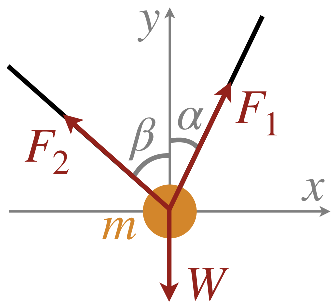
Solutions to Problem Set 3

Free body diagrams

PHYS-101(en)

1. Balancing forces

1. The forces exerted on the ball are the weight $W = mg$, the tension force exerted by the right cable F_1, and the tension force exerted by the left cable F_2.



2. The forces in the $\hat{x}$ direction are

$$W_x = 0$$
$$F_{1x} = F_1 \sin\alpha$$
$$F_{2x} = -F_2 \sin\beta$$

and the forces in the $\hat{y}$ direction are

$$W_y = -W = -mg$$
$$F_{1y} = F_1 \cos\alpha$$
$$F_{2y} = F_2 \cos\beta.$$

3. The ball undergoes no acceleration, so Newton's second law is $\Sigma\vec{F} = 0$ and we have

$$\vec{W} + \vec{F}_1 + \vec{F}_2 = 0. \tag{1}$$

We project this in the $\hat{x}$ direction to get

$$F_1 \sin\alpha - F_2 \sin\beta = 0.$$

Rearranging, we find

$$F_1 = F_2 \frac{\sin\beta}{\sin\alpha}. \tag{2}$$

We then project equation (1) in the $\hat{y}$ direction to get

$$F_1 \cos\alpha + F_2 \cos\beta - mg = 0. \tag{3}$$

Substituting (2) into (3) gives

$$F_2 \frac{\sin\beta}{\sin\alpha}\cos\alpha + F_2\cos\beta - mg = 0 \quad \Rightarrow \quad F_2\left(\frac{\sin\beta}{\sin\alpha}\cos\alpha + \cos\beta\right) = mg.$$

Solving for F_2 gives

$$F_2 = \frac{mg}{\frac{\sin\beta}{\sin\alpha}\cos\alpha + \cos\beta} = \frac{mg\sin\alpha}{\sin\beta\cos\alpha + \sin\alpha\cos\beta} = mg\frac{\sin\alpha}{\sin(\alpha+\beta)},$$

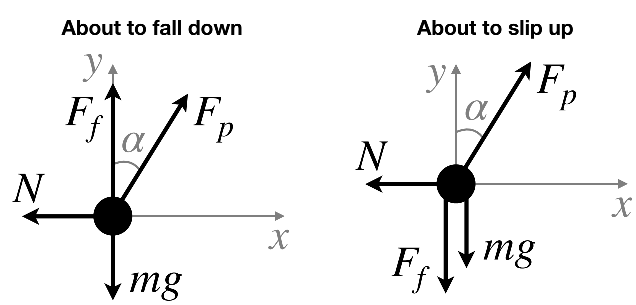
where in the last step we have used the sine angle sum trigonometric identity. By substituting this into equation (2) we find the final answer for F_1 of

$$F_1 = mg\frac{\sin\beta}{\sin(\alpha+\beta)}.$$

From equation (2), we see that, if $\alpha = \beta$, then $F_1 = F_2$ as would be expected from the symmetry of the problem.

2. Pushing a book against a wall

1. Let m, μ_s, and α be defined as in the problem and let $\vec{F}_f$ represent the friction force. Additionally, let $\vec{F}_p$ be the force of your push on the book. We will define our coordinate system such that $x > 0$ corresponds to the wall, so that you are pushing from the $x < 0$ side. The free body diagrams for both cases are given below. Note that the static friction force opposes the direction that the book is almost moving in.



