Swiss
Plasma
Center

General Physics: Mechanics

PHYS-101(en)

Lecture 8b: Kinetic energy and work

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Conceptual question

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Compared to the amount of energy required to accelerate a car from rest to 10 km/h, the work required to accelerate the same car from 10 km/h to 20 km/h is…

A. the same.
B. twice as much.
C. three times as much.
D. four times as much.

$$\Delta K_1 = \frac{1}{2} m_c v_f^2 - \frac{1}{2} m_c \cancel{v_0^2} = \frac{1}{2} m_c \left(10 \frac{km}{h}\right)^2 = \frac{1}{2} m_c \cdot 100 \frac{km^2}{h^2}$$

$$W_2 = \Delta K_2 = \frac{1}{2} m_c v_f^2 - \frac{1}{2} m_c v_0^2 = \frac{1}{2} m_c \left(20 \frac{km}{h}\right)^2 - \frac{1}{2} m_c \left(10 \frac{km}{h}\right)^2$$

$$= \frac{1}{2} m_c \left(400 \frac{km^2}{h^2} - 100 \frac{km^2}{h^2}\right)$$

$$= \frac{1}{2} m_c \cdot 300 \frac{km^2}{h^2}$$

$$= 3 \Delta K_1$$

Example: Marbula One racing

