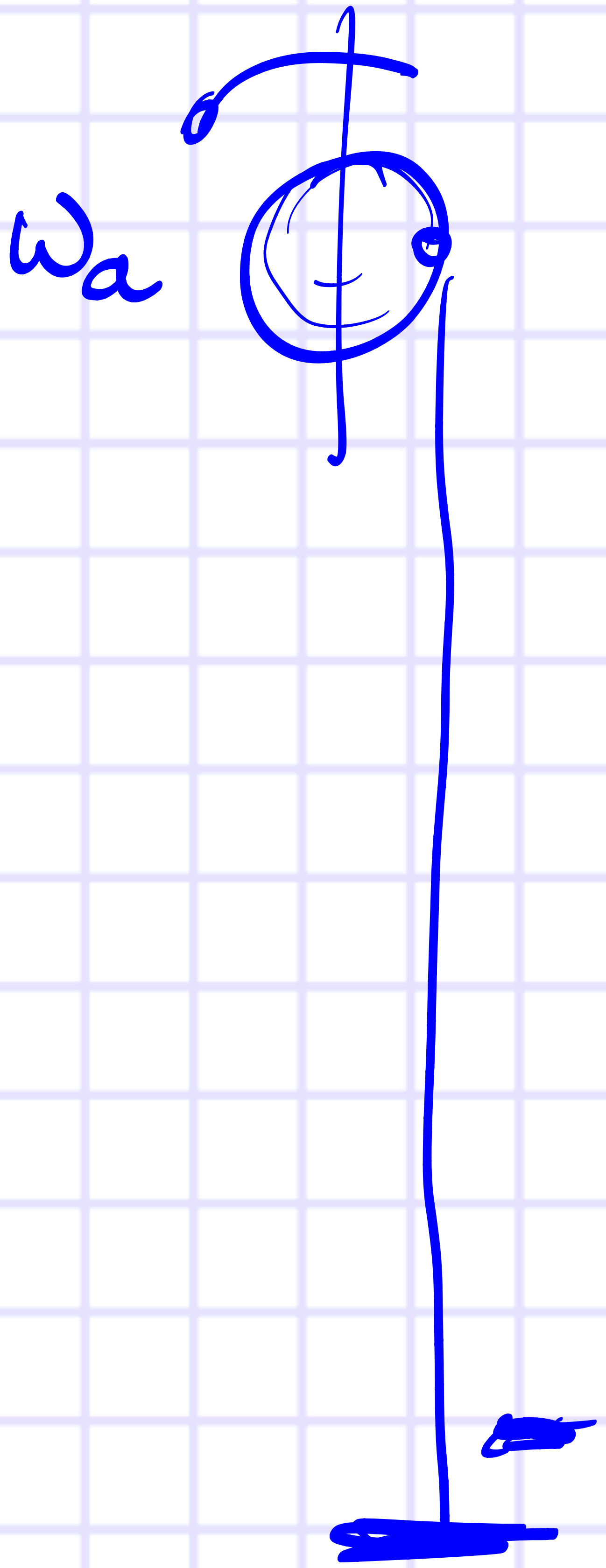
18-19 Novembre 2025

Manip oscillateur forcé









ω_a vitesse angulaire de rotation

