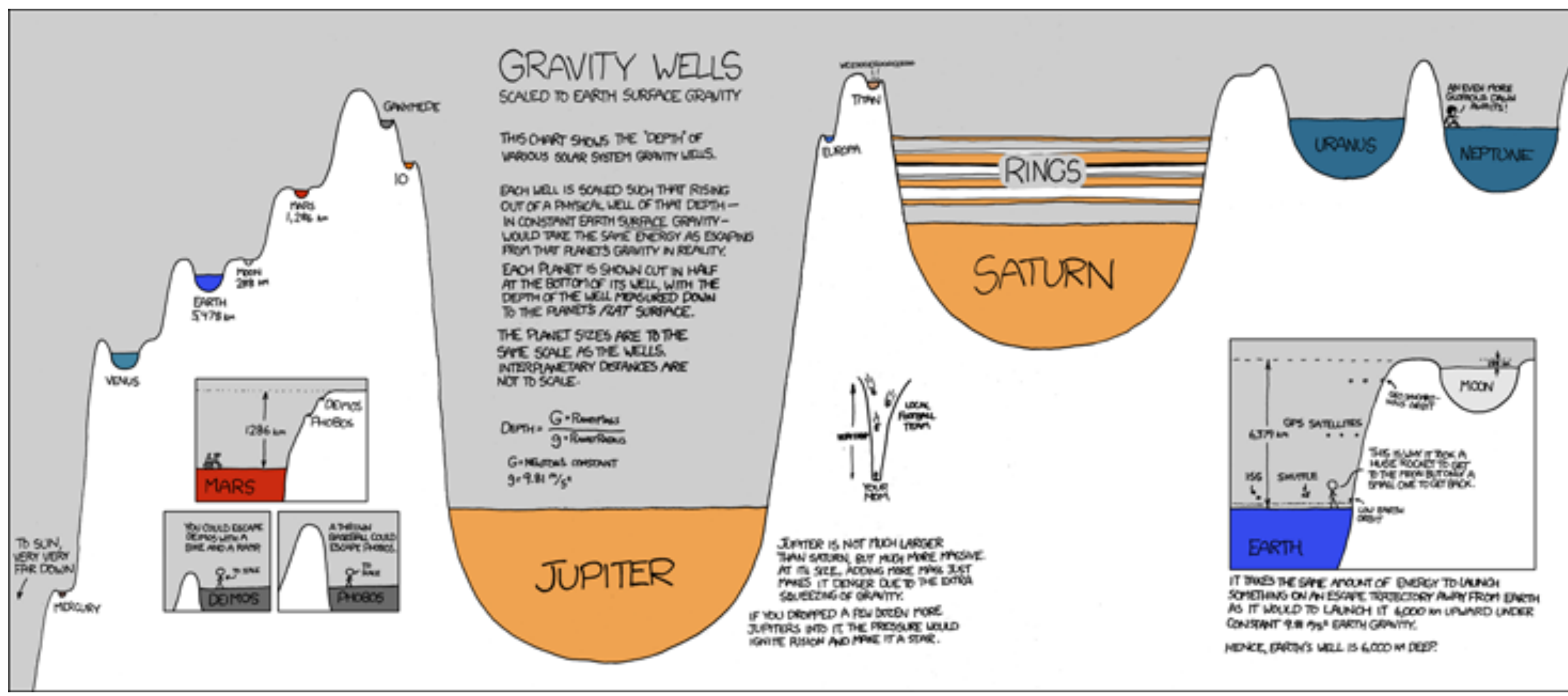
General Physics: Mechanics

PHYS-101(en)

Lecture 9a:
Potential energy, energy conservation



https://xkcd.com/681

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November 10th, 2025

EPFL
Swiss Plasma Center

Today's agenda (Serway 7-8, MIT 13-14)

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1. A few words on Mock exam 1
2. Power
3. Energy and work
   - Potential energy
   - Conservation of mechanical energy

Mock exam takeaways

Swiss Plasma Center

- A total of 138 mock exams were graded
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    - The exam was long. Better reference: **7.6/14**

Mock exam takeaways

- Common pitfall 1: Not describing your answer with enough level of detail to show that you understand what you are doing.
  - Physics is **not** about just showing some formulas
  - You will miss possible partial credit
  - As an example of what you could do, check the solutions on the Moodle.

Mock exam takeaways

- Common pitfall 1: Not describing your answer with enough level of detail to show that you understand what you are doing.
  - Physics is **not** about just showing some formulas
  - You will miss possible partial credit
  - As an example of what you could do, check the solutions on the Moodle. However,
- Common pitfall 2: Writing too much!
  - Writing too many details makes you spend time unnecessarily

Mock exam takeaways

- Common pitfall 3: Lingering on a question and not moving on.
  - Part (b) was not needed in later parts
  - Part (e) was actually quite simple

Today's agenda (Serway 7-8, MIT 13-14)

1. A few words on Mock exam 1

2. **Power**

3. Energy and work

    - Potential energy

    - Conservation of mechanical energy

From last week — kinetic energy and work

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- Units of work and energy are Joules (J)

Power from work

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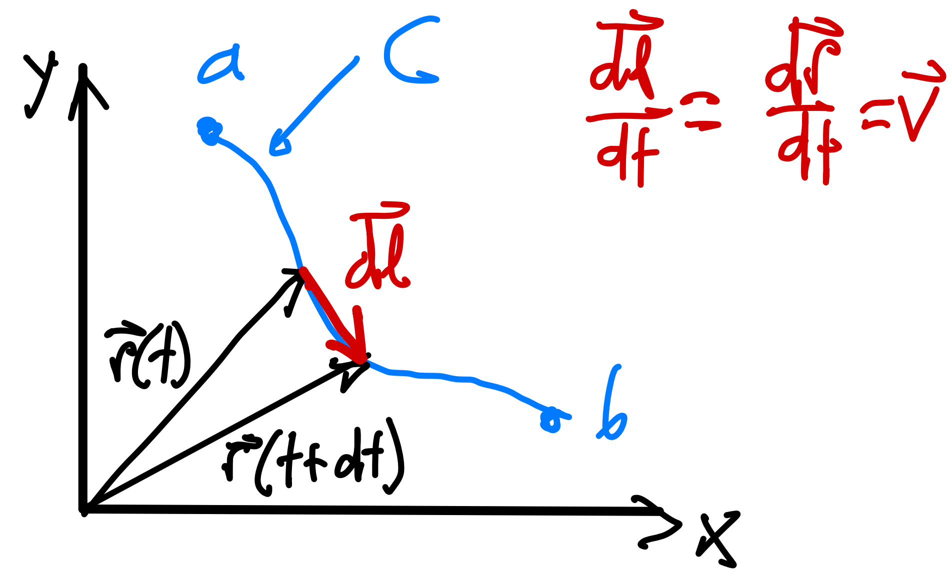
- Using the work-kinetic energy theorem, power is also the amount of energy transferred or converted per unit time
- Humans operate at roughly 100 W

Power from force

- For a constant force, power can be written as

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F}\cdot\vec{\ell}\right) = \vec{F}\cdot\frac{d\vec{\ell}}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$



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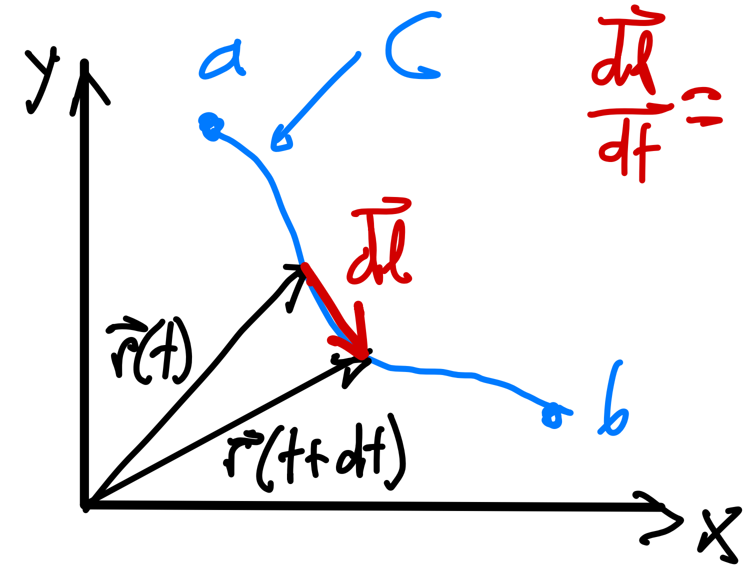
- What about a variable force?

$$P = \frac{d}{dt}W = \frac{d}{dt}\left[\int_C \vec{F}\cdot d\vec{l}\right] = \frac{d}{dt}\left[\int_{t_0}^{t} \vec{F}\cdot\vec{v}\,dt\right] = \vec{F}\cdot\vec{v}$$

$$d\vec{l} = \vec{v}\,dt$$

$$\frac{d\vec{l}}{dt} = \frac{d\vec{r}}{dt} = \vec{v}$$



Conceptual question

A particle starts from rest at $x = 0$ and moves to $x = L$ while experiencing the variable force $F(x)$ shown in the figure. What is the particle's kinetic energy at $x = L/2$ and at $x = L$?