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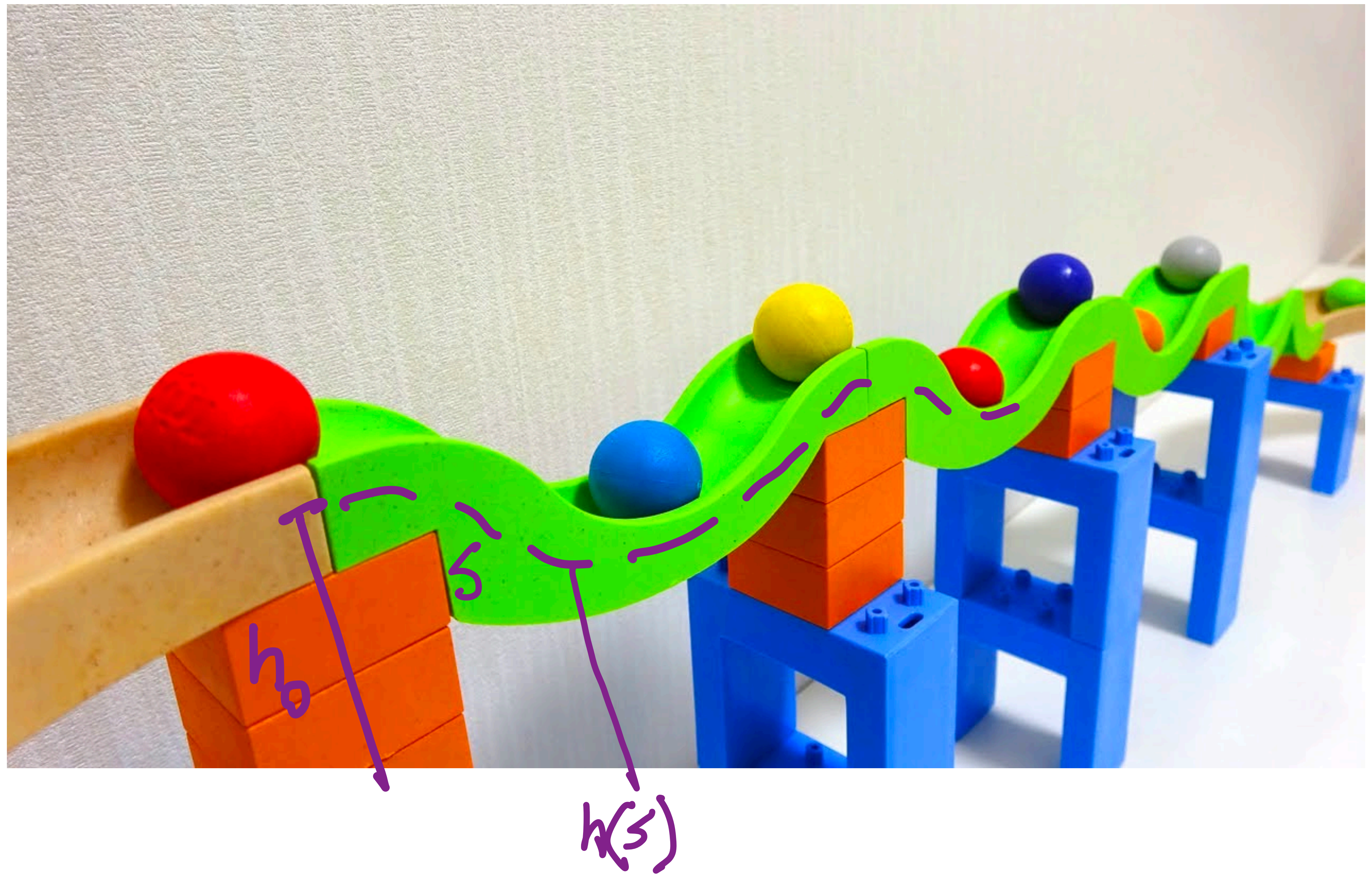
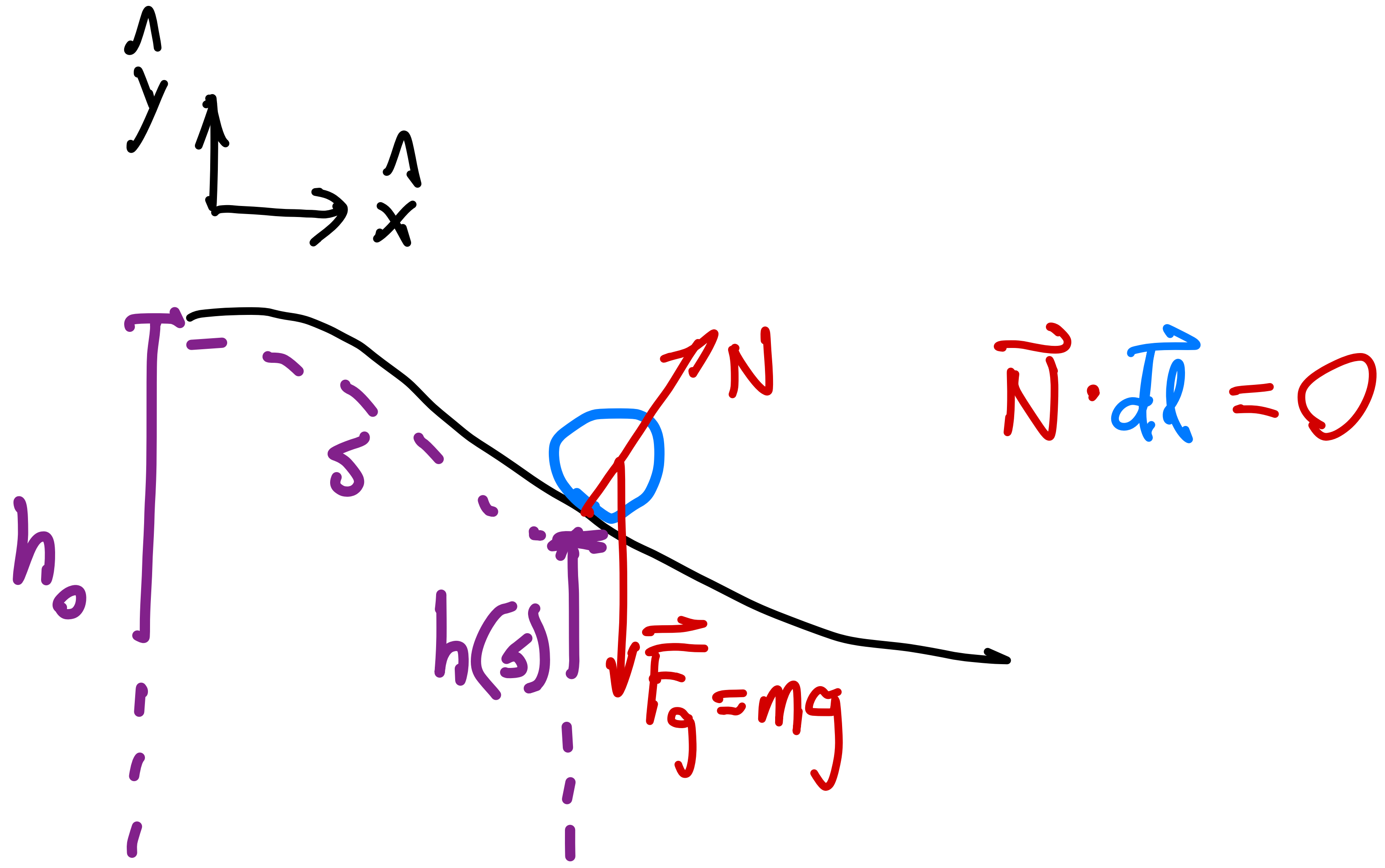
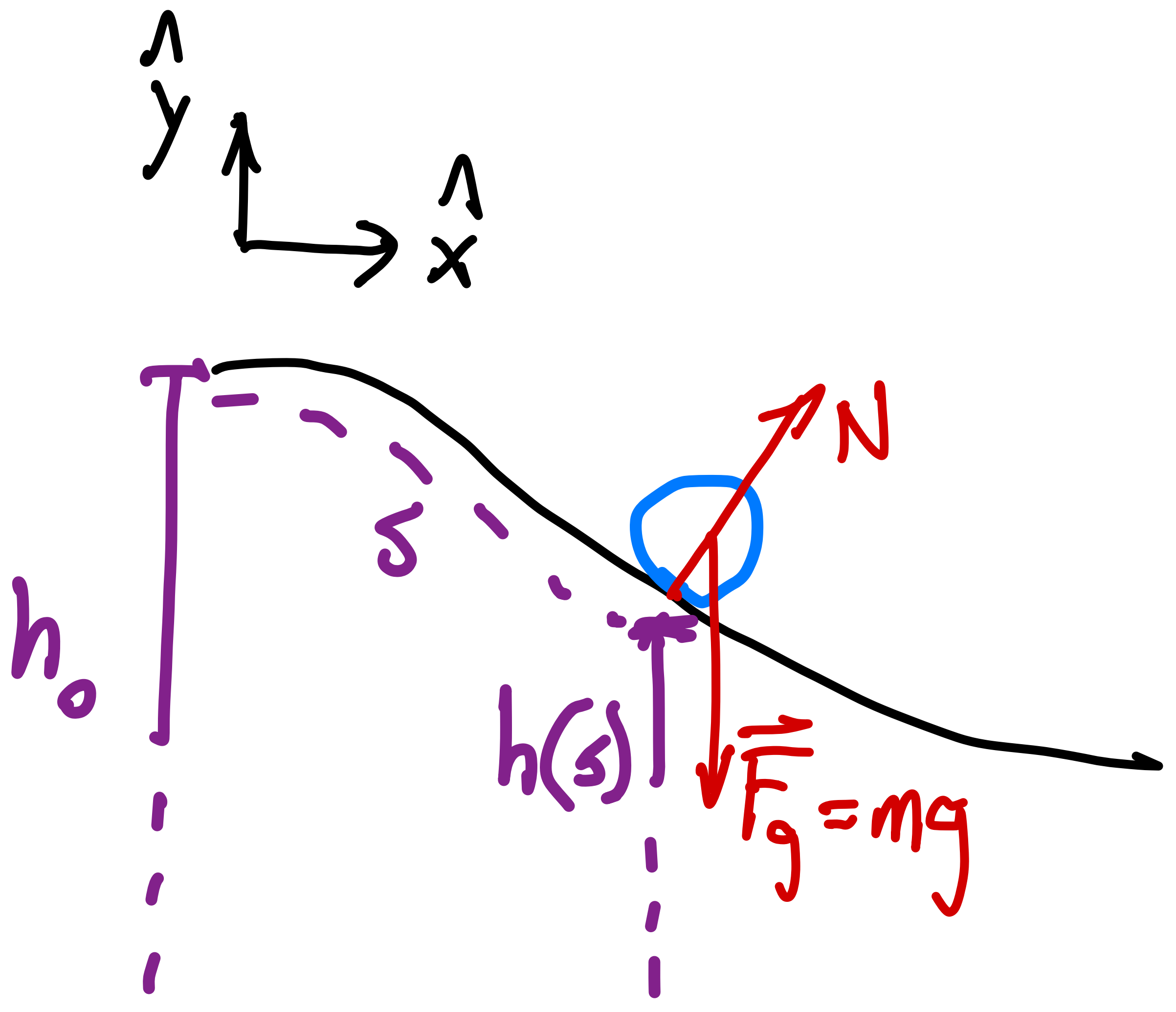
You're designing a *marble racing* track! The course is straight (i.e. no left or right turns) and the marbles are released from rest at a height h_0. Given the height $h(s)$ as a function of the distance s along the track, you need to calculate the marble's speed $v(s)$ to ensure they can make it to the bottom. Ignore friction and drag.





