EPFL

Multiple Choice Questions (4 points)

1. *What is the magnitude of the torque about the point S?* **(1 point)**

   (a) FL
   (b) $FL\sin(\theta)$
   (c) $FL\cos(\theta)$
   (d) $FL\tan(\theta)$
   (e) None of the above.

Explanation: The magnitude of a torque about a pivot point is $|\vec{\tau}| = rF\sin(\phi)$. Here, the magnitude of the position vector is $|\vec{r}| = L$, and the magnitude of the force vector is $|\vec{F}| = F$. However, the angle θ given in the problem statement is NOT the angle between $\vec{r}$ and $\vec{F}$. Rather, $\phi = \pi/2 - \theta$ and the correct answer is then $|\vec{\tau}| = FL\cos(\theta)$.

2. *At how many of the labeled points will the velocity be zero?* **(1 point)**

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

Explanation: The total mechanical energy of a system is defined as the sum of its kinetic and potential energies: $E_m = U + K$. When $E_m = U$, the kinetic energy (and therefore velocity) of the particle will be zero. This occurs at two points: a and g.

3. *The greatest acceleration of the center of mass (CM) of the baseball bat will be produced by pushing with a force F at which point?* **(1 point)**

   (a) Position 1
   (b) Position 2
   (c) Position 3
   (d) All positions are the same
   (e) The CM does not accelerate

Explanation: The acceleration of the CM of an object is just given by the sum of external forces on the object divided by its mass: $\vec{a}_{\text{CM}} = \vec{F}_{\text{ext}}/m$. As a result, $\vec{a}_{\text{CM}}$ is the same regardless of where the force is applied.

4. *Which objects collide at $t = 1.5\,s$?* **(1 point)**

   (a) Cart B and the spring
   (b) Cart B and the motion sensor
   (c) Carts A and B
   (d) Cart A and the spring
   (e) Cart A and the motion sensor

Explanation: Initially, the motion sensor tracks cart A as it rolls to the right, as indicated by the linear relationship between distance and time from 0.2 s to 1.0 s. Cart A then collides with cart B, stopping its motion and resulting in no observed change in distance from 1.0 s to 1.5 s. During this time, cart B rebounds off the spring and rolls back towards cart A. Finally, at 1.5 s, cart B collides with cart A sending it back towards the motion sensor, reflected by the shrinking distance with time from 1.5 s onward.

5. Big Air at the Winter X Games (10 points)

a. (**2.0 points**) *Verify that the work done by friction on the skier in the flat section is given by:* $W_f = -\mu\, m\, g\, d$.

Use a free-body diagram to correctly find the friction force (0.5 points):

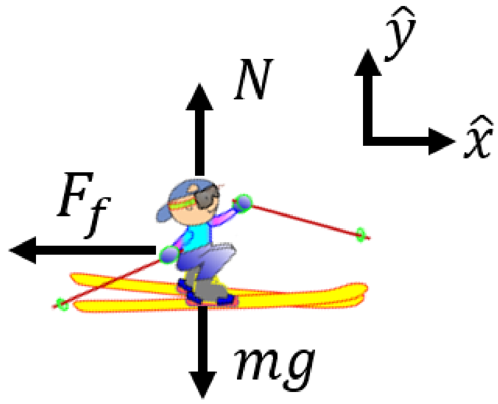

Figure 1: Free-body diagram for the skier.

From the free-body diagram in the flat part (figure 1):

$$\Sigma F_y = N - mg \;\Rightarrow\; N = mg$$

From this, $F_f = \mu N = \mu mg$ and $\vec{F_f} = -\mu mg\,\hat{x}$. As a result, friction is constant and acts along a total displacement of $\vec{l} = d\,\hat{x}$.

Correctly calculate the work done by friction (1.0 points):

We can then express the work done by friction over the flat section of the ramp as:

$$W_f = \vec{F_f}\cdot\vec{l} = (-\mu mg\,\hat{x})\cdot(d\,\hat{x}) = -\mu mgd$$

Why is this quantity negative?