A. $\frac{F_{max}L}{2}, F_{max}L$

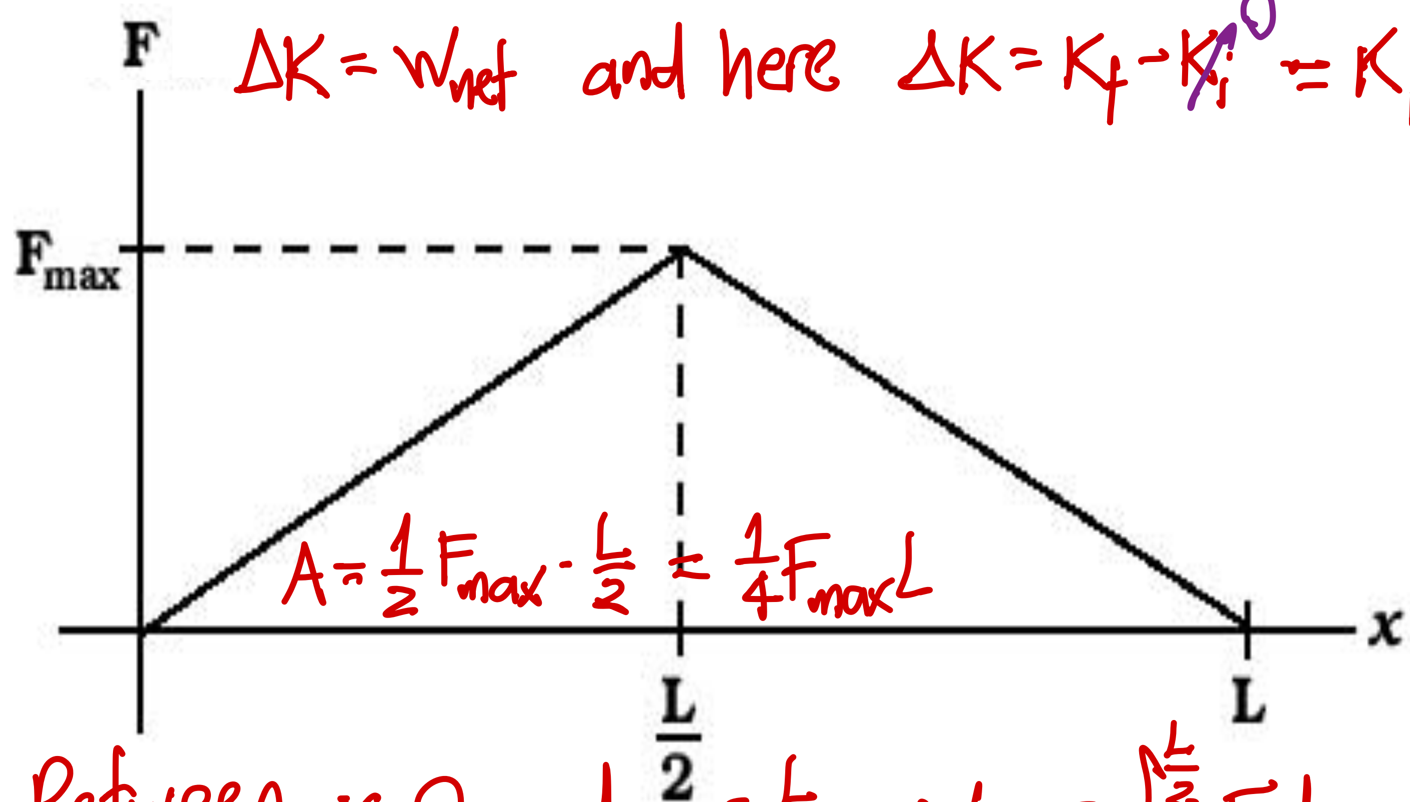
B. $\frac{F_{max}L}{2}, 0$

C. $F_{max}L, 0$

D. $\frac{F_{max}L}{4}, \frac{F_{max}L}{2}$

E. $\frac{F_{max}L}{2}, \frac{F_{max}L}{4}$

$\Delta K = W_{net}$ and here $\Delta K = K_f - K_i = K_f$ (with $K_i = 0$)



Between $x=0$ and $x=\frac{L}{2}$, $W_{net} = \int_0^{\frac{L}{2}} F\,dx = A$

$x=0$ and $x=L$, $W_{net} = \int_0^{\frac{L}{2}} F\,dx + \int_{\frac{L}{2}}^{L} F\,dx = 2A$

Conceptual question

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A sports car accelerates from zero to 30 km/h in 1.5 s. How long does it take for it to accelerate from zero to 60 km/h, assuming the power of the engine to be independent of velocity and neglecting friction?

A. 2 s
B. 3 s
C. 4.5 s
D. 6 s
E. 9 s

$W = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2$ ($K_i = 0$)

constant power

$P = \frac{dW}{dt} = \frac{\Delta W}{\Delta t} = \frac{1}{\Delta t}\left(\frac{1}{2} m v_f^2\right)$

- For 0 to 30 $\frac{km}{h}$: $P = \frac{1}{\Delta t_1}\left(\frac{1}{2} m v_{30}^2\right)$

- For 0 to 60 $\frac{km}{h}$: $P = \frac{1}{\Delta t_2}\left(\frac{1}{2} m (2 v_{30})^2\right)$

$= \frac{1}{\Delta t_2} \cdot 4 \cdot \frac{1}{2} m v_{30}^2 = \frac{1}{\Delta t_2} \cdot 4 \cdot P \cdot \Delta t_1$

$\Rightarrow \Delta t_2 = 4 \Delta t_1 = 6 s$

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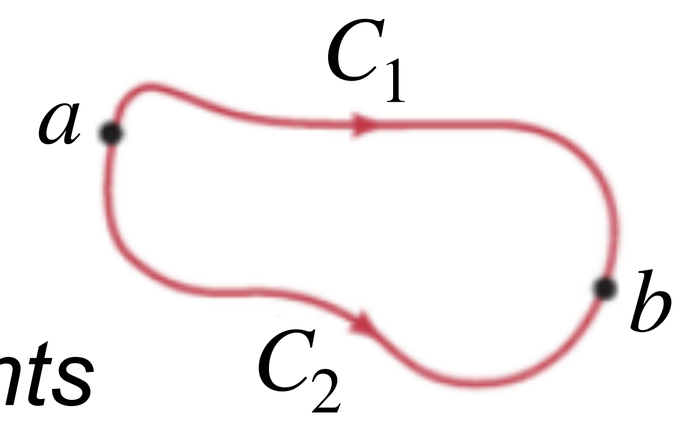
From last week — conservative forces

- A force is called *conservative* if



The work done by the force on
a particle moving between any two points
is independent of the path taken by the particle.

or equivalently

The net work done by the force on
a particle moving around any closed path is zero.

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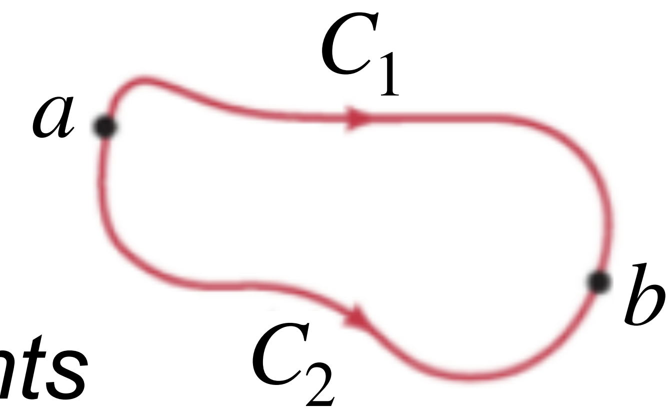

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- Otherwise, the force is *non-conservative*
- Work-kinetic energy theorem applies to both

Conservative and nonconservative forces

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- Conservative examples:
  - Work by gravity only depends on Δy and work by springs only depends on Δx

Conservative and nonconservative forces

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- Conservative examples:
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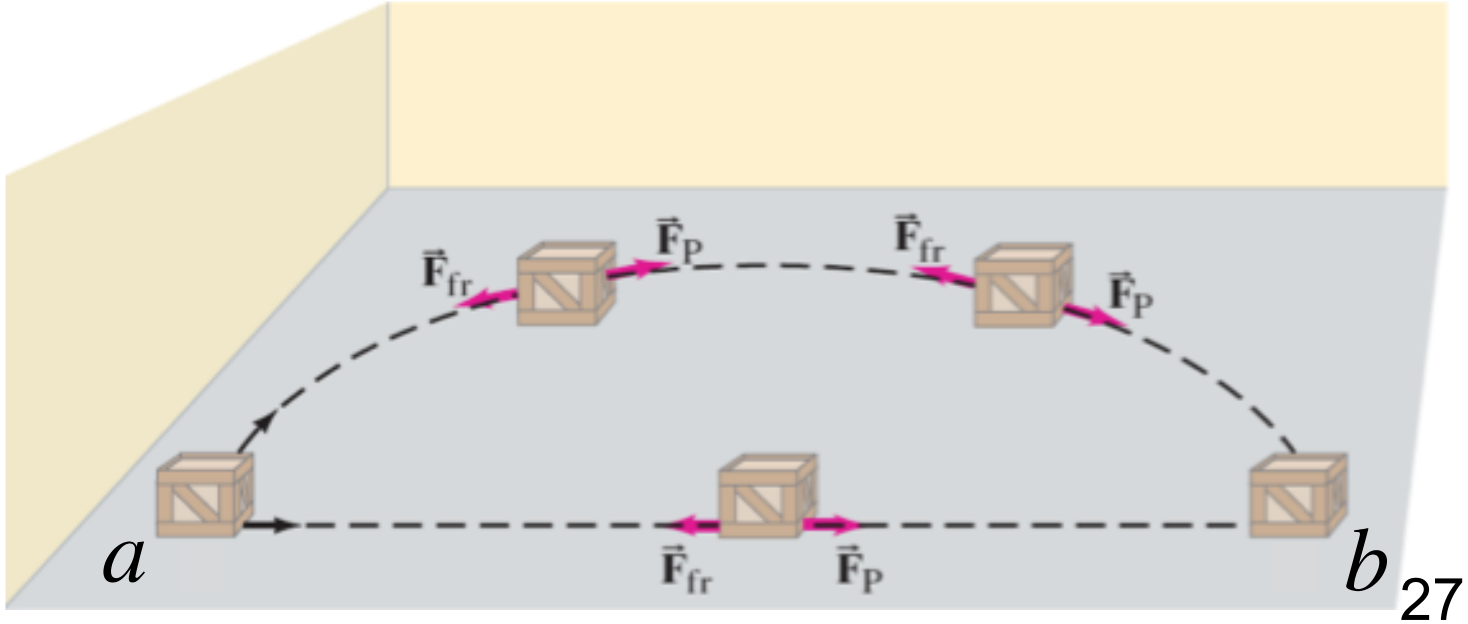
- Nonconservative example:
  - When friction is present, the work done depends not only on the starting and ending points, but also on the path taken
  - The longer path has more dissipation



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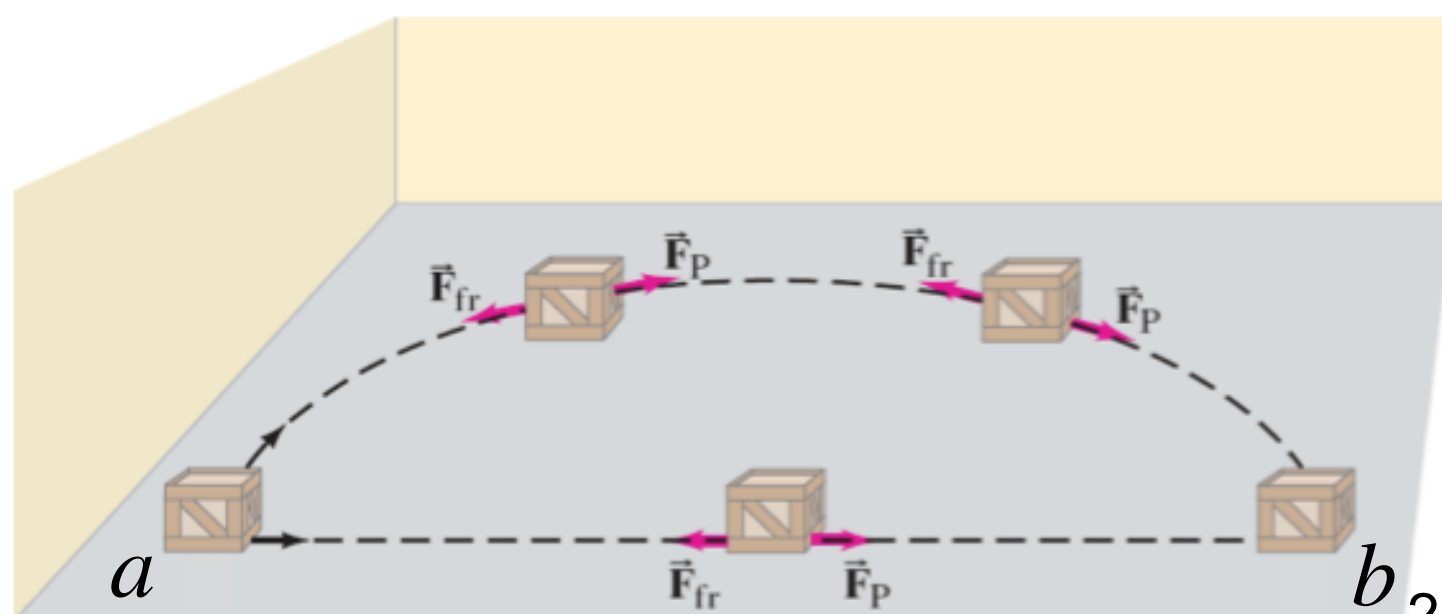
Conservative Force	**Nonconservative forces**
Gravitational	Friction
Elastic (spring)	Air resistance (drag)
Electric	Tension in rope
	Motor or rocket propulsion
	Push or pull by a person



Potential energy

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The change in potential energy is the energy "stored" by an object as the result of work done by a conservative force.

