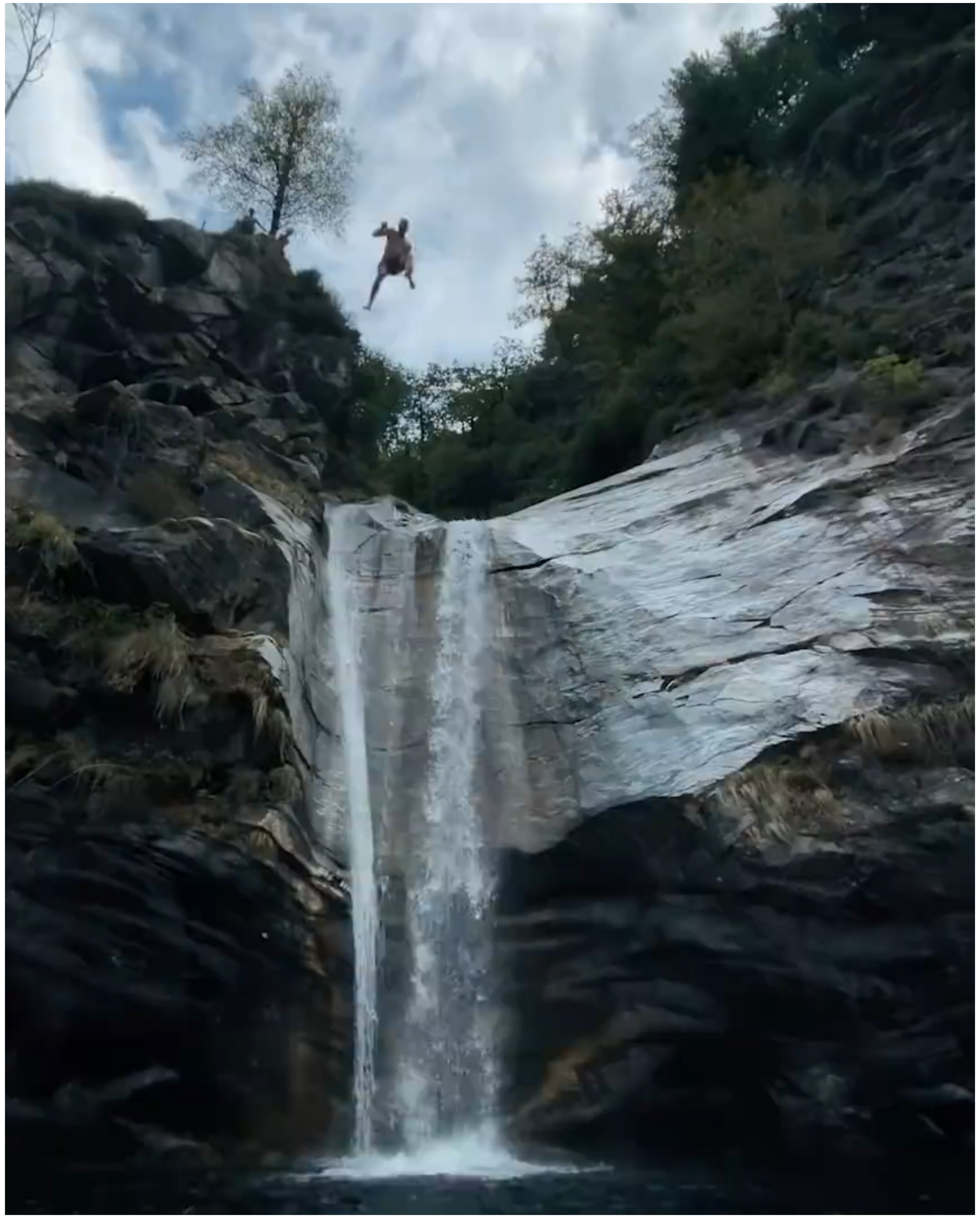
General Physics: Mechanics



Swiss Plasma Center

PHYS-101(en)