Example: Marbula One racing



$\vec{N} \cdot \vec{dl} = 0$

$$W_{net} = \int_{RT} \vec{F}_{net} \cdot \vec{dl} = \int_{RT} \vec{F}_g \cdot \vec{dl} + \cancel{\int_{RT} \vec{N} \cdot \vec{dl}}^{\,0} = \int_{RT} -mg\hat{y} \cdot (\hat{x}dx + \hat{y}dy)$$

$$= \int_{RT} -mg\,dy = \int_{h_0}^{h(s)} -mg\,dy = -mg\int_{h_0}^{h(s)} dy = -mg[h(s) - h_0]$$

$$= mg[h_0 - h(s)]$$

$$W_{net} = \Delta K \Rightarrow \cancel{m}g[h_0 - h(s)] = \frac{1}{2}\cancel{m}v^2(s) - \cancel{\frac{1}{2}mv^2(0)}^{\,0}$$

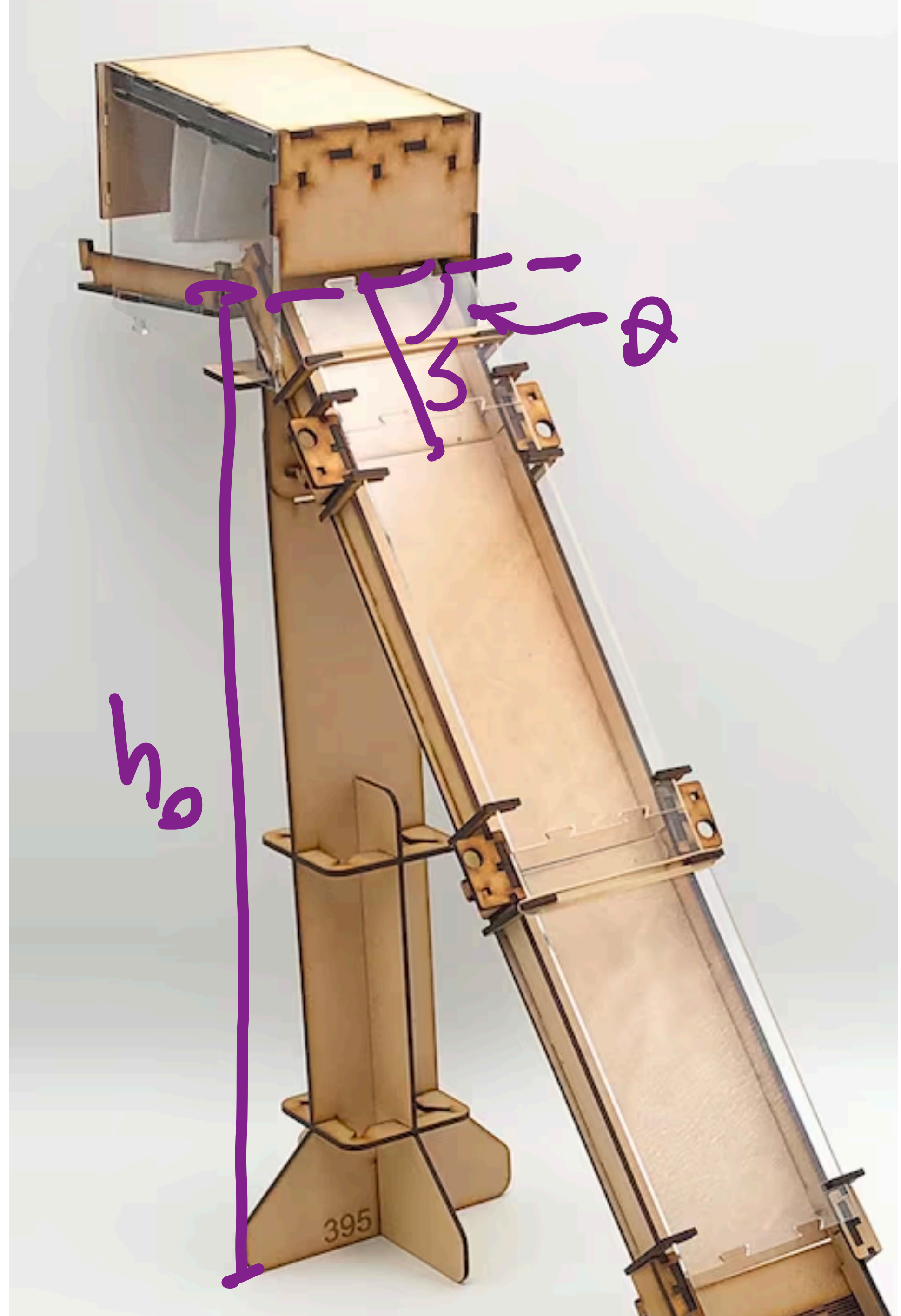
$$\Rightarrow v^2(s) = 2g[h_0 - h(s)]$$

$$\Rightarrow \boxed{v(s) = \sqrt{2g[h_0 - h(s)]}}$$

We require $h_0 > h(s)$ for $v(s)$ to exist and $v(s) \neq 0$