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$$\Delta U = U_b - U_a = -W = -\int_a^b \overrightarrow{F} \cdot d\overrightarrow{\ell}$$

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$$\Delta U = U_b - U_a = -W = -\int_a^b \overrightarrow{F} \cdot d\overrightarrow{\ell}$$

- Thus, there is gravitational potential energy and spring potential energy, but not friction potential energy

- The motivation for this definition is so that energy is conserved (as we will see)

Gravitational potential energy

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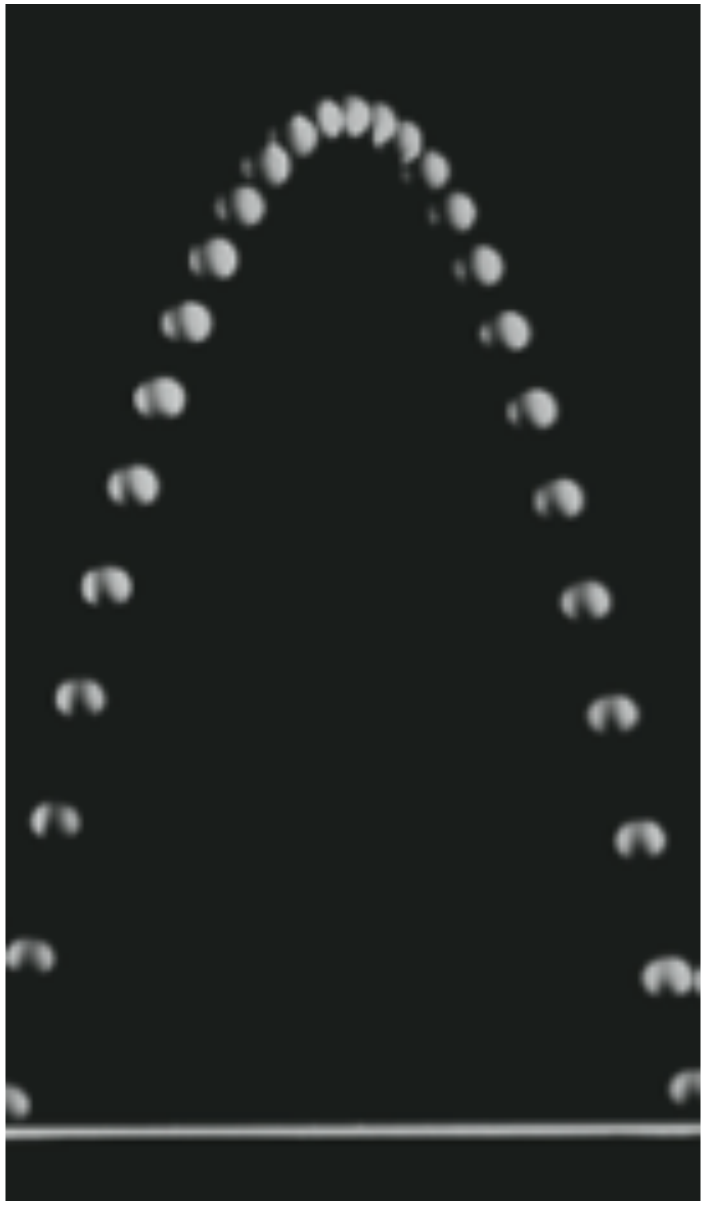
$$\Delta U = -W = -\int_a^b \overrightarrow{F} \cdot d\overrightarrow{\ell}$$



Gravitational potential energy

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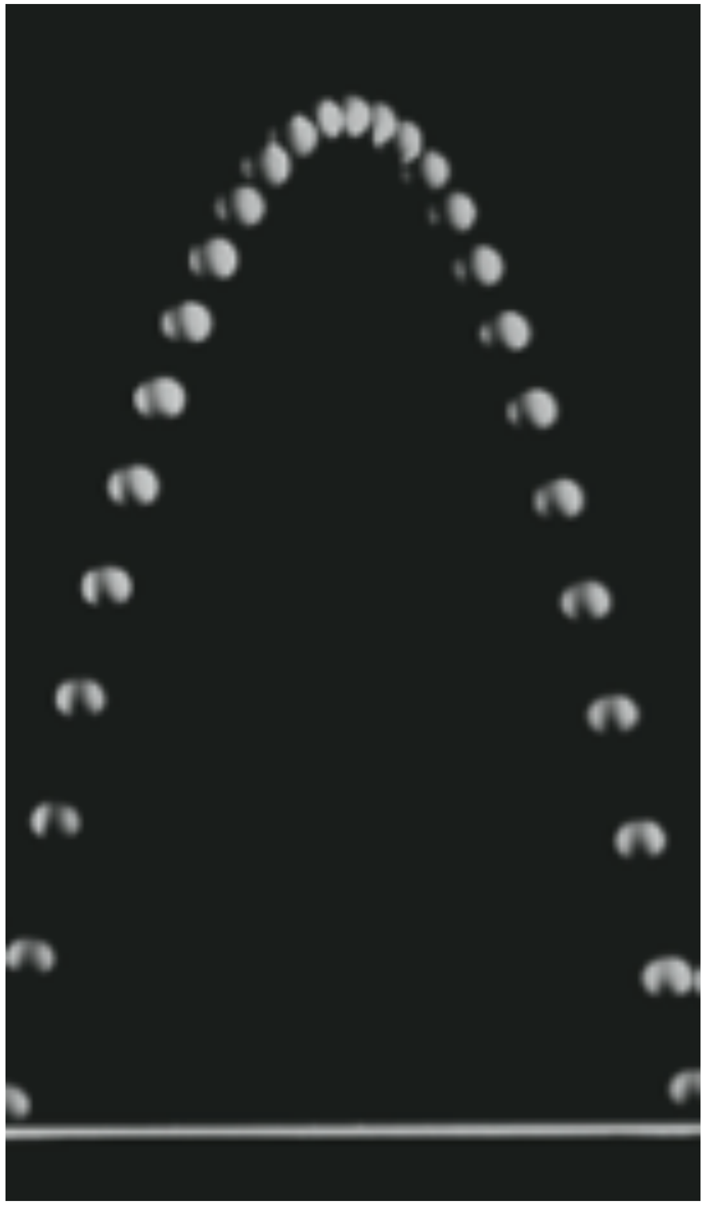
- When a ball is thrown upwards, gravity initially does negative work, meaning the kinetic energy decreases and the potential energy increases



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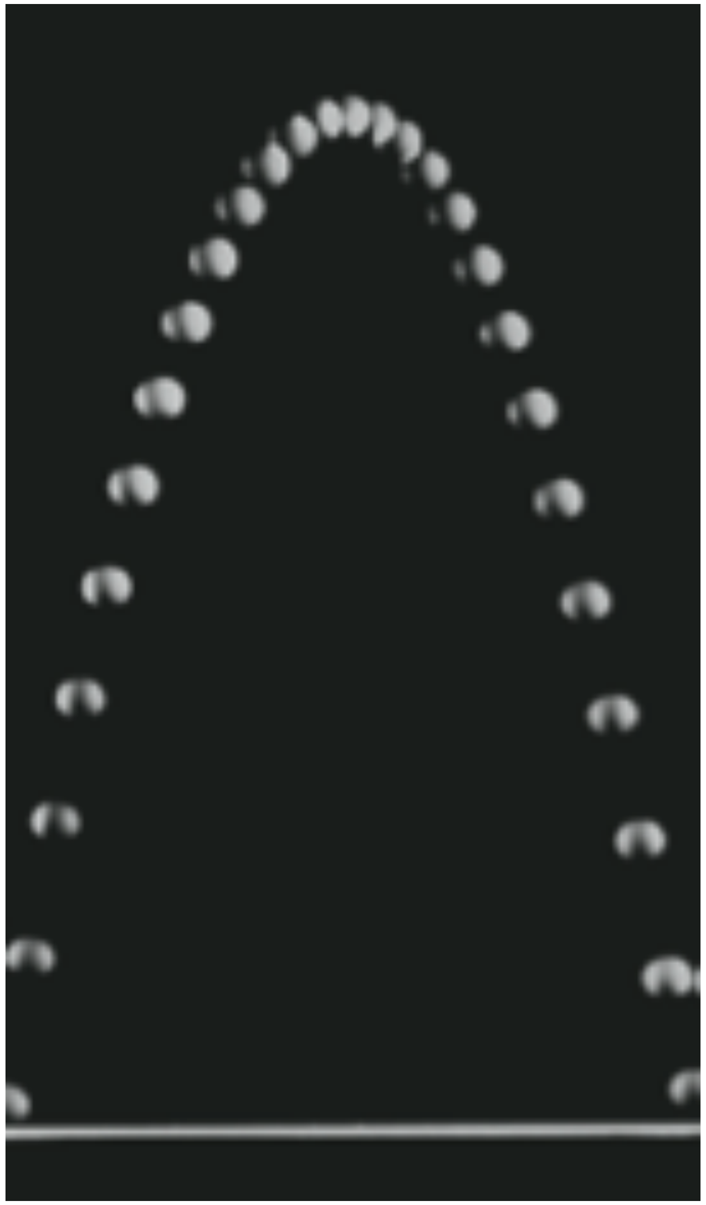