Lecture 9b: Potential energy, energy conservation

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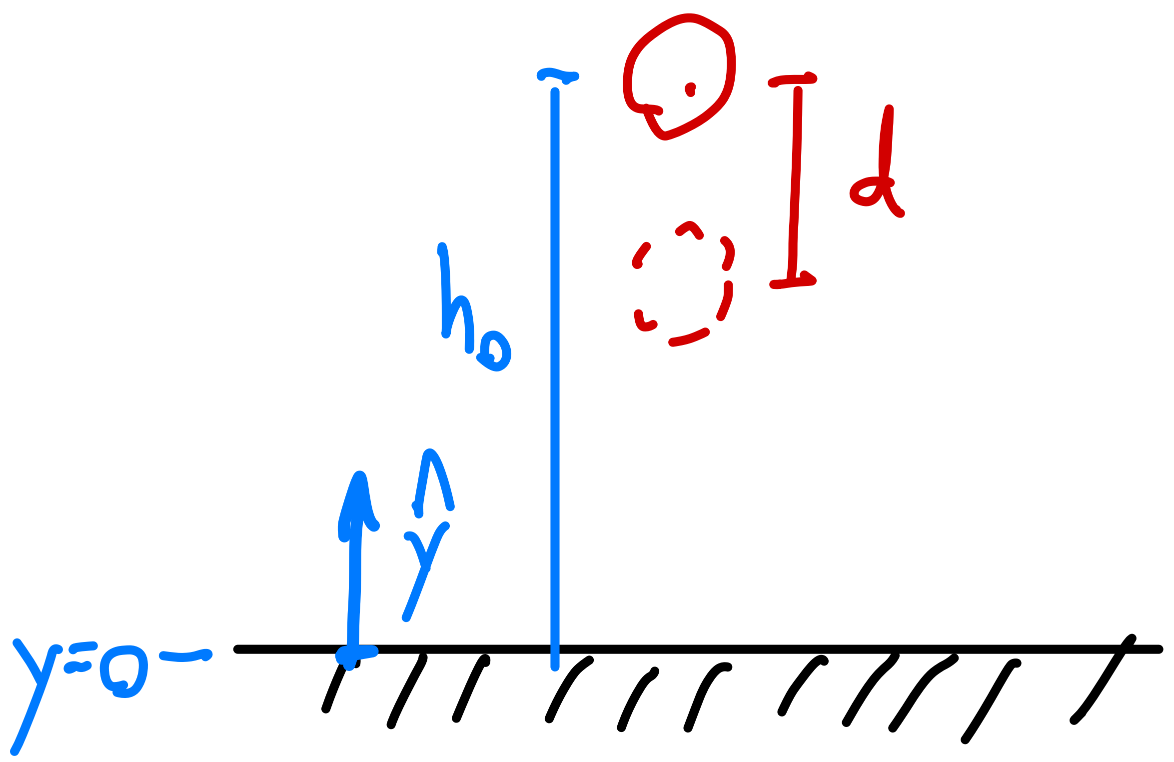
Example: The power of gravity

What is the power of gravity? In other words, if an object of mass m starts at rest near the surface of earth, what is the instantaneous power after it falls a distance d? You may ignore the effects of air resistance.

$$U_g(y) = mgy + Q$$



$$P_g = \vec{F}_g \cdot \vec{v} \qquad \vec{F}_g = -mg\hat{y} \qquad \vec{v} = v_y\hat{y} = -|v_y|\hat{y}$$

$$\Rightarrow P_g = (-mg\hat{y})\cdot(-|v_y|\hat{y}) = mg|v_y|\underbrace{(\hat{y}\cdot\hat{y})}_{=1} = mg|v_y|$$

$$W_{non} = \Delta E_m \Rightarrow \Delta E_m = 0 \Rightarrow K_i + U_{gi} = K_d + U_{gd} \quad (W_{non} = 0,\ K_i = 0)$$