Example: Marbula One racing (w/ friction)

You're designing a *marble racing* track! The course is straight (i.e. no left or right turns) and the marbles are released from rest at the top. Given a **linear height profile that makes an angle θ *below* the horizon,** $h(s) = -\sin\theta\ s + h_0$**,** you need to calculate the marble's speed $v(s)$ to ensure they can make it to the bottom. Let the friction coefficient be μ_k and ignore drag.



$$\Delta K = W_{net} \qquad \Delta K = \tfrac{1}{2} m v^2(s) - \cancel{\tfrac{1}{2} m v^2(0)}^{\,0\ \text{(rest)}}$$

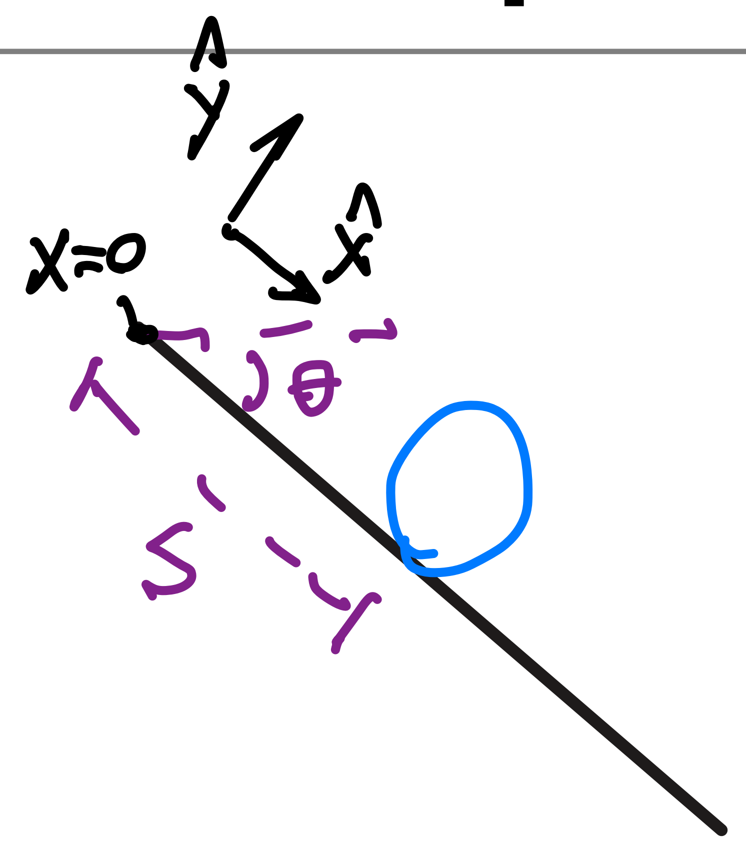
$$\vec{F}_{net} = \vec{F}_g + \vec{N} + \vec{F}_K$$