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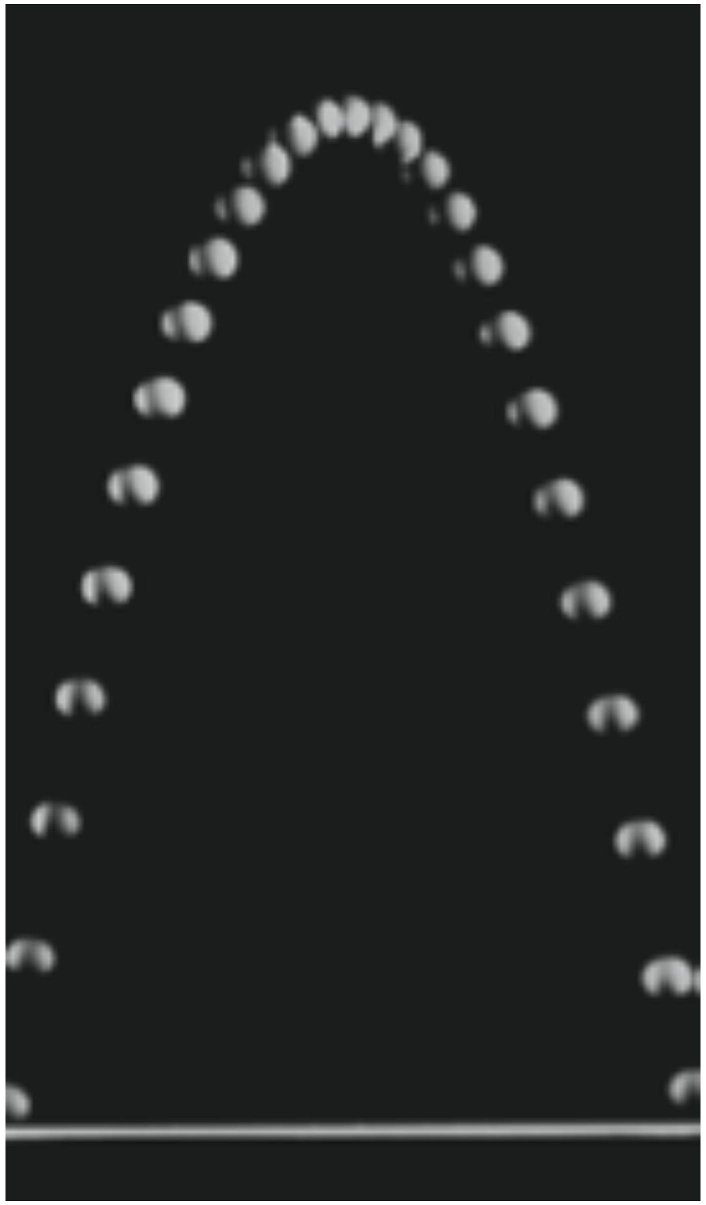

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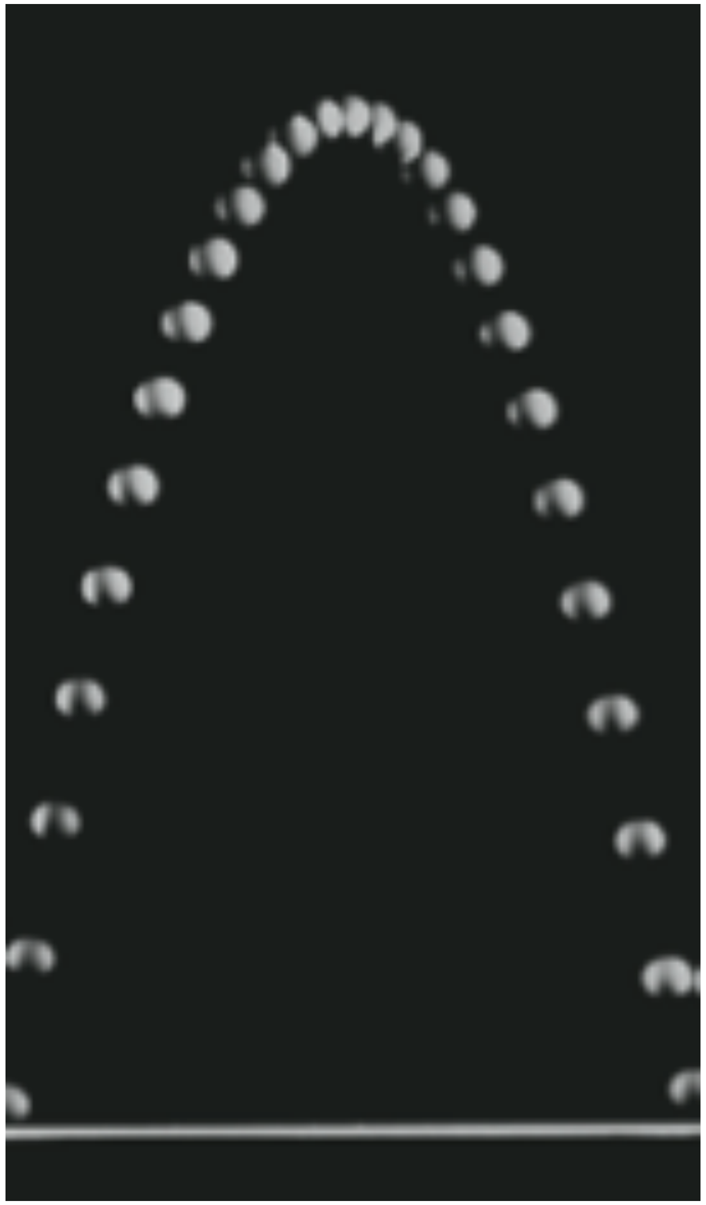


- When the ball returns to its initial height, it will have the same kinetic energy as when it was thrown

Calculating gravitational potential energy

$$\Delta U = -W = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

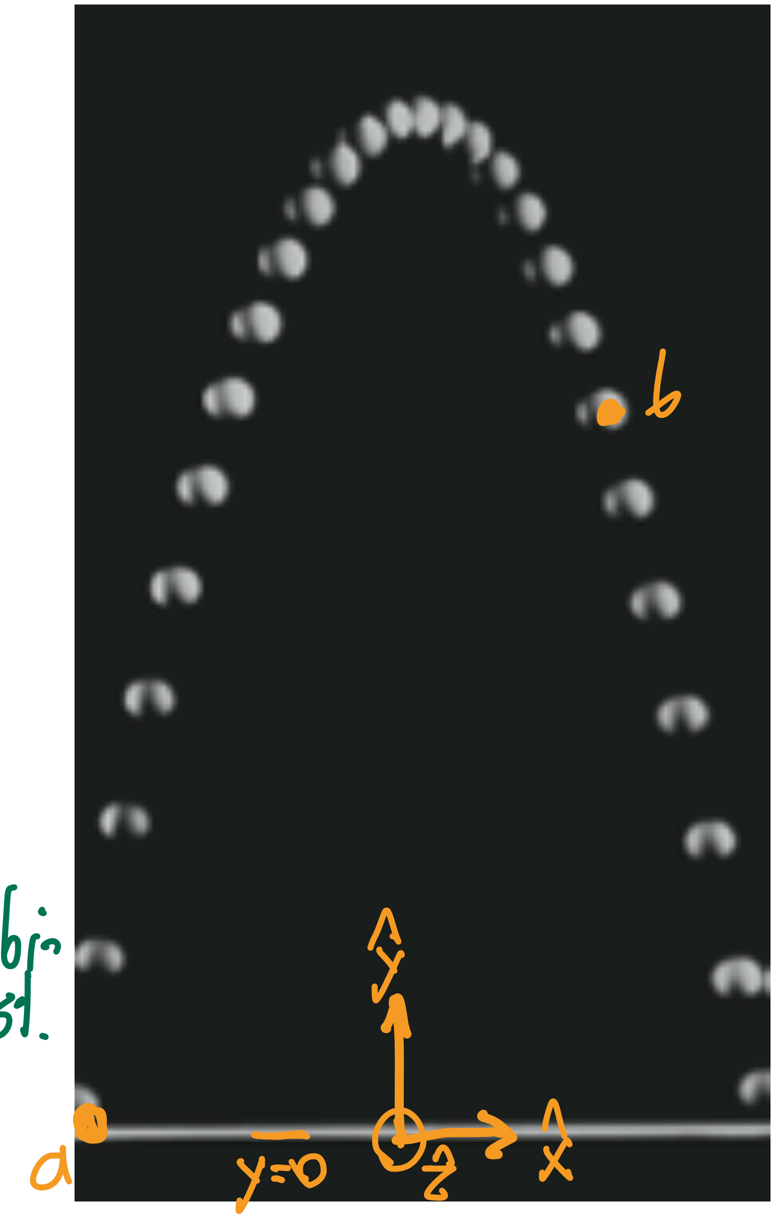
$\vec{F} = \vec{F}_g = -mg\hat{y}$ $\quad d\vec{\ell} = \hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$

$\vec{F}\cdot d\vec{\ell} = -mg\hat{y}\cdot(\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz) = -mg\,dy$

$\Delta U_{ab} = -W = -\int_a^b \vec{F}\cdot d\vec{\ell} = -\int_{y_a}^{y_b} -mg\,dy = mg(y_b - y_a)$

$\Rightarrow \Delta U_{ab} = U_b - U_a = (mgy_b + Q) - (mgy_a + Q)$ "Some" arbitrary const.

$\Rightarrow$ $\boxed{U_g(y) = mgy + Q}$



Force and potential energy

$$\Delta U = U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

- Given the potential energy, how do we find the force?

Force and potential energy

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$$\Delta U = U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

- Given the potential energy, how do we find the force?
- In 1D, it is straightforward — just take the derivative

For example along y: $\vec{F} = F_y \hat{y}$ $\quad d\vec{\ell} = \hat{y}\,dy \Rightarrow \vec{F}\cdot d\vec{\ell} = F_y\,dy$

$\Delta U = U(y) - U(y_{ref}) = -\int \vec{F}\cdot d\vec{\ell} = -\int_{y_{ref}}^{y} F_y\,dy$

$\frac{d}{dy}\left[U(y) - U(y_{ref})\right] = \frac{d}{dy}\left[-\int_{y_{ref}}^{y} F_y\,dy\right]$

$\Rightarrow \frac{dU}{dy} = -F_y \Rightarrow$ $\boxed{F_y = -\frac{dU}{dy}}$

For gravity:

$U_g(y) = mgy + Q$

$\frac{dU_g}{dy} = mg$

$-\frac{dU_g}{dy} = -mg = F_y$

Force and potential energy

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$$\Delta U = U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

- Given the potential energy, how do we find the force?
- In 3D, we must use the *gradient* $\vec{\nabla}$

$$\vec{F} = -\vec{\nabla} U = -\left(\hat{x}\frac{\partial U}{\partial x} + \hat{y}\frac{\partial U}{\partial y} + \hat{z}\frac{\partial U}{\partial z}\right)$$

Potential energy diagrams

- A *potential energy diagram* is just a plot of potential energy versus position