$\sim R \sin(\omega_a t)$

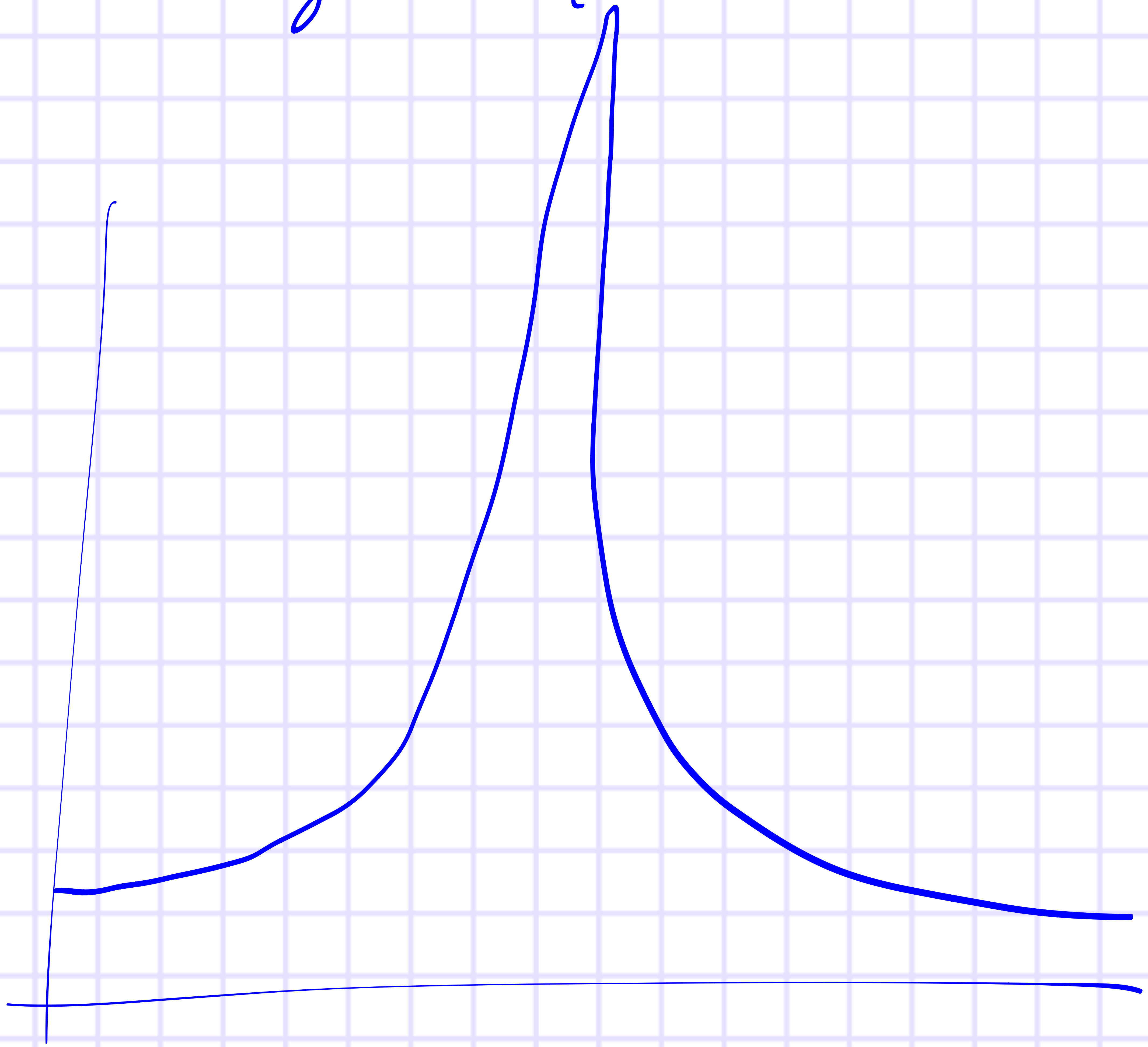
$\propto$ impose une force

$\propto \sin(\omega_a t)$