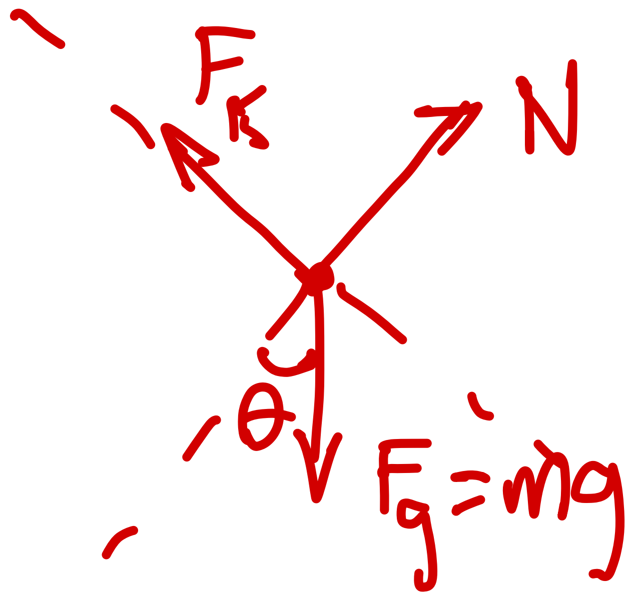
$$W_{net} = \int_{RT} \vec{F}_{net} \cdot \vec{dl} = \int_{RT} (\vec{F}_g + \vec{N} + \vec{F}_K) \cdot dl$$

$$= \underbrace{\int_{RT} \vec{F}_g \cdot \vec{dl}} + \int_{RT} \vec{F}_K \cdot \vec{dl}$$

$$mg[h_0 - h(s)] = mg[\cancel{h_0} - (-\sin(\theta)s + \cancel{h_0})] = mgs \cdot \sin(\theta)$$

Example: Marbula One racing (w/ friction)





$$\vec{F}_K = \mu_K N(-\hat{x}) = -\mu_K mg\cos(\theta)\hat{x}$$

$$\Sigma F_y: \; N - mg\cos(\theta) = 0 \Rightarrow N = mg\cos(\theta)$$

$$W_{F_K} = \int_{RT} \vec{F}_K \cdot d\vec{l} = \int_{RT} -\mu_K mg\cos(\theta)\hat{x}\cdot(\hat{x}\,dx) = \int_0^s -\mu_K mg\cos(\theta)\,dx = -\mu_K mg\cos(\theta)\overbrace{\int_0^s dx}^{=(s-0)=s}$$

$$= -\mu_K mg\cos(\theta)s$$

$$\Delta K = W_{net} = W_g + W_{F_K} \Rightarrow \frac{1}{2}\cancel{m}v^2(s) = \cancel{m}gs\sin(\theta) - \mu_K\cancel{m}gs\cos(\theta) = \cancel{m}gs[\sin(\theta)-\mu_K\cos(\theta)]$$