$$\Rightarrow K_d = U_{gi} - U_{gd} = mgd \Rightarrow \tfrac{1}{2} m v_d^2 = mgd \Rightarrow |v_d| = \sqrt{v_d^2} = \sqrt{2gd} \qquad \left(K_d = \tfrac{1}{2} m v_d^2\right)$$

$$\boxed{P_g = mg\sqrt{2gd} = \sqrt{2}\, m g^{3/2} \sqrt{d}}$$

Example: Cliff jumping

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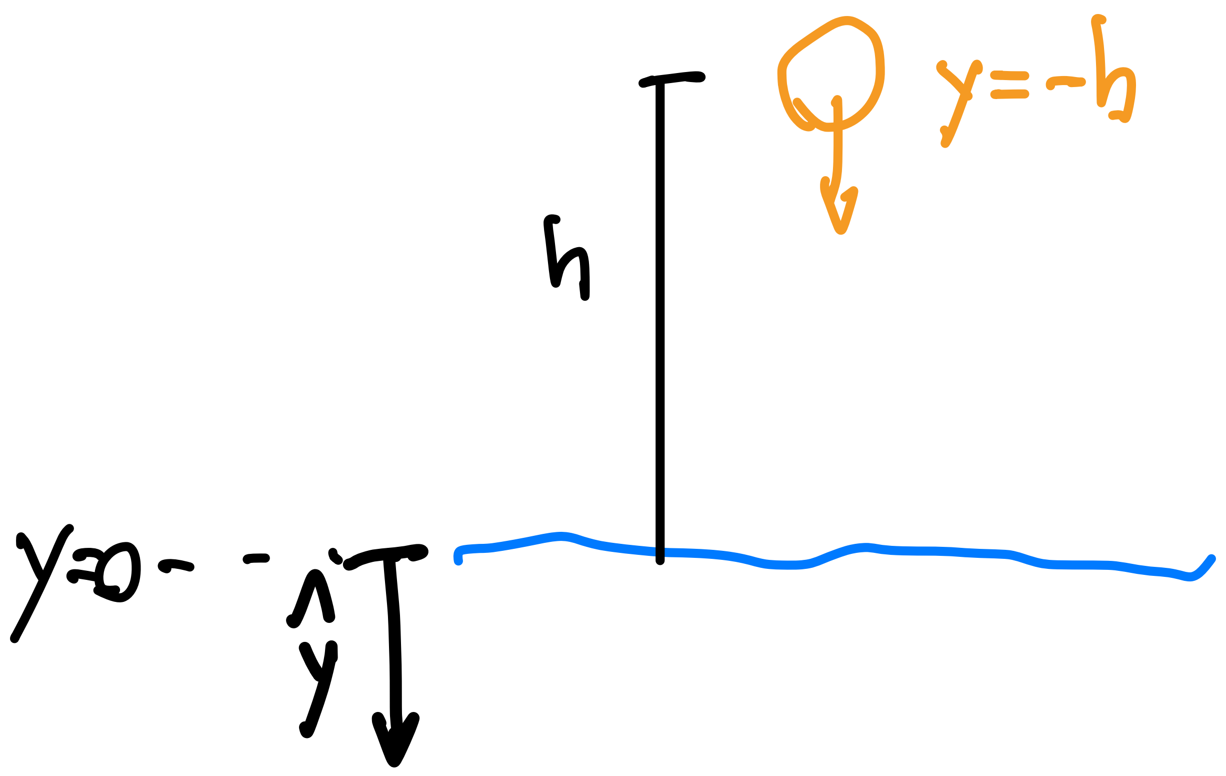
You’re on vacation and want to jump from a cliff into water. If humans can temporarily withstand about 10 g’s of force without much discomfort, how high can you jump from? How deep must the water be? Neglect gravity when you’re in the water and ignore *air* resistance, but assume your drag coefficient in water is $\beta \approx 15$ kg/m.