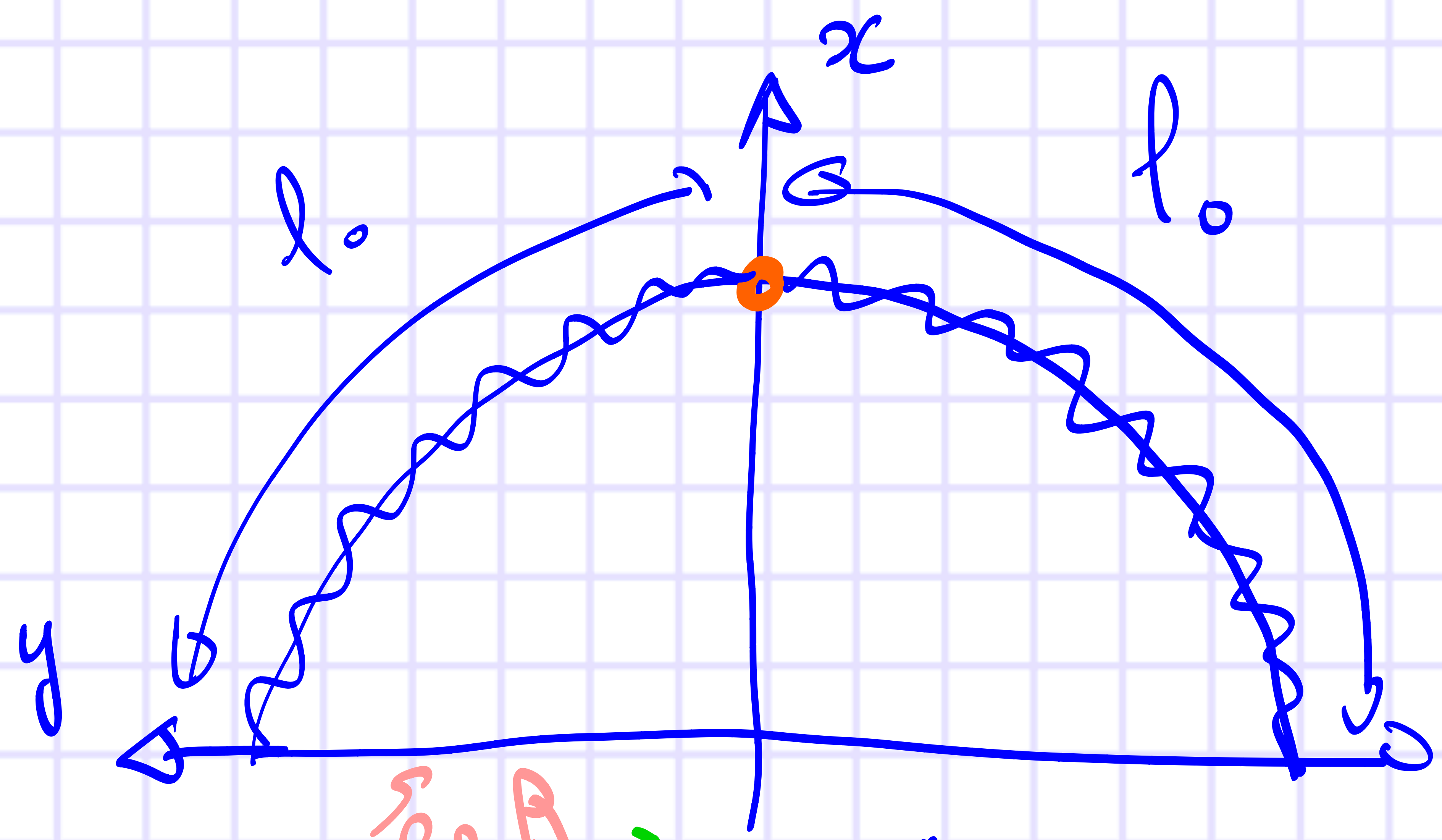
ω_e

Exercice amphi

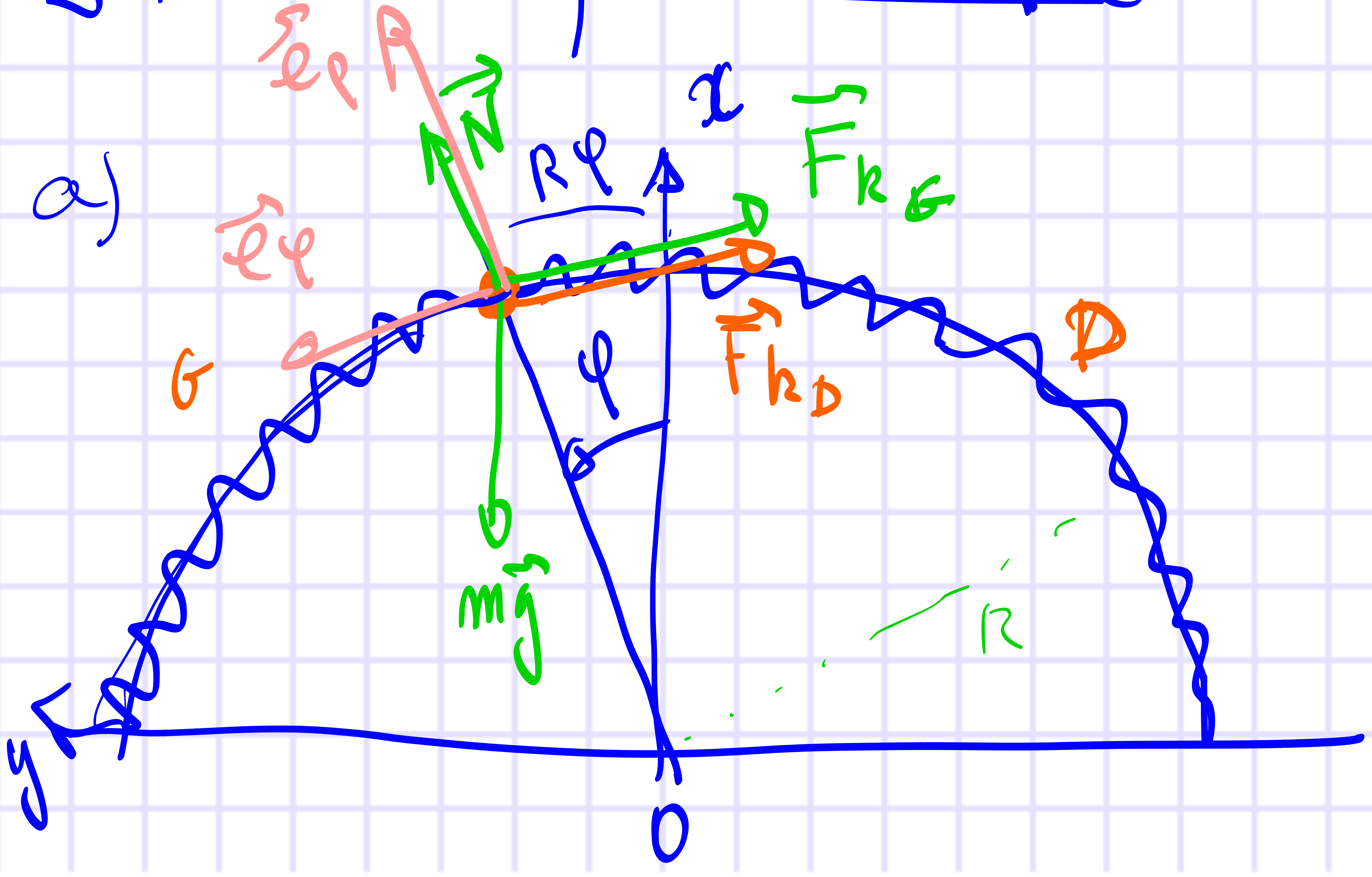
$A(\omega_e)$ $\varphi(\omega_e)$ régime forcé



