Problem Set 4

Circular motion

PHYS-101(en)

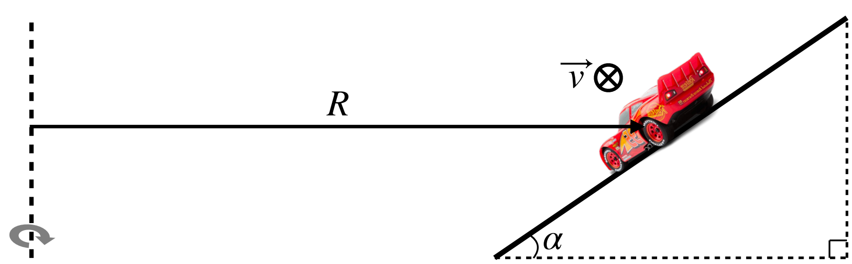
1. Rotating space station

In order to create an Earth-like environment for its inhabitants, a cylindrical space station of radius R is rotated about its central axis. People standing on the inner rim of the station experience a normal force from the wall that plays the role of "artificial gravity."

1. Find the angular velocity ω required for the centripetal acceleration at the rim to equal the magnitude of the gravitational acceleration g on Earth.
2. Express the period of rotation T in terms of R and g.
3. Derive an expression for the tangential speed v of a person at the rim in terms of R and g, and determine by what multiplicative factor v changes when the station radius is increased from R to $2R$.

2. Banked turn

A car of mass m is going around a circular turn of radius R, that is banked at an angle α with respect to the ground. The coefficient of static friction between the tires and the road is μ_s. Let g be the magnitude of the gravitational acceleration. You may neglect kinetic friction (i.e. the car's tires do not slip).

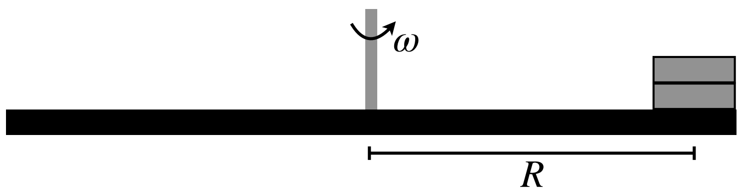


1. At what speed should the car enter the banked turn if the road is very slippery (i.e. $\mu_s \to 0$) in order not to slide up or down the banked turn? Call this speed v_0.
2. What is the minimum speed v_{min} the car needs so that it does not slide **down** the banked turn? You can assume that $\mu_s < \tan \alpha$.
3. What is the maximum speed v_{max} the car can have so that it does not slide **up** the banked turn? You can assume that $\mu_s \tan \alpha < 1$.
4. Suppose the car enters the turn with a speed v such that $v_{max} > v > v_0$. Find an expression for the magnitude of the friction force.

3. Angular speed of coins

Two identical coins, each of mass m, are stacked on top of each other at the rim of a turntable (a distance R from the center). The turntable turns at constant angular speed ω and the coins ride it without slipping. Suppose the coefficient of static friction between the turntable and the bottom coin is given by μ_1 and the coefficient of static friction between the two coins is given by μ_2, where $\mu_2 < \mu_1$. Let g be the gravitational constant.

1. What is the magnitude of the radial friction force exerted by the turntable on the bottom coin?
2. If the angular speed is very slowly increased, which coin slips first or do they both slip at the same instant? What is the maximum angular speed ω^{max} such that no slipping occurs?



4. Circular motion of the earth

The earth is spinning about its axis with a period of 23 hours 56 min and 4 sec. The equatorial radius of the earth is 6.38×10^6 m. The latitude of Lausanne is $46°31'$N.

1. Find the velocity of a person at EPFL as they undergo circular motion about the earth's axis of rotation.
2. Find the person's centripetal acceleration.

5. Spiral motion of a point mass

A point mass P with mass m is represented in polar coordinates. The motion of P is determined by the vector sum of the following two external forces acting on it

$$\vec{F}_1 = -mk^2\vec{r}$$

and

$$\vec{F}_2 = -2m\lambda\vec{v},$$

where $k > \lambda > 0$. Note that we neglect gravity. The force $\vec{F}_1$ is spring-like (i.e. proportional and opposite to $\vec{r}$, the displacement from the equilibrium position at the origin) and $\vec{F}_2$ is a viscous friction-type force (i.e. proportional and opposite to the velocity v).

In this problem you are given that:

- $\dot{\phi} \neq 0$, $\ddot{\phi} = 0$, and $\ddot{\rho} \neq 0$,
- the initial conditions at $t = 0$: $\phi = 0$ and $\rho = \rho_0$,

- the formulas for the velocity $\vec{v}$ and the acceleration $\vec{a}$ in polar coordinates:

$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi}$$
$$\vec{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{\phi},$$

- the solution of the equation $\dot{\rho} = -\lambda\rho$ has the form $\rho\left(t\right) = Ce^{-\lambda t}$, where C is an integration constant.

1. Represent the system graphically in polar coordinates.
2. Write down the equations of motion in the form of differential equations, without solving them.
3. From the equations of motion, determine
   - the radial position $\rho\left(t\right)$,
   - the angle $\phi\left(t\right)$ and use it to find $\rho\left(\phi\right)$ from $\rho\left(t\right)$, and
   - the speed of the particle $v\left(t\right)$.