Explain that friction is negative because it opposes the motion of the skier (0.5 points).

b. (**2.0 points**) *What is the minimum value of* ρ_0 *for the skier to barely make it across the flat part?*

Correctly apply the work–kinetic energy theorem (1.0 points):

Using the work–kinetic energy theorem:

$$\Delta K = W_{\text{net}} = W_g + W_f = -\Delta U_g + W_f \;\Rightarrow\; \Delta K + \Delta U_g = W_f$$

$$W_f = (K_f + U_f) - (K_i + U_i)$$

Identify appropriate values for the kinetic and potential energies and solve for the correct ρ_0 (1.0 points):

We can now choose the "initial" point as the top of the circular ramp and the "final" point as the end of the flat part. If we define $U_g = 0$ at $h = 0$ and $K_f = 0$ for the skier to "barely" make it across the flat part, we can plug in the values for $U_i = mg\rho_0$, $K_i = 0$, and the result from (a) to find ρ_0:

$$(0 + 0) - (0 + mg\rho_0) = W_f = -\mu mgd \;\Rightarrow\; \rho_0 = \mu d$$

c. (**1.0 points**) *What is the minimum value of ρ_0 for the skier to barely reach the top of the ramp (height h in the launch section)?*

As in (b), properly apply the work–kinetic energy theorem (0.5 points):

Using the work–kinetic energy theorem:

$$\Delta K = W_{\text{net}} = W_g + W_f = -\Delta U_g + W_f \;\Rightarrow\; \Delta K + \Delta U_g = W_f$$

$$W_f = (K_f + U_f) - (K_i + U_i)$$

As in (b), identify appropriate values for the kinetic and potential energies and solve for the correct ρ_0 (0.5 points):

Now, we are asked to find the value of ρ_0 for the skier to reach the top of the ramp. This only requires changing the value of final potential energy of the skier to $U_f = mgh$:

$$(0 + mgh) - (0 + mg\rho_0) = W_f = -\mu mgd \;\Rightarrow\; \rho_0 = \mu d + h$$

d. (**2.0 points**) *If ρ_0 is sufficiently large for the skier to take off from the ramp, compute the maximum height that the skier will reach in the air as a function of the parameters of the problem.*

Use the work–kinetic energy theorem to find the takeoff velocity of the skier (1.0 points):

To determine the maximum height the skier will reach in the air, we first find the takeoff velocity. To do so, we can again use the work–kinetic energy theorem by replacing the final kinetic energy with $K_f = \frac{1}{2}mv^2$:

$$(\frac{1}{2}mv^2 + mgh) - (0 + mg\rho_0) = W_f = -\mu mgd \;\Rightarrow\; v^2 = 2g(\rho_0 - h - \mu d)$$

Use conservation of energy or projectile motion to solve for $h_{\max}$ (1.0 points):

In the air, the skier is only subject to the force of gravity, a conservative force, so conservation of mechanical energy holds:

$$K_h + U_{g,h} = K_{\max} + U_{g,\max}$$

Where K_h and $U_{g,h}$ denote the kinetic and potential energies at the takeoff position at height h. At the top of the skier's trajectory (height $h_{\max}$), the vertical component of the velocity is 0, but the horizontal component is not. However, we know that the horizontal component of the velocity remains constant during ballistic motion. We can then write equations for the

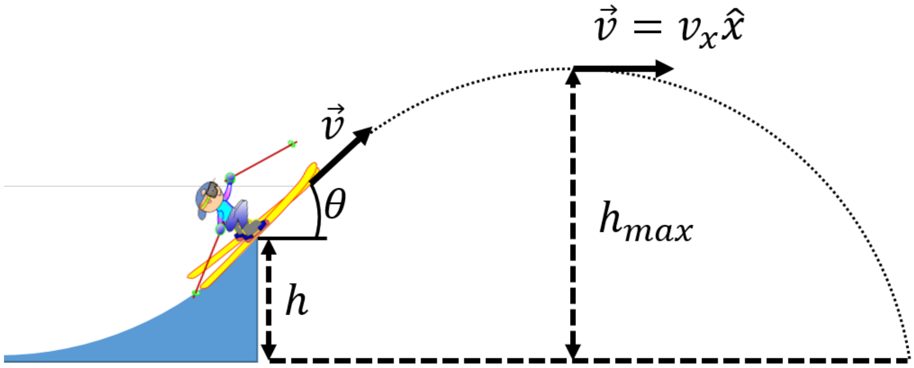

Figure 2: Trajectory of the skier.

potential and kinetic energy of the skier at $h_{\max}$ and substitute these into the conservation of energy equation:

$$U_{g,\max} = mgh_{\max} \qquad \text{and} \qquad K_{\max} = \frac{1}{2}mv_x^2 = \frac{1}{2}m(v\cos(\theta))^2 = \frac{1}{2}mv^2\cos^2(\theta)$$

$$K_h + U_{g,h} = \frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2\cos^2(\theta) + mgh_{\max}$$