Exercice amphi



a)



$$l_0 = \frac{\pi}{2} R$$

b) polaire : $\vec{a}(t) = (\cancel{\ddot{\rho}} - \rho\dot{\varphi}^2)\vec{e}_\rho + (\cancel{2\dot{\rho}\dot{\varphi}} + \rho\ddot{\varphi})\vec{e}_\varphi$

$\rho = R = cte$ $\quad \dot{\rho} = 0 \quad \ddot{\rho} = 0$

$$\vec{a}(t) = -R\dot{\varphi}^2\vec{e}_\rho + R\ddot{\varphi}\,\vec{e}_\varphi$$

c) $\sum \vec{F} = m\vec{a} = \vec{N} + m\vec{g} + \vec{F}_{kG} + \vec{F}_{kD}$

$= N\vec{e}_\rho + \vec{F}_{kG} + \vec{F}_{kD} + m\vec{g}$

$\vec{F}_{kG} = -k(-R\varphi)(-\vec{e}_\varphi) = -kR\varphi\vec{e}_\varphi$

$\vec{F}_{kD} = -k(R\varphi)\,\vec{e}_\varphi = -kR\varphi\vec{e}_\varphi$

Exercice amphi

$$m\vec{g} = -mg\cos\varphi\,\vec{e}_\rho + mg\sin\varphi\,\vec{e}_\varphi$$

$$\Sigma\vec{F} = (N\vec{e}_\rho - mg\cos\varphi\,\vec{e}_\rho) + (mg\sin\varphi - 2kR\varphi)\,\vec{e}_\varphi$$

$$= -mR\dot{\varphi}^2\,\vec{e}_\rho \quad + mR\ddot{\varphi}\,\vec{e}_\varphi$$

$$\boxed{\text{sur } \vec{e}_\rho : N - mg\cos\varphi = -R\dot{\varphi}^2 m \; ; \; \text{sur } \vec{e}_\varphi \; mg\sin\varphi - 2kR\varphi = mR\ddot{\varphi}}$$

d) si $\varphi = 0$ $\vec{F}_{kD} = \vec{F}_{kG} = \vec{0}$ forces : $m\vec{g}$ et $\vec{N}$ $\Rightarrow \vec{N} = N\vec{e}_\rho$ $m\vec{g} = -mg\,\vec{e}_\rho$

en $\varphi = 0$ pas de forces sur $\vec{e}_\varphi$ $a_\tau = 0$ $a_\tau = \dfrac{dV}{dt} = 0$

si on prend $V = 0$ en $\varphi = 0$ comme $\dfrac{dV}{dt} = 0 \Rightarrow V = cte = 0$