$\Delta E_m = W_{non} = 0$

$\Rightarrow K_i + U_{gi} = K_f + U_{gf}$

$\Rightarrow K_f = U_{gi} - U_{gf} = mgh$, with $K_f = \frac{1}{2} m v_w^2$

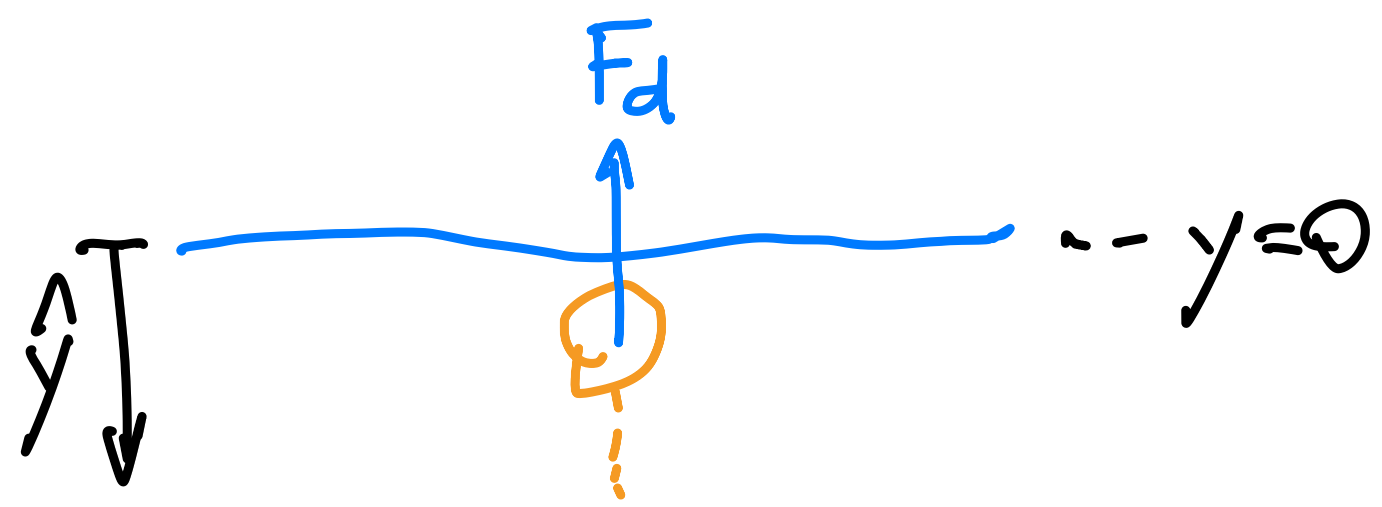
$\Rightarrow v_w^2 = 2gh$

Cliff jumping safety

1. Be a good swimmer
2. Never go alone
3. Make sure the water is deep enough
4. Prepare for intense water pressure
5. Wear shoes
6. Don’t jump headfirst
7. Keep your body streamlined
8. Blow out through your nose
9. Be of sound mind
10. Confidence is key

Example: Cliff jumping

If humans can temporarily withstand about 10 g's of force without much discomfort, how high can you jump from?



$$\vec{F}_d = \beta v^2(-\hat{v}) = -\beta v_y^2 \hat{y} \Rightarrow |\vec{F}_d|^{max} = \beta v_w^2 = \beta 2gh$$