Potential energy diagrams

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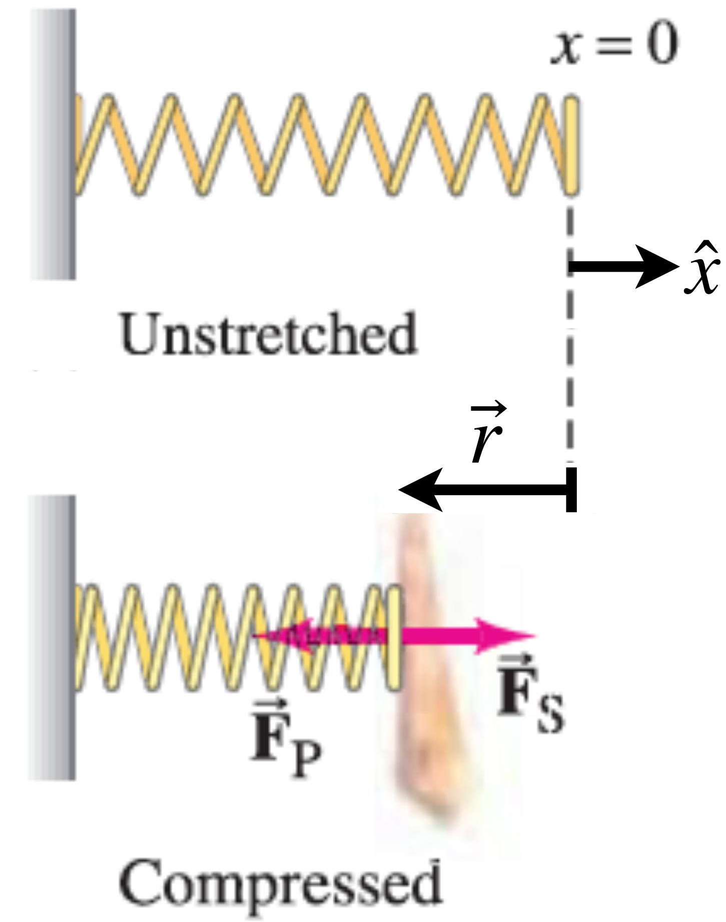
- Draw the potential energy diagram for a spring



$$\vec{F}_s = -k(\vec{r} - \vec{r}_0) = -k(x\hat{x}) = -kx\hat{x}$$

$$\vec{dl} = \hat{x}\,dx \Rightarrow \vec{F}_s \cdot \vec{dl} = -kx(\hat{x}\cdot\hat{x})\,dx = -kx\,dx$$

$$\Delta U_s = -W_s = -\int_{comp} \vec{F}_s \cdot \vec{dl} = +\int_{x_0}^{x_c} kx\,dx = k\int_{x_0}^{x_c} x\,dx$$

$$= k\left[\frac{x^2}{2}\right]_{x_0}^{x_c} = \frac{1}{2}kx_c^2 - \frac{1}{2}kx_0^2 = U_s(x_c) - U_s(x_0)$$

$$\boxed{U_s(x) = \frac{1}{2}kx^2 + Q}$$

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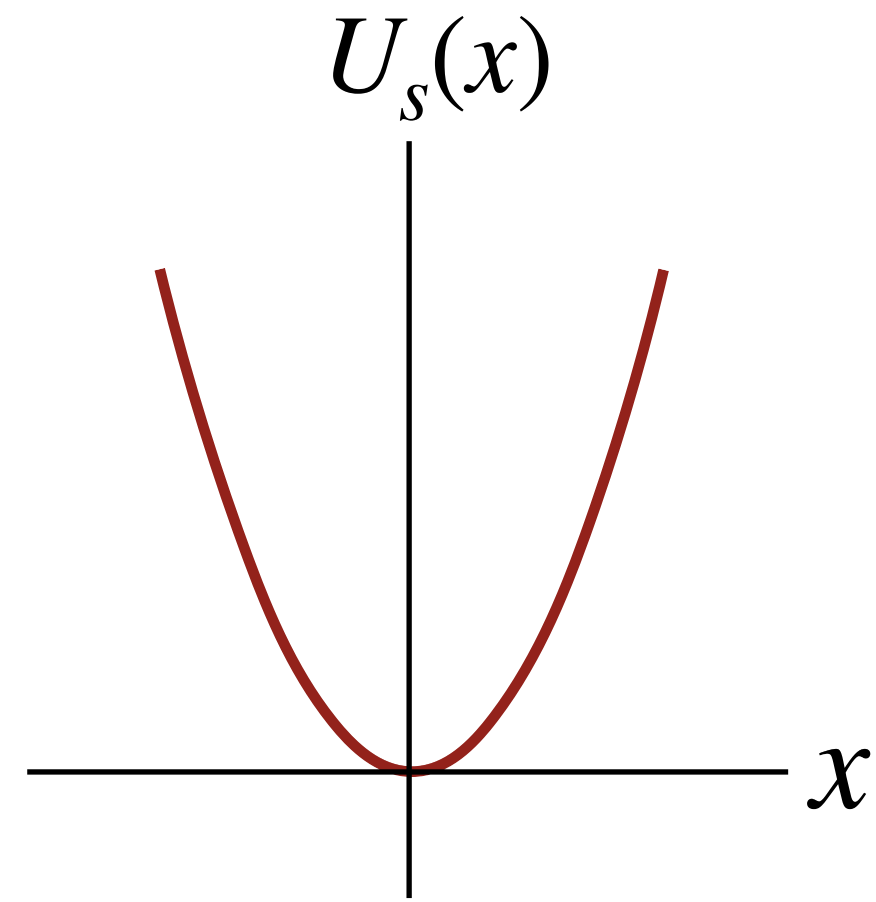

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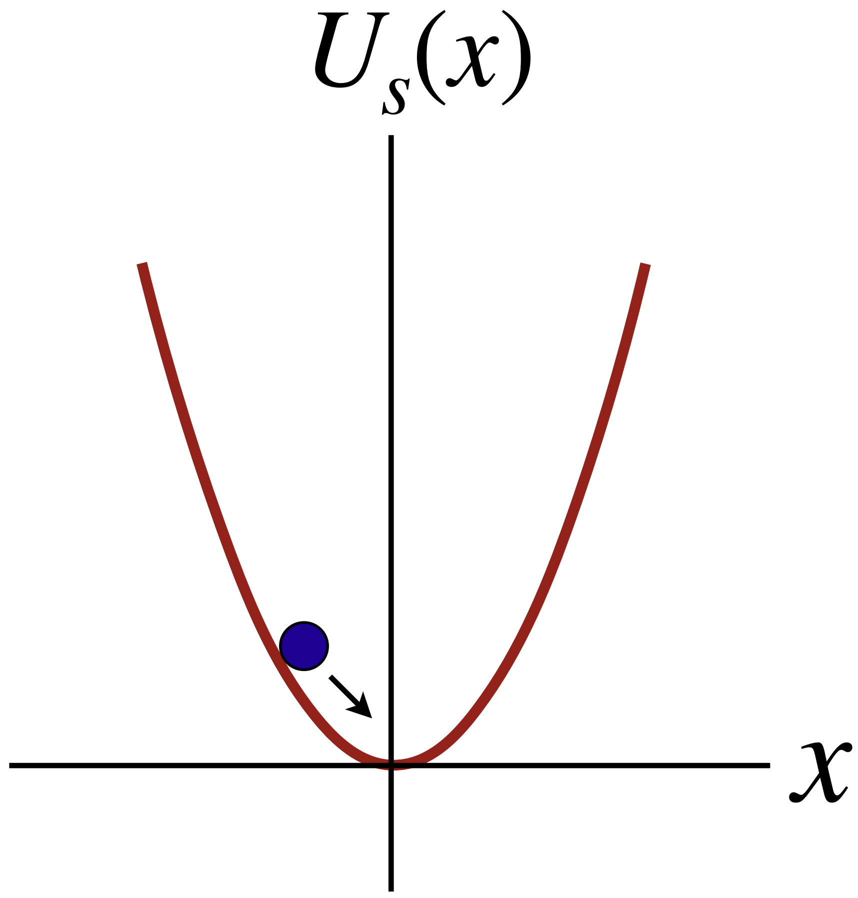

Potential energy diagram for a spring

$$F_s = -\frac{dU}{dx}$$

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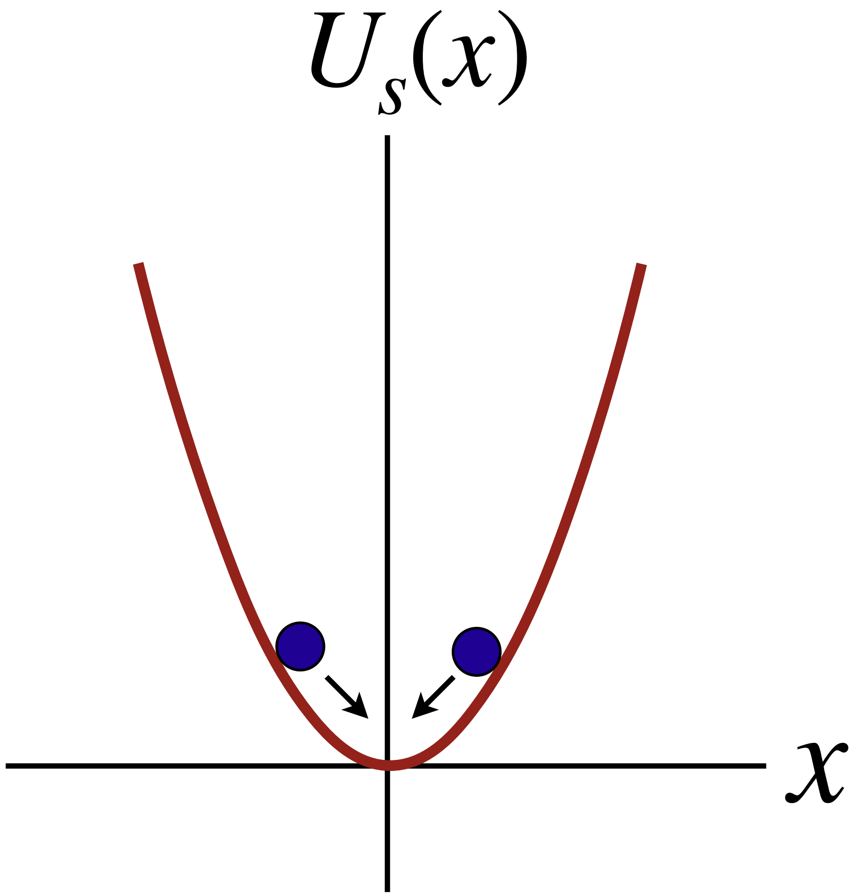