2. We first consider the case when the book is almost falling down. The frictional force points up and has its maximum value, meaning it has a **norm** given by

$$F_f = \mu_s N.$$

Applying Newton's second law and requiring equilibrium (i.e. $\vec{a} = 0$) gives

$$N = F_p \sin\alpha$$

in the $\hat{x}$ direction and

$$\mu_s N + F_p\cos\alpha - mg = 0$$

in the $\hat{y}$ direction. Using these two equations to eliminate the normal force N and solve for F_p gives the solution of

$$F_p = \frac{mg}{\cos\alpha + \mu_s\sin\alpha}.$$

Next we consider the case when the book is about to slip up. The frictional force points down and has its maximum value, meaning it has a **norm** given by

$$F_f = \mu_s N.$$

Applying Newton's second law and requiring equilibrium (i.e. $\vec{a} = 0$) gives

$$N = F_p \sin\alpha$$

in the $\hat{x}$ direction and

$$-\mu_s N + F_p \cos\alpha - mg = 0$$

in the $\hat{y}$ direction. Using these two equations to eliminate the normal force N and solve for F_p gives the solution of

$$F_p = \frac{mg}{\cos\alpha - \mu_s \sin\alpha}.$$

3. To find the force for which the friction is zero, we can take the limit that $\mu_s \to 0$ in either of the solutions to part 2. This gives

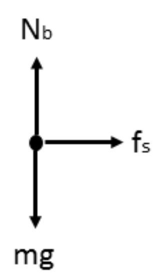
$$F_p = \frac{mg}{\cos\alpha}.$$

Alternatively, one could draw the free body diagrams without the friction force and solve the resulting components of Newton's second law, which gives the same answer.

When $\alpha = 0$, $F_p = mg/\cos(0) = mg$ and when $\alpha = 90°$, $F_p = mg/\cos(90°) \to \infty$, which are both consistent with our intuition.

3. Force with friction

The free body diagram for the books on their own is shown below, where we have the normal force of the table on the books $\vec{N}_b$, the static friction force from the table on the books $\vec{f_s}$, and the weight of the books $m\vec{g}$.



There is no motion in the $\hat{y}$ direction, so Newton's second law tells use that the weight is balanced by the normal force N_b according to

$$N_b - mg = 0 \quad \Rightarrow \quad N_b = mg. \tag{4}$$

The only horizontal force on the books is the static friction force f_s, which is equal to its maximum value of $f_s = \mu_s N_b$ when Carl is applying the maximum force for which the books do not slide. By applying Newton's second law in the $\hat{x}$ direction, we find

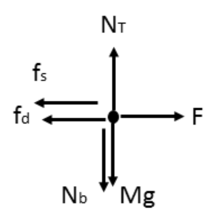
$$\sum F = ma \quad \Rightarrow \quad f_s = ma \quad \Rightarrow \quad \mu_s N_b = ma \quad \Rightarrow \quad a = \frac{\mu_s N_b}{m}.$$

Therefore, using equation (4) we see that the acceleration is

$$a = \mu_s g. \tag{5}$$

Now consider the table, whose free body diagram is shown below and includes a lot of forces. There is the normal force from the ground $\vec{N}_T$, the force applied by Carl $\vec{F}$, the kinetic friction force from the floor $\vec{f}_d$, the weight $m\vec{g}$, the normal force from the books on the table $-\vec{N}_b$, and the static friction force from the books $-\vec{f}_s$. Note that the static friction force on the table is an action-reaction pair with the static friction force in the free body diagram for the books, so it must be equal in magnitude and opposite in direction. Similarly the normal force from the books on the table is an action-reaction pair with the normal force in the free body diagram for the books.

In order to avoid sliding, the table and books must accelerate identically. The kinetic friction force $\vec{f}_d$ between the table and the floor has a magnitude of $f_d = \mu_d N_T$.