Un objet placé en $\varphi = 0$ avec $V = 0$ y reste $\Rightarrow$ position d'équilibre

e) sur $\vec{e}_\varphi$ car pas de N : $mg\sin\varphi - 2kR\varphi = mR\ddot{\varphi}$; $\underline{(mg - 2kR)\varphi = mR\ddot{\varphi}}$

φ faible $\Rightarrow \sin\varphi \simeq \varphi$

Exercice amphi

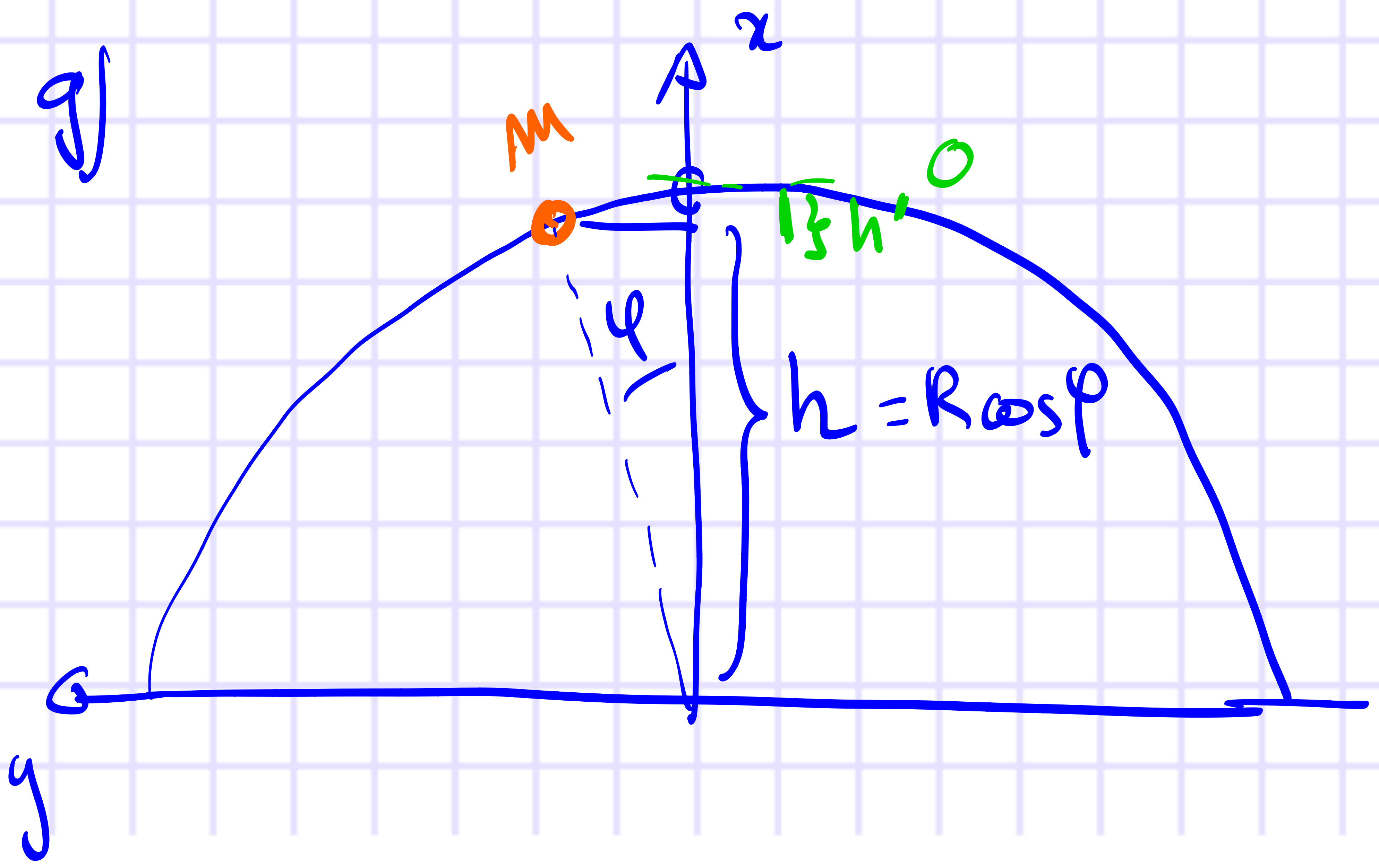
f) $(mg - 2kR)\varphi = mR\ddot{\varphi}$

$$\ddot{\varphi} + \underbrace{\frac{(2kR - mg)}{mR}}_{\Omega_0^2 \text{ doit être positif !}} \varphi = 0$$