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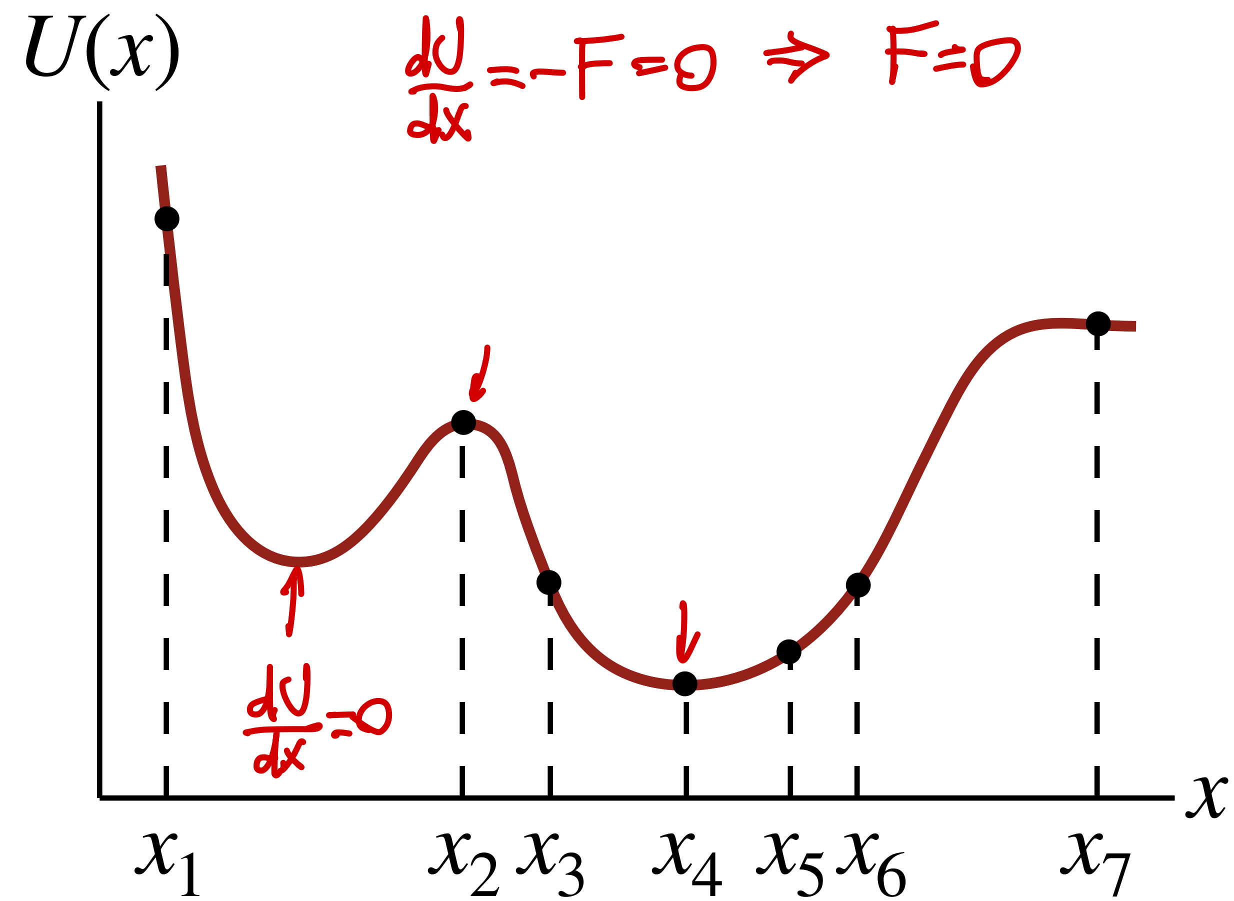

Potential energy diagram for a particle moving under the influence of a conservative force

Potential energy diagrams

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- Equilibrium points:

  Points for which $F = 0$

  $$\Rightarrow \frac{dU}{dx} = 0$$

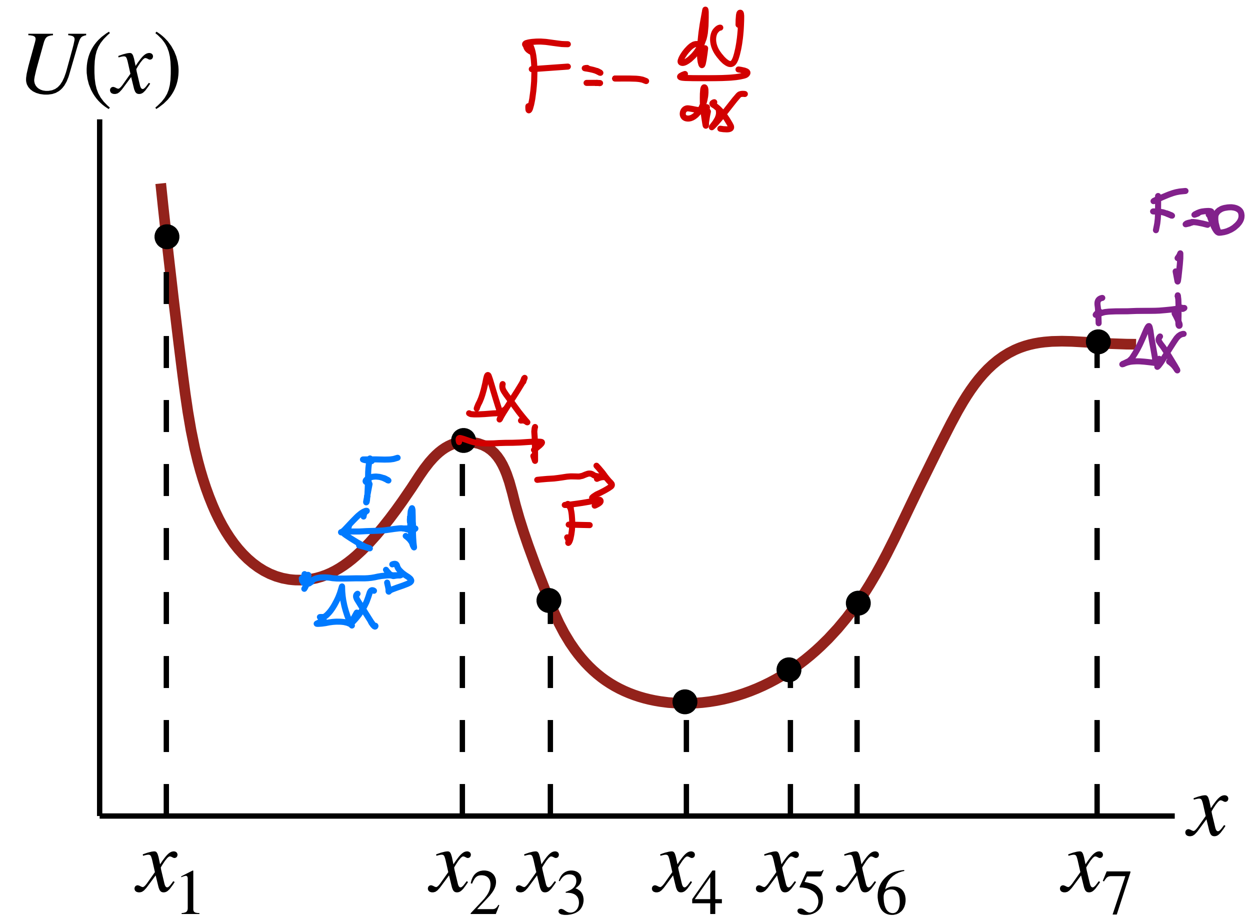
  - Equilibrium is **stable** if:

    $$\frac{dF}{dx} < 0 \Leftrightarrow \frac{d^2U}{dx^2} > 0$$

    $$= \frac{d}{dx}\left[-\frac{dU}{dx}\right] = -\frac{d^2U}{dx^2}$$

  - Equilibrium is **unstable** if:

    $$\frac{dF}{dx} > 0 \Leftrightarrow \frac{d^2U}{dx^2} < 0$$



Potential energy diagram for a particle moving under the influence of a conservative force

$$\frac{dF}{dx} = 0 \Leftrightarrow \frac{d^2U}{dx^2} = 0$$

Neutrally stable

Conceptual question

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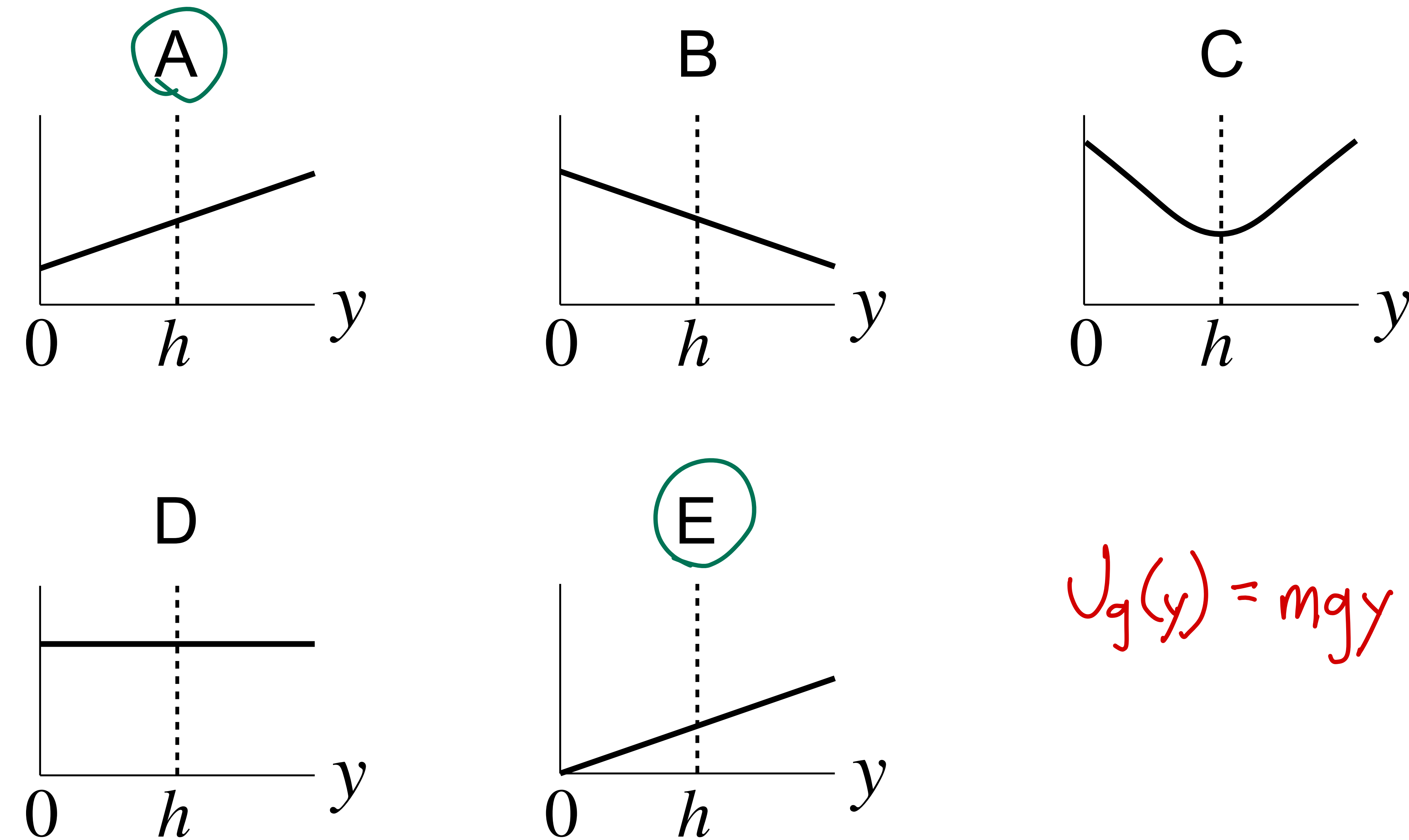
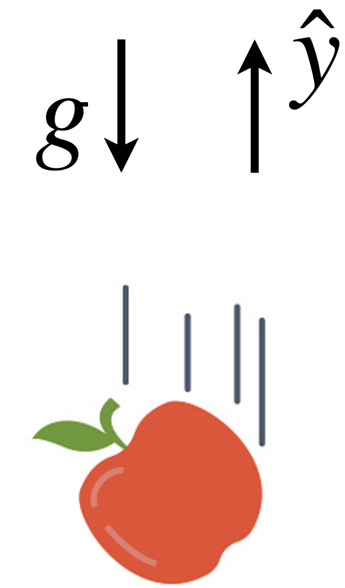
An apple at height h falls from a tree. What is the potential energy diagram $U(y)$ for this situation?