Since the table does not accelerate in the $\hat{y}$ direction, Newton's second law gives

$$N_T - N_b - Mg = 0 \quad \Rightarrow \quad N_T = N_b + Mg = (m + M)\, g, \tag{6}$$

where we have used equation (4). In the $\hat{x}$ direction, Newton's second law for the table is

$$F - f_d - f_s = Ma.$$

By substituting equations (4) through (6) and the forms of the friction forces (i.e. $f_s = \mu_s N_b$ and $f_d = \mu_d N_T$) from above, we obtain

$$F - \mu_d N_T - \mu_s N_b = M\mu_s g \quad \Rightarrow \quad F - \mu_d (m + M)\, g - \mu_s mg = M\mu_s g.$$

Solving for this equation for F gives the final answer of

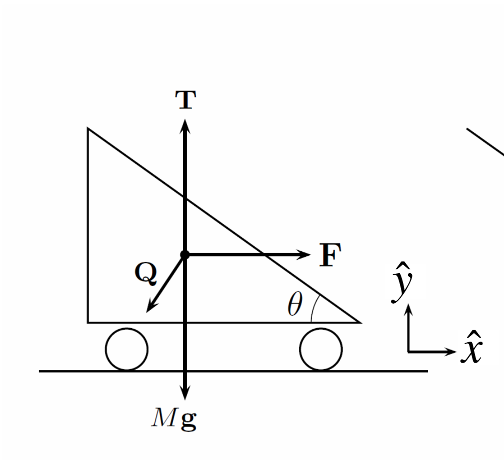
$$F = \mu_d (m + M)\, g + \mu_s (m + M)\, g \quad \Rightarrow \quad F = (\mu_d + \mu_s)(M + m)\, g$$

and we can plug in numbers to find

$$F = 159 \text{ N}.$$

4. Triangular trolley

1. The free body diagrams for both trolleys are shown below. The forces on the small trolley are the weight $m\vec{g}$ and the normal force from the triangular trolley $\vec{S}$. The forces on the triangular trolley are the weight $M\vec{g}$, the normal force from the small trolley $\vec{Q}$, the normal force from the ground $\vec{T}$, and the external force $\vec{F}$.



2. The forces on the small trolley are the weight $m\vec{g}$ and the normal force $\vec{S}$ from the triangular trolley acting on the small trolley. Thus, from Newton's second law the acceleration of the small trolley $\vec{a}$ is

$$m\vec{g} + \vec{S} = m\vec{a} \quad \Rightarrow \quad \vec{a} = \vec{g} + \frac{\vec{S}}{m}.$$

Projecting this in the $\hat{x}$ and $\hat{y}$ directions gives

$$a_x = \frac{S\sin\theta}{m}$$

and

$$a_y = \frac{S\cos\theta}{m} - g.$$

The forces on the triangular trolley are the weight $M\vec{g}$, the externally applied force $\vec{F}$, the normal force $\vec{T}$ from the ground acting on the triangular trolley, and the normal force $\vec{Q}$ from the small trolley acting on the triangular trolley.

We can recognize that $\vec{S}$ and $\vec{Q}$ are action-reaction pairs. Thus, from Newton's third law we know that

$$\vec{Q} = -\vec{S}.$$

Using this, Newton's second law for the triangular trolley becomes

$$M\vec{g} + \vec{T} + \vec{F} + \vec{Q} = M\vec{A} \quad \Rightarrow \quad \vec{A} = \vec{g} + \frac{\vec{T}}{M} + \frac{\vec{F}}{M} - \frac{\vec{S}}{M}.$$

Projecting this in the x and y directions gives

$$A_x = \frac{F}{M} - \frac{S\sin\theta}{M}$$

and

$$A_y = -g + \frac{T}{M} - \frac{S\cos\theta}{M},$$

where F and T are the norms of $\vec{F}$ and $\vec{T}$ respectively.
Since the triangular trolley is not accelerating vertically, we can take $A_y = 0$ to show that

$$T = Mg + S\cos\theta.$$

We want to find the force that leaves the small trolley immobile on the larger one, so we require

$$A_x = a_x$$

$$A_y = a_y,$$

which corresponds to

$$\frac{F}{M} - \frac{S\sin\theta}{M} = \frac{S\sin\theta}{m}$$

$$0 = \frac{S\cos\theta}{m} - g$$

respectively. From the second equation we see that

$$S = \frac{mg}{\cos\theta},$$

which can be substituted into the first to find the final answer,

$$F = g\,(M+m)\tan\theta.$$