We can then write an expression for $h_{\max}$ and substitute in the value for v^2 found earlier:

$$h_{\max} = h + \frac{v^2(1-\cos^2(\theta))}{2g} = h + \frac{v^2\sin^2(\theta)}{2g}$$

$$h_{\max} = h + (\rho_0 - h - \mu d)\sin^2(\theta) = h(1-\sin^2(\theta)) + (\rho_0 - \mu d)\sin^2(\theta)$$

$$h_{\max} = h\cos^2(\theta) + (\rho_0 - \mu d)\sin^2(\theta)$$

e. (**3.0 points**) *What is the g-force experienced by the skier during the initial (circularly-shaped) descent? Write your answer as a function of the angle β. Note that the g-force is the magnitude of the force exerted by the surface on the skier, divided by the weight of the skier.*

Correctly find the normal force acting on the skier using centripetal acceleration (1.0 points):

From the free-body diagram during the circular descent (figure 3), if cylindrical coordinates are chosen with an origin at the center of the circle, the forces along $\hat{\rho}$ will be:

$$\Sigma F_\rho : mg\cos(\theta) - N = m\,a_{\text{cent}} = -m\frac{v^2(\beta)}{\rho_0}$$

$$N = m\frac{v^2(\beta)}{\rho_0} + mg\cos(\theta)$$

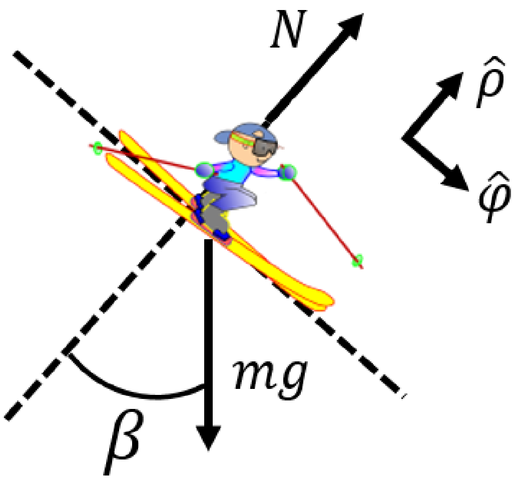

Figure 3: Free-body diagram for the skier while descending the ramp.

Use conservation of energy to find the expression for the normal force in terms of m, g, and β (1.0 points):

Since only gravity is doing work on the skier, conservation of mechanical energy holds:

$$K_i + U_{g,i} = K_\beta + U_{g,\beta} \Rightarrow 0 + mg\rho_0 = \frac{1}{2}mv^2(\beta) + mgh(\beta)$$

$$\frac{1}{2}mv^2(\beta) = mg(\rho_0 - h(\beta))$$

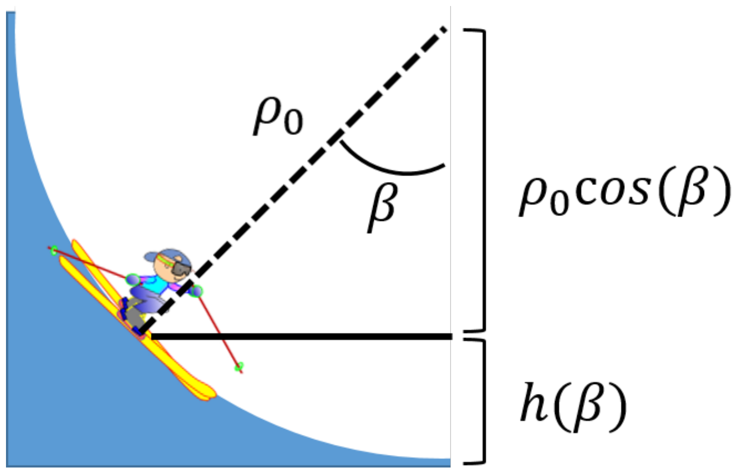

Figure 4: The height of the skier while descending the ramp.

To find the height of the skier with respect to the floor, $h(\beta)$, we can use the geometry of the problem (see figure 4) which tells us that $\rho_0 = h(\beta) + \rho_0 \cos(\beta)$ and therefore $h(\beta) = \rho_0 - \rho_0 \cos(\beta)$. We can then substitute $h(\beta)$ into the previous expression to find the g-factor:

$$\frac{1}{2}mv^2(\beta) = mg\left[\rho_0 - \rho_0 + \rho_0\cos(\beta)\right] = mg\rho_0\cos(\beta)$$

$$N = 2mg\cos(\beta) + mg\cos(\beta) = 3mg\cos(\beta)$$

Divide N by mg to find the g-force (0.5 points):

$$\frac{N}{mg} = 3\cos(\beta)$$

Where is the g-force largest?

Explain that the g-force is largest when the skier reaches the bottom of the ramp (0.5 points):

The g-force is largest when $\beta = 0$, in which case $N/mg = 3$. This occurs at the bottom of the ramp.