$$\Rightarrow \boxed{v(s) = \sqrt{2gs[\sin(\theta)-\mu_K\cos(\theta)]}}$$

We need $\sin(\theta) > \mu_K\cos(\theta)$

$\Rightarrow \tan(\theta) > \mu_K$

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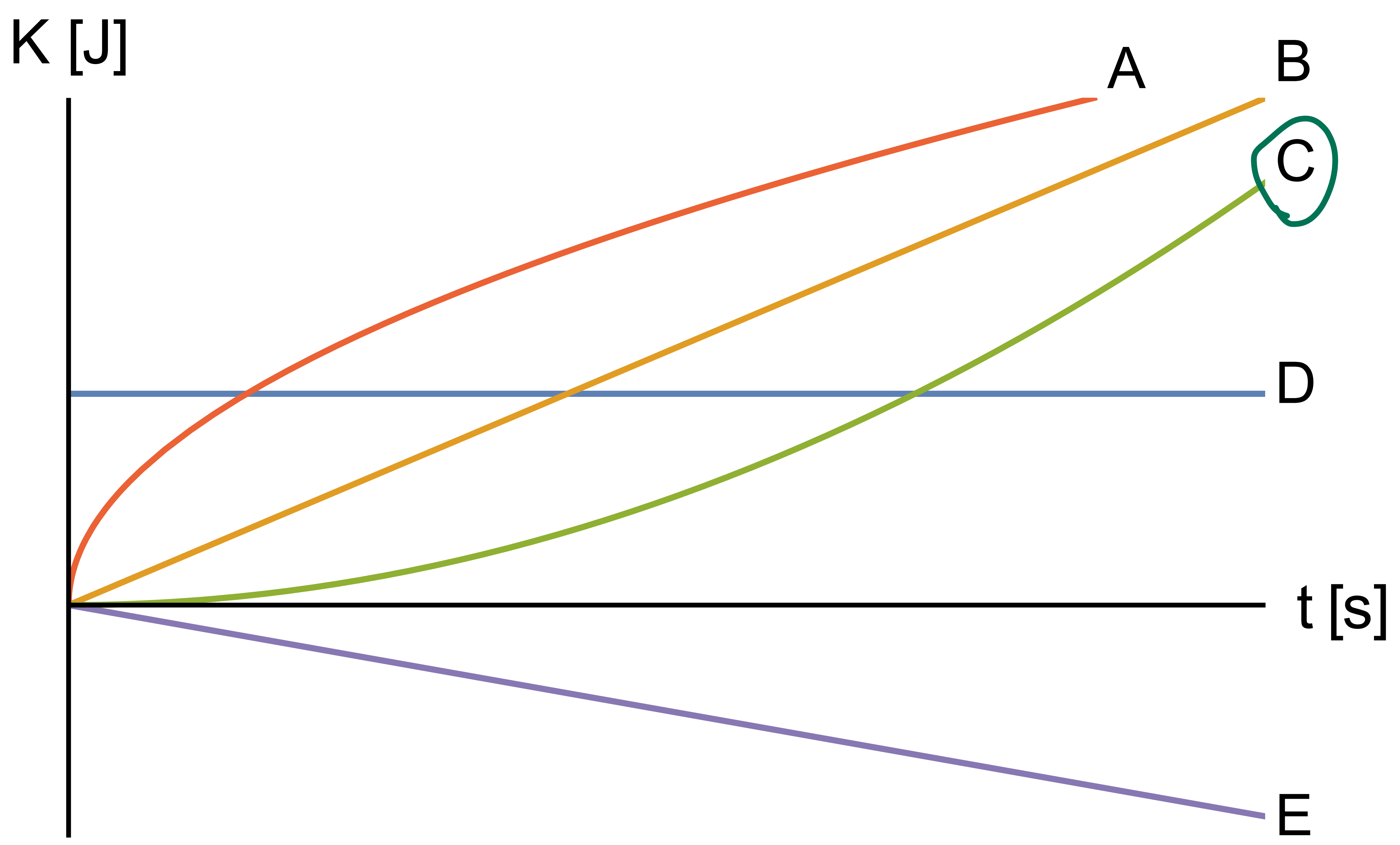
An acorn falls to the earth from a tree. Which of the following graphs best represents the time dependence of the acorn's kinetic energy? Neglect air resistance.



$v_y(t) = \cancel{v_{y0}}^{0} - gt = -gt$

$v(t)^2 = |\vec{v}(t)|^2 = |v_y(t)|^2 = (-gt)^2 = g^2t^2$

$K = \frac{1}{2}mv^2(t) = \frac{1}{2}mg^2t^2$