$U_g(y) = mgy + Q$ (const)

Conceptual question

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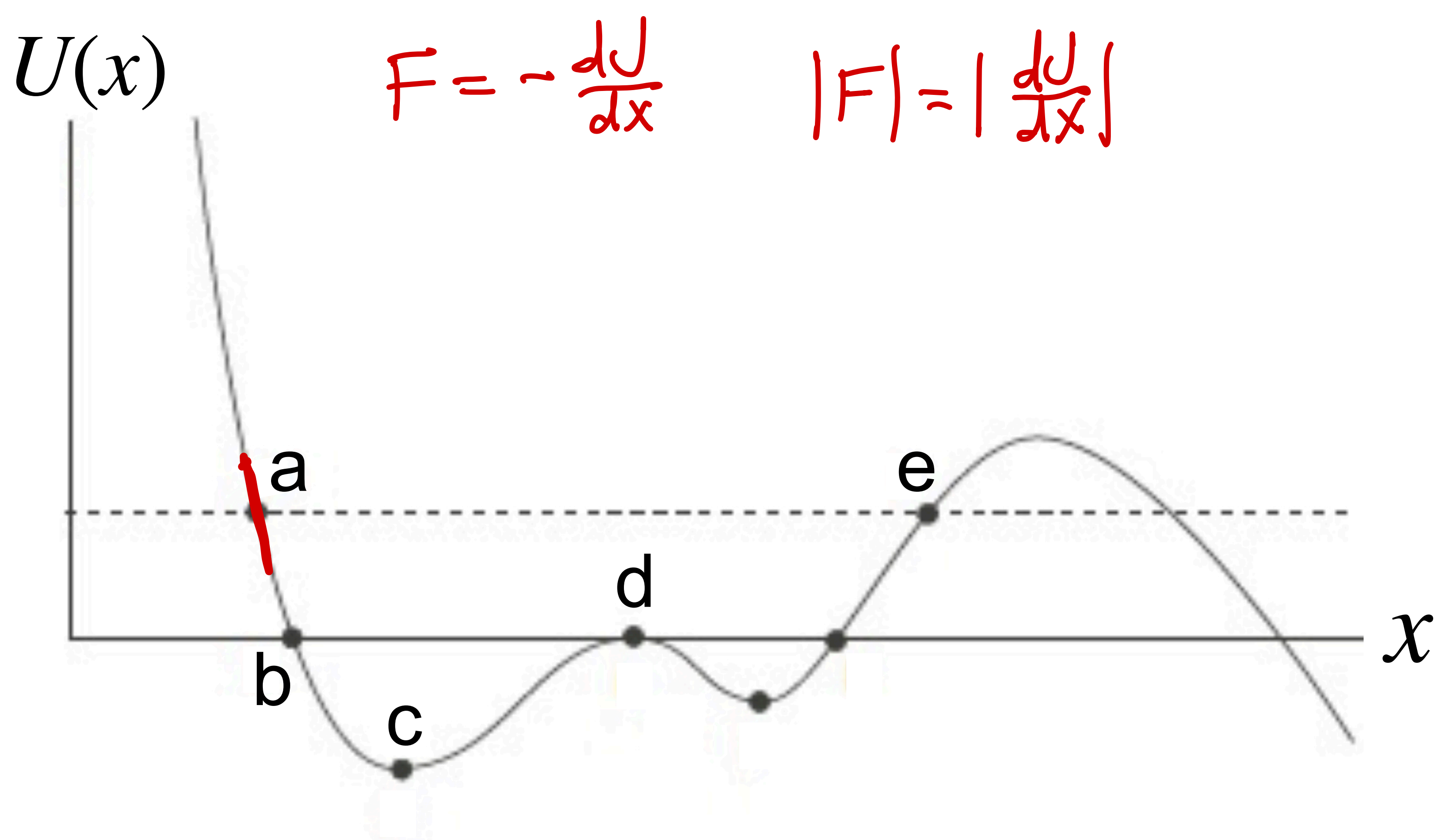
The figure below shows the potential energy diagram for a particle executing one dimensional motion between points “a” and “e”.

At which point will the magnitude of the force be the largest?

A. a
B. b
C. c
D. d
E. e



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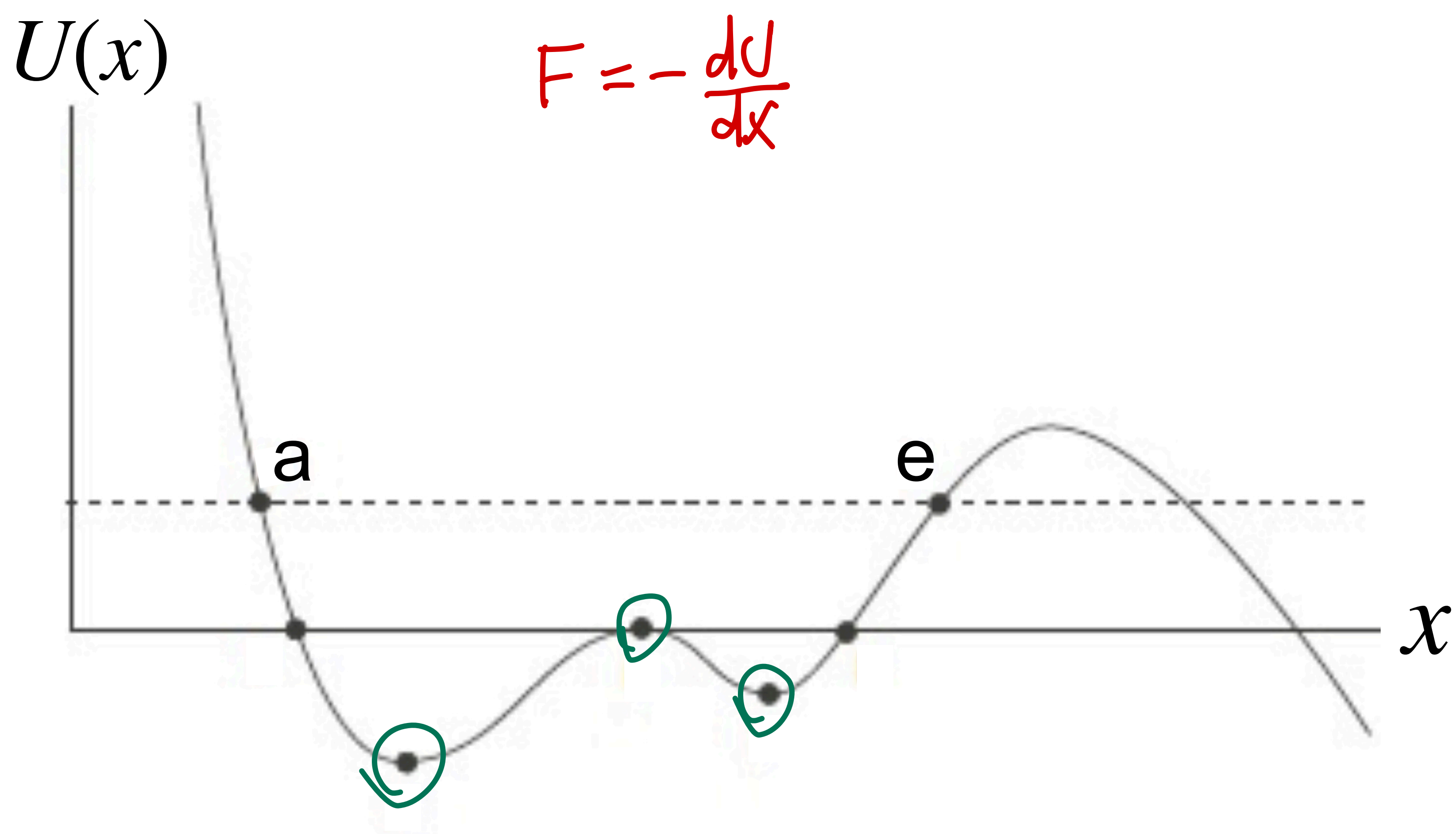
The figure below shows the potential energy diagram for a particle executing one dimensional motion between points "a" and "e".

At how many of the points will the force be zero?

A. 0
B. 1
C. 2
D. 3
E. 4



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   - **Conservation of mechanical energy**

Conservation of energy

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The total energy is neither increased nor decreased in any process.

- Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant

- Whenever it seems that energy is disappearing, we always find that it is actually just hiding in a different form

- No known exceptions

Energy has many different forms

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- Mechanical
  - Kinetic
    - Translational

Energy has many different forms

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- Thermal
- Chemical
- Nuclear
- Electrical
- Light (electromagnetic)
- and more!

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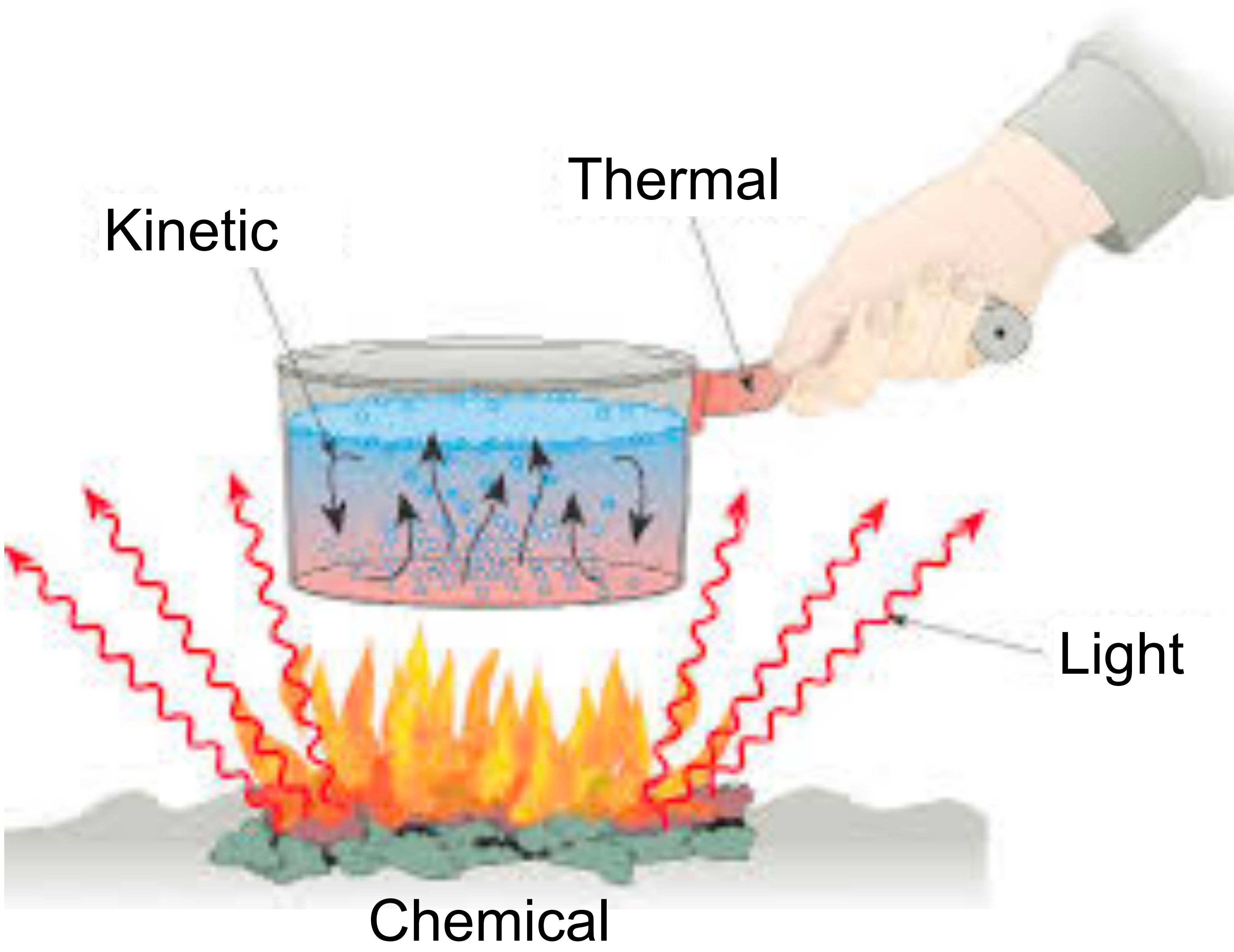