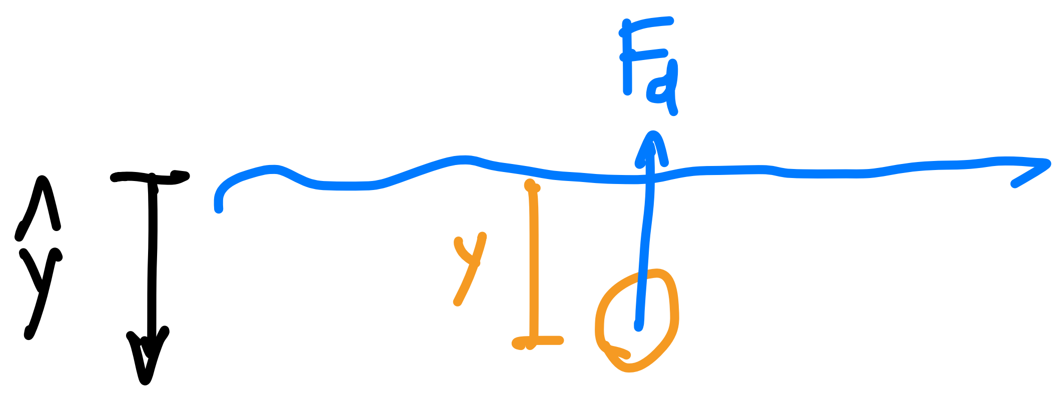
$|\vec{F}_d|^{max} \lesssim 10mg$ for "comfort"

$$\Rightarrow 2\beta g h \lesssim 10 m g \Rightarrow h \lesssim 5\frac{m}{\beta}$$

For $m = 60\,\text{kg}$, $\boxed{h \lesssim 5\,\frac{60\,\text{kg}}{15\,\frac{\text{kg}}{\text{m}}} \approx 20\,\text{m}}$

Example: Cliff jumping

How deep must the water be? Neglect gravity when you're in the water and ignore *air* resistance, but assume your drag coefficient in water is $\beta \approx 15$ kg/m.



$\vec{F}_d = -\beta v_y^2 \hat{y}$ $\quad d\vec{l} = \hat{y}\, dy$ $\quad \Rightarrow \quad \vec{F}_d \cdot d\vec{l} = -\beta v_y^2\, dy$

$$\Rightarrow \quad W_d = \int_0^y -\beta v_y^2\, dy = -\beta \int_0^y v_y^2\, dy$$

$$\Delta K = W_{net} = \cancel{W_g}^{\,0} + W_d = -\beta \int_0^y v_y^2\, dy$$

$$\Delta K = \frac{1}{2} m v_y^2 - \frac{1}{2} m v_w^3$$

$$\frac{d}{dy}\left[\frac{1}{2} m v_y^2 - \frac{1}{2} m v_w^3\right] = \frac{d}{dy}\left[-\beta \int_0^y v_y^2\, dy\right] \quad \Rightarrow \quad \frac{1}{2} m \frac{d}{dy}\left[v_y^2\right] = -\beta \frac{d}{dy}\left[\int_0^y v_y^2\, dy\right] = -\beta v_y^2$$

$$\Rightarrow \quad \frac{d}{dy}\underbrace{\left[v_y^2\right]}_{s} = -\frac{2\beta}{m}\underbrace{\left(v_y^2\right)}_{s} \quad \Rightarrow \quad \frac{ds}{dy} = -\frac{2\beta}{m} s$$

Example: Cliff jumping

$$\frac{ds}{dy} = -\frac{2\beta}{m} s \Rightarrow \int \frac{ds}{s} = \int -\frac{2\beta}{m} dy \Rightarrow \ln|s| = -\frac{2\beta}{m} y + C_1 \Rightarrow \underbrace{s}_{v_y^2} = e^{-\frac{2\beta}{m} y} e^{C_1}$$

$$= C_2 e^{-\frac{2\beta}{m} y}$$

We know that $v_y^2(0) = v_w^3 = 2gh \Rightarrow v_y^2(0) = C_2 e^0 = C_2 = 2gh$

$$\Rightarrow v_y^2 = 2gh\, e^{-\frac{2\beta}{m} y} \Rightarrow \boxed{|v_y| = \sqrt{2gh}\, e^{-\frac{\beta}{m} y}}$$

Typical "decay" distance is

$$\frac{m}{\beta} \approx \frac{60\,\text{kg}}{15\,\frac{\text{kg}}{\text{m}}} = 4\,\text{m}$$