Problem Set 2

Ballistics

PHYS-101(en)

1. Vectors

1. Let $\vec{v}_1 = \hat{x} + 2\hat{y}$ and $\vec{v}_2 = \hat{x} - \hat{y}$. Draw the following vectors in a 2D coordinate space:
   a. $\vec{v}_1$
   b. $\vec{v}_2$
   c. $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$
   d. $\vec{v}_4 = 2\vec{v}_1 - \vec{v}_2$
2. Start a vector $\vec{a}$ from the end of a vector $\vec{b}$ and denote the angle between them as θ. Then complete the triangle with the vector $\vec{c} = \vec{a} + \vec{b}$. Using a drawing and basic trigonometric identities of right triangles, find an expression for the magnitude (i.e. length) of $\vec{c}$ in terms of the magnitude of $\vec{a}$, the magnitude of $\vec{b}$, and θ. Notice that you have re-derived the "law of cosines", relating the lengths of the sides of a triangle to the cosine of one of its angles.
3. Find an expression for the angle α, defined as the angle between $\vec{a}$ and $\vec{c}$, as a function of the magnitude of $\vec{b}$, the magnitude of $\vec{c}$, and the angle θ.

2. The tortoise and the hare revisited

The tortoise and the hare are racing a distance L along a straight track. The tortoise and hare start with constant velocities, v_t and v_h respectively. The hare is not taking this race seriously, so its velocity is less than the tortoise's. Once the tortoise arrives at a bridge a distance $L' < L$ from the start, the hare realizes it could lose and accelerates with a constant acceleration a. Express this problem mathematically. Find the condition on a that will allow the hare to win the race. Check your result in terms of units. Brainstorm limiting cases for which you have clear intuition about the solution. In these limiting cases, does your expression for a make sense?

3. Sherlock Holmes

Sherlock Holmes visits France. He climbs to the top of the Eiffel tower and wonders what its height is. Since he cannot speak French, he formulates an alternate plan — he drops his magnifying glass over the handrail! After 9 seconds, he <u>hears</u> it hit the ground. How tall is the Eiffel tower? You may assume the magnifying glass undergoes standard projectile motion (thereby ignoring air drag).

Some hints/steps to follow:

a. Make a drawing and define your variables

b. Solve the equations symbolically

c. Check the units of your result

d. Eliminate the non-physical solution by considering the limiting case where the speed of sound v_s is infinite

e. Substitute the numerical values $v_s = 320$ m/s and $g = 10$ m/s^2 to calculate the physical solution

4. Reference frames

A ship sails due southeast with a speed of $v = 18$ km/h. A boy standing on the deck throws a ball in the east direction with an initial speed (relative to the ship) of $v_b = 15$ m/s and initial angle of $\theta = 30$ degrees upwards (with respect to horizontal). Write an expression for the trajectory of the ball in the reference frame associated with (a) the ship and (b) the ocean (laboratory) reference frame.

5. Dropping a stone from a sailboat

Perched at the top of a sailboat mast, a girl drops a stone. The mast, of height h, is perfectly vertical. The initial distance between the top of the mast and the stone is assumed to be zero.

1. The sailboat is docked on land. The girl drops the stone, imparting no initial velocity. How long does it take for the stone to hit the foot of the mast? How far from the foot of the mast does the stone land? Ignore air resistance, and assume the deck of the boat is level with the water.

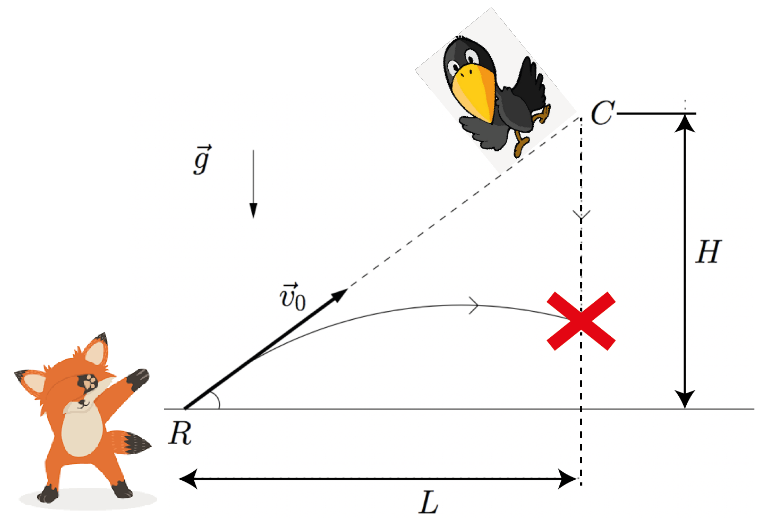
2. The girl takes the boat onto Lake Geneva and cruises with a constant horizontal velocity $\vec{v}_0$. She repeats her stone dropping experiment from part 1. How long does it take for the stone to hit the deck or water? How far from the foot of the mast does the stone land? Ignore air resistance, and assume the deck of the boat is level with the water.

3. It is 2300, and the girl and her sailboat can travel to space! During her interstellar space exploration, the boat is subject to a constant acceleration $\vec{a}$, which is parallel to the mast (i.e. "upwards") with the magnitude of $|\vec{a}| = a = 9.81$ m/s^2. The girl repeats her stone dropping experiment as before. At the moment when the stone is dropped, the velocity of the vessel relative to the stars points along the mast and has a norm v_0. Note that the dropped stone is subject to no acceleration. Again find the time for the stone to hit the deck and the distance it lands from the foot of the mast. *Hint: write the equation of motion for the stone as well as for the foot of the mast.*

6. The fox and the crow

Several days after their adventures described in the well-known fable, the fox finds the crow, again with a piece of cheese. The crow is no longer susceptible to flattering words from the fox. Wishing to steal the cheese for himself, the fox throws a stone, releasing it exactly in the direction of the crow. This scares the crow, who simultaneously flies away and drops his piece of cheese.

The crow sat in a tree with height H, which has a base that is a distance L horizontally from the fox. The initial velocity of the stone is $\vec{v}_0$.



1. At what time does the collision between the stone and the cheese occur? You can assume that the initial speed of the stone $v_0 = |\vec{v}_0|$ is sufficiently large to ensure a collision (until you reach part 4).
2. Show that the stone and the cheese always collide.
3. At what position does the collision occur?
4. What is the minimum speed v_0 required to ensure the stone and the cheese collide?

7. Challenge (optional): Rugby up-and-under play

A rugby technique called *up-and-under* consists of kicking the ball far and wide and then running under its path in order to catch it before it lands. The player can run with a maximum speed of v_p^{max} and can give the ball an initial maximum speed of v_{bi}^{max}. Assume the ball's trajectory is parabolic and that the player runs at a constant velocity starting the instant that the ball is in the air.

1. What angle with the ground α should she kick the ball at to maximize the distance at which she catches the ball (i.e. the distance between where the kick occurs and where the catch occurs)? What is this maximum distance?
2. Considering that the attacking player kicks the ball as described in part 1. How far should the defense be placed (from the opposing player) to counter her shot? The height of the defensive player's hand in the air is h.