DEMO (483, 86, 764 and 38)

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Conservation of energy

Conservation of mechanical energy

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- *Mechanical energy* refers to the energy of an object's motion (i.e. kinetic) and the energy that is "stored" in an object by its position (i.e. potential)

$$E_m = K + U$$

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- If all forces doing work on a system are *conservative*, then its mechanical energy is conserved

- Can be proved from the work-kinetic energy theorem applied to conservative forces

Conservation of mechanical energy

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$$\Delta K = W_{net} = W_{cons} + W_{non}$$

$$= -\Delta U + W_{non}$$

$$W_{cons} = -\Delta U$$

$$\Rightarrow W_{non} = \Delta K + \Delta U = (K_f - K_i) + (U_f - U_i) = \underbrace{(K_f + U_f)}_{E_{mf}} - \underbrace{(K_i + U_i)}_{E_{mi}}$$

$$\Rightarrow \boxed{W_{non} = E_{mf} - E_{mi} = \Delta E_m}$$

If there are no non-conservative forces doing work then $W_{non} = 0$

$$\Rightarrow \boxed{\Delta E_m = 0}$$

Conceptual question

responseware.eu
Session ID: epflphys101en

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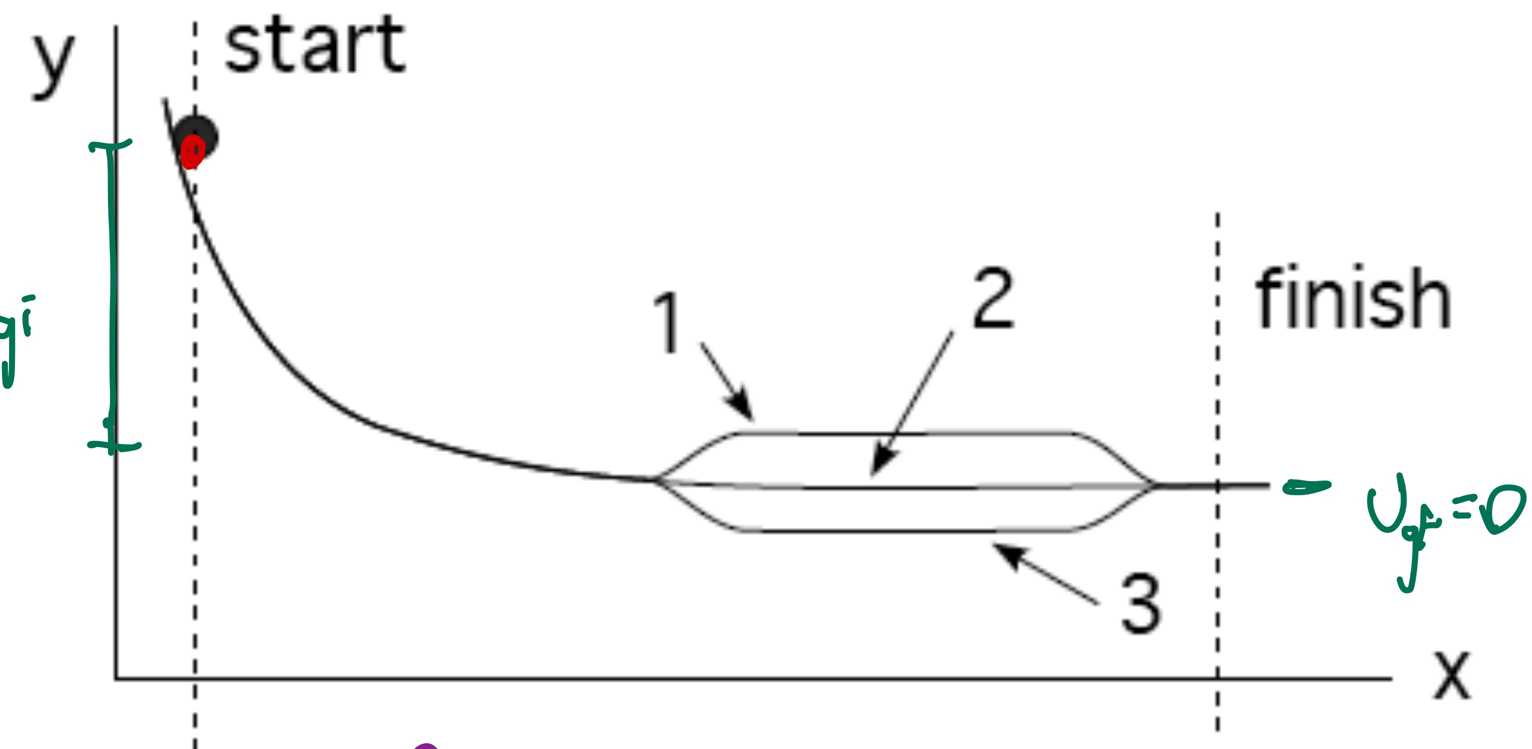
An object starts from rest and slides down a frictionless hill without air drag. Which path leads to the highest speed at the finish?

A. 1
B. 2
C. 3
D. All are the same.

$mgh = U_{gi}$



$$0 = W_{non} = W_{AD} + W_{fr} + W_N = \Delta E_m = E_{mf} - E_{mi}$$

$$E_{mf} = E_{mi}$$

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2} m v_f^2 = mgh$$

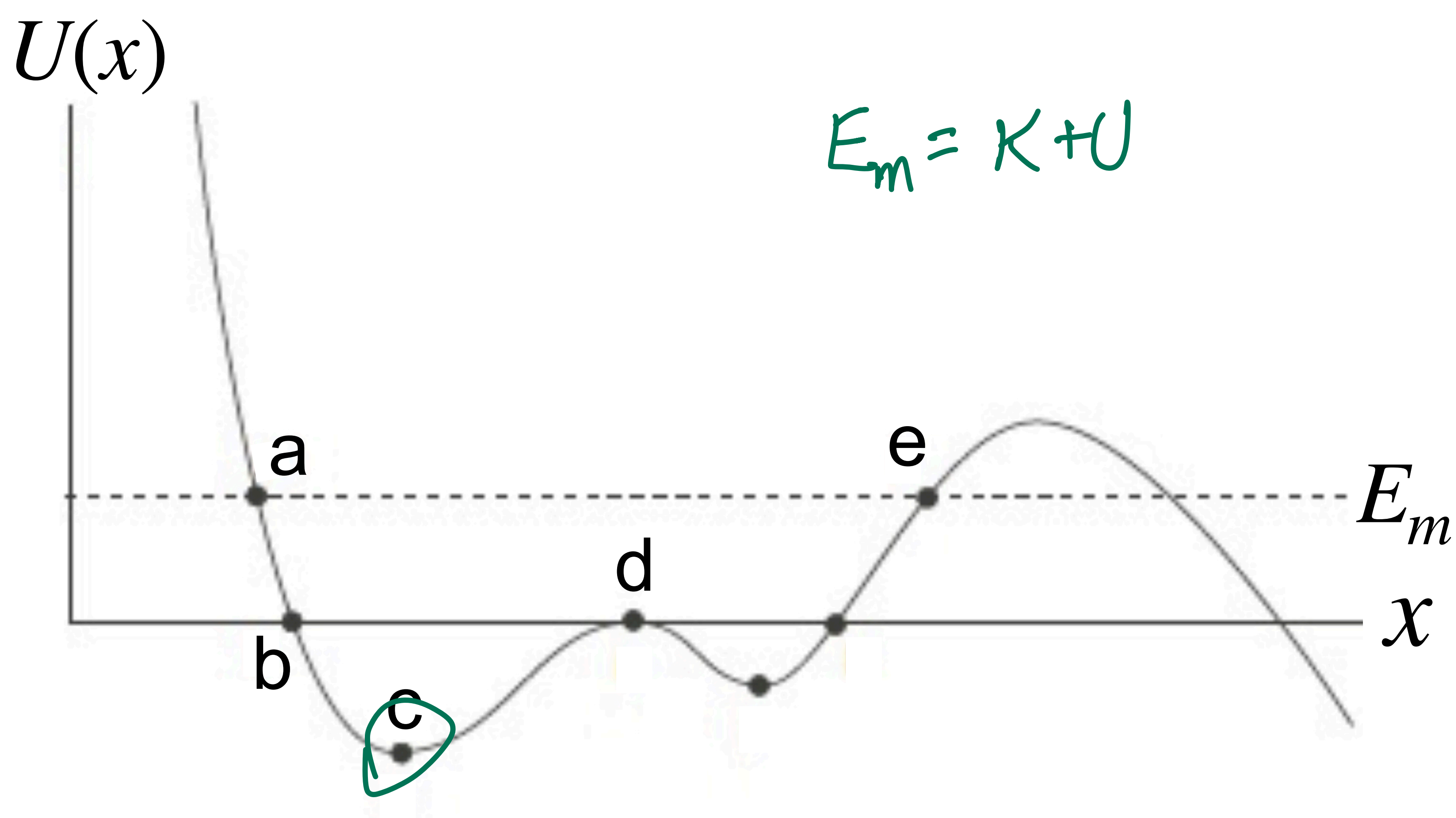
$$\Rightarrow v_f = \sqrt{2gh}$$

Conceptual question

The figure below shows the potential energy diagram for a particle executing one dimensional motion between points “a” and “e”. The total mechanical energy of the system is indicated by the dashed line.

At which point will the kinetic energy be the largest?

A. a
B. b
C. c
D. d
E. e



Summary

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- Power is rate at which work is done (or energy is transferred/converted)
- The potential energy, which is only associated with conservative forces, is given by

$$\Delta U = -W, \quad \text{e.g.} \quad U_g = mgy \quad \text{and} \quad U_s = kx^2/2$$

- Force related to the derivative (or gradient) of the potential
- **Total energy, considering all forms, is *always* conserved**
- Total mechanical energy is the sum of kinetic energy and potential energy
- If all forces doing work are conservative, then mechanical energy is conserved

See you tomorrow for cliff jumping

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