$\ddot{\varphi} + \Omega_0^2 \varphi = 0$ oscillateur harmonique

$\boxed{2kR - mg > 0}$ $\boxed{2kR > mg}$

g)



pesanteur 0 de E_p en $x = 0$

$E_p^{\vec{g}} = mgh = mgR\cos\varphi$

avec 0 de E_p en $\varphi = 0$ $(x = R)$

$E_p^{\vec{g}} = mgh' = -R(1 - \cos\varphi)mg$

$= mgR\cos\varphi - mgR$

Exercice amphi

$E_p^k = \frac{1}{2} k (\Delta l)^2$ gauche : $\frac{1}{2} k (-R\varphi)^2 = \frac{1}{2} k R^2 \varphi^2$ droite : $\frac{1}{2} k (R\varphi)^2$ $\frac{1}{2} k R^2 \varphi^2$

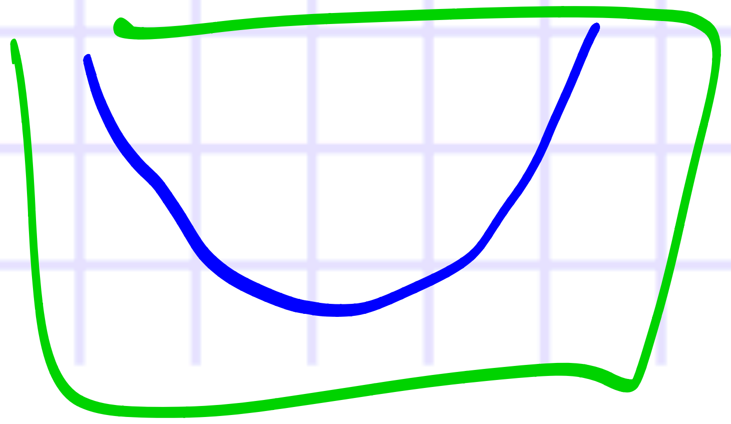
2 ressorts $E_p^k = kR^2\varphi^2$

$E_p^{tot} = mgR\cos\varphi + kR^2\varphi^2$ φ petit $\cos(\varphi) \simeq 1 - \frac{\varphi^2}{2}$

si $\varphi \ll 1$ $E_p^{tot} = mgR\left(1 - \frac{\varphi^2}{2}\right) + kR^2\varphi^2$

$E_p^{tot} = mgR + \boxed{\left(kR^2 - \frac{mgR}{2}\right)}\varphi^2$

oscillateur harmonique ⇒ position d'équilibre stable min de E_p



+ potentiel "quadratique" ✓

$A > 0$ $kR^{\cancel{2}} - \frac{mg\cancel{R}}{2} > 0$

$\boxed{2kR - mg > 0}$

Exercice amphi

"bonus" : $E_p = mgR + \left(kR^2 - \frac{mgR}{2}\right)\varphi^2$

$\frac{dE_p}{d\varphi} = 0 + \left(kR^2 - \frac{mgR}{2}\right) 2\varphi$ en $\varphi = 0$ $\frac{dE_p}{d\varphi} = 0$

donc ~~est~~ $\varphi = 0$ position d'équilibre

i) $E_{mec} = E_p^{tot} + \frac{1}{2} m v^2 = E_p^{tot} + \frac{1}{2} m (R\dot{\varphi})^2$

$E_{mec} = mgR\cos\varphi + kR^2\varphi^2 + \frac{1}{2} m R^2 \dot{\varphi}^2 = \text{constante}(t)$

$\frac{dE_{mec}}{dt} = 0 = mgR(-\cancel{\dot{\varphi}}\sin\varphi) + kR^2 2\cancel{\dot{\varphi}}\varphi + \cancel{\frac{1}{2}} m R^2 \cancel{2}\cancel{\dot{\varphi}}\ddot{\varphi}$

$-mg\cancel{R}\sin\varphi + 2kR^{\cancel{2}}\varphi + mR^{\cancel{2}}\ddot{\varphi} = 0$

$$\boxed{mR\ddot{\varphi} + 2kR\varphi - mg\sin\varphi